THE ASSOCIATION BETWEEN THE HOME NUMERACY ENVIRONMENT AND EARLY MATH SKILLS: MATH LANGUAGE AS A MODERATOR

by

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ABSTRACT

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A number of studies provide evidence that the home numeracy environment (HNE) is important for the development of early numeracy skills. There is also evidence that preschoolers understanding of math language is a strong predictor of numeracy skills. However, there is limited research on the role of math language knowledge in the relation between the HNE and early numeracy skills. The purpose of this study was to investigate the relation between different aspects of the HNE (direct vs. indirect numeracy activities) and numeracy skills while considering math language as a moderator. Participants included 125 children between 3.01 to 5.17 years ($M = 4.09$) and their parents. Parents reported on the frequency of engaging children in home numeracy activities. Children were assessed in the fall and spring of their preschool year on their numeracy skills and math language knowledge. Hierarchical multiple regressions were conducted to test if prior math language knowledge would moderate the relation between each component of the HNE (indirect and direct numeracy activities) and later numeracy skills. Results indicate that math language was not a moderator of these relations. However, supplemental analyses suggest that math language mediates the relation between direct HNE and numeracy skills. These findings provide evidence that the relation between the direct HNE and early numeracy may be explained by preschooler’s math language knowledge.

INTRODUCTION

Children's early math skills are an important predictor of later academic success (Duncan et al., 2007). These early math skills that children develop during the preschool years, both at school and at home, set the foundation for later mathematics skills (Blevins-Knabe, 2012; Hill, 2001; Kleemans, Peeters, Segers, & Verhoeven, 2012; LeFevre et al., 2009). Previous research indicates that young children often acquire numeracy skills in informal settings, including during interactions with parents in the home environment (Baroody & Wilkins, 1999; Dickinson & Tabors, 2001; H. Ginsburg, 1977; Melhuish et al., 2008). A growing body of research has shown that the home numeracy environment (HNE), a construct that encompasses parent-child engagement in numeracy-related activities, predicts preschoolers' numeracy development (Blevins-Knabe & Musun-Miller, 1996; Kleemans et al., 2012; LeFevre et al., 2009). However, certain aspects of the HNE (i.e., indirect vs. direct activities) have been inconsistently predictive of numeracy skills and it is possible that this previously inconsistent association may depend on child characteristics. As parent-child interactions that fall within the HNE are often language based (Gunderson & Levine, 2011; Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010; Susperreguy & Davis-Kean, 2016), and children's knowledge of math language is a strong predictor of their early numeracy skills (Purpura & Logan, 2015; Toll & Van Luit, 2014b), it is possible that children's math language knowledge moderates the association between certain HNE components and children's development of numeracy skills.

Given that both math language knowledge and the HNE are predictors of numeracy skills, it is important to consider that children's understanding of math language may play a role in the association between the HNE and numeracy skills, which could inform ways that parents may foster children's understanding of math language so that children can benefit from different aspects of the HNE. However, no published studies have investigated the role of preschoolers' math language knowledge on the direction and/or strength of the relation between the HNE and early numeracy skills. The present study will address this gap by investigating the moderating role of prior math language on the relations between the HNE and child later numeracy.

Early Numeracy

Early numeracy skills, such as counting and identifying quantities, are important for students' later understanding of formal mathematics, such as written formal addition and subtraction skills, which are typically learned when children are in school (Baroody, Gannon, Berent, & Ginsburg, 1983). Early numeracy skills are also learned prior to school entry or during daily social interactions with caregivers (Ginsburg, 1977). Early numeracy is comprised of three broad domains (Purpura & Lonigan, 2013) and these skills develop through three primary phases (Krajewski & Schneider, 2009; Purpura, Baroody, & Lonigan, 2013).

Early numeracy domains include numbering, numerical relations, and arithmetic operations (Purpura & Lonigan, 2013). Numbering refers to understanding the counting sequence and identifying quantity, and consists of skills such as verbal counting, one-to-one counting, understanding cardinality, and subitizing (i.e., ability to recognize a small amount of objects and know how many there are without counting). The second domain, numerical relations, comprises a child's understanding of how numbers on the mental number line are associated with each other and the verbal and nonverbal aspects of quantity. Numerical relations consist of comparing sets and numerals, ordering numbers, recognizing numerals, and connecting numerals to quantities. The final domain, arithmetic operations, involves a basic understanding of addition and subtraction, and consists of addition and subtraction with (e.g., using blocks) or without objects (e.g., story problems), composing and decomposing quantities, and basic number (addition) combinations (Purpura & Lonigan, 2013).

Within those domains, early numeracy skills appear to develop in three phases (Krajewski & Schneider, 2009). The first phase is characterized by children's ability to distinguish between different quantities, recite number words, and verbally count in sequence, which can be thought of as foundational numerical skills (Krajewski & Schneider, 2009; Purpura et al., 2013). The second phase is characterized by children's ability to verbally count a fixed set of objects in sequence, link imprecise number words (e.g., a bit, much) to quantities, and link precise number words (e.g., two, four) to quantities, suggesting that a focus on understanding language specific to math is important during this phase (Krajewski & Schneider, 2009; Purpura et al., 2013). During the third phase, children understand the relation between quantities and number words, the composition and decomposition of quantities (e.g., the quantity 3 is composed of 1 and 2), and differences between numbers (e.g., the difference between 3 and 4 is 1; Krajewski & Schneider, 2009; Purpura et al., 2013). National U.S. standards for early mathematics have recognized that the informal concepts and skills learned in these phases are important precursors to students' understanding of more advanced math skills such as formal addition, subtraction, and place value (National Association for the Education of Young Children [NAEYC] & National Council of Teachers of Mathematics [NCTM], 2003; National Mathematics Advisory Panel [NMAP], 2008; National Research Council [NRC], 2009).

Importance of early numeracy in math development. A growing body of evidence suggests that early numeracy skills are important for the development of students' later math success. For example, longitudinal studies show that children with higher numeracy skills during preschool and kindergarten also had higher scores of math achievement in middle and high school (Duncan et al., 2007; Watts, Duncan, Siegler, & Davis-Kean, 2014). Further, some evidence suggests that numeracy skills during preschool predict growth and achievement in math throughout elementary school (Duncan et al., 2007; Geary, 2011; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Watts, Duncan, Clements, & Sarama, 2018).

The National Research Council (2009) report indicates that young children must use math reasoning processes to master early math skills such as simple addition or story problems. For example, to be able to figure out the story problem "If you have three cookies and you eat one, how many are left?" children must reason that "eating one cookie" means that they will have fewer cookies than the larger number so the answer must be fewer than the initial whole. Additionally, preschoolers must learn to make connections and communicate about early math concepts by describing their thinking, talking about the patterns they see, and learning to use the language of math objects, situations, and symbols (Clements, 2001; NRC, 2009). The NRC also states that preschoolers' informal math experiences, which include exploration, problem solving, and language, provide an important foundation for the development of later formal math skills.

Beyond students' future academic success, early numeracy skills are related to other areas as well. For example, early numeracy skills are important for mastering math activities in everyday situations, such as keeping track of time, counting money, and measuring ingredients (LeFevre et al., 2009). Understanding the composition and decomposition of quantities during the preschool years is not only important for learning to add and subtract fractions and decimals in school, but is also important for everyday math, such as understanding how to convert a recipe from metric to U.S. standard units (e.g., cups, tablespoon; (Blevins-Knabe, 2012). Additionally, numeracy knowledge is necessary for students' career attainment, especially in the science, technology, engineering, and math (STEM) fields (NMAP, 2008). Ultimately, early numeracy skills are not only important for academic success but are also beneficial for everyday problem solving.

The Relation Between Language and Early Numeracy Skills

Language skills are important for successfully acquiring early math skills (LeFevre et al., 2010; Sowinski et al., 2015). Notably, many studies have demonstrated a strong relation between children's general language and math skills (Hooper, Roberts, Sideris, Burchinal, & Zeisel, 2010; Purpura & Ganley, 2014; Purpura, Hume, Sims, & Lonigan, 2011; Purpura & Napoli, 2015; Romano, Babchishin, Pagani, & Kohen, 2010). Prior work has shown that young children's expressive vocabulary is strongly related to most early numeracy skills (Purpura & Ganley, 2014) and that preschoolers with higher vocabulary knowledge had higher scores on numeracy tasks one year later (Purpura et al., 2011).

Early math language. Despite the close relation between general language and numeracy skills, recent evidence indicates that content-specific math language is a stronger predictor of math skills than is general language (Toll & Van Luit, 2014a). Math language is content-specific language that consists of terms used to describe quantitative and spatial relations, but does not include direct references to specific numbers (Purpura & Logan, 2015). Quantitative math language includes words such as "more," "less," "many," and "fewer" (Barner, Chow, & Yang, 2009; Purpura, Napoli, Wehrspann, & Gold, 2017). Children must understand quantitative math language to make and describe comparisons between words that describe quantities and groups of objects or numbers. For example, when a preschooler understands the word "fewer" they should know that it refers to a particular group of objects being smaller than another group of objects. Spatial math language consists of words, such as "near" and "above" that are associated with locations, directions, and shapes (Cannon, Levine, &

Huttenlocher, 2007). Children must understand spatial math language to make and describe comparisons between spatial terms and objects' sizes or locations (Pruden, Levine, & Huttenlocher, 2011). For example, when a preschooler understands the word "above" they should know that it refers to an object being higher in location than something else.

Children's math language knowledge in preschool and kindergarten is one of the strongest predictors of their numeracy skills, above and beyond general language knowledge (Purpura & Logan, 2015; Toll & Van Luit, 2014b). Further, Purpura et al., (2017) conducted an intervention that provided evidence that increasing preschoolers' exposure to math language by using storybooks resulted in an increase in math language knowledge and numeracy skills. In another study, Toll and Van Luit (2014b) demonstrated that math language mediates the relation between general language and early numeracy skills, suggesting that content-specific math language may be the mechanism by which children use their language knowledge to understand and solve numeracy tasks. Together these studies suggest two key theories: math language may causally underlie numeracy development and children's numeracy skills are highly language-dependent.

Children's Acquisition of Early Math

It is often assumed that children learn most early numeracy skills in school (Ginsburg, Lee, & Boyd, 2008). Several intervention studies have indicated that children's math skills significantly improve when their teachers implement a math-focused curriculum in their classrooms (Bobis et al., 2005; Clements & Sarama, 2007; Clements & Sarama, 2008; Clements & Sarama, 2011; Clements, Sarama, Spitler, Lange, & Wolfe, 2011; Sarama & Clements, 2004). Considering the importance of mathematics throughout formal schooling and beyond, national education and research panels have emphasized the importance of comprehensive math curricula to help equip preschool-aged children with the math knowledge that is necessary throughout life (NAEYC & NCTM, 2002; NRC, 2009). A joint statement from NAEYC and NCTM provides a guide for 3- to 6-year-olds' typical early math achievement, such as numeracy skills, and strategies on how to promote students' success in mathematics. Suggested teaching strategies for early numeracy skills include teachers organizing an environment that lends itself to math exploration, guiding children's counting in everyday situations, telling real life stories involving numbers, and asking word problems using "how many" (Clements, 2001; NAEYC & NCTM, 2002; Piasta, Pelatti, & Miller, 2014; Sarama & Clements, 2004). Partially as a result of national standard recommendations, math activities in some preschools have increased from an average of 4 minutes to 24 minutes a day over the last decade (Connor, Morrison, & Slominski, 2006; Piasta et al., 2014).

Complementary work suggests that young children also learn numeracy skills when their parents engage them in math-focused activities at home (Baroody & Wilkins, 1999; Blevins-Knabe & Musun-Miller, 1996; H. Ginsburg, 1977; Manolitsis, Georgiou, & Tziraki, 2013; Melhuish et al., 2008; Niklas & Schneider, 2014; Starkey & Klein, 2000). The HNE is an everyday context for this early development where parents provide children with resources and activities that may facilitate the development of early numeracy skills such as counting objects, comparing sizes, and understanding terms like "more" and "less" (Blevins-Knabe, 2012). However, in contrast to the work examining the preschool math environment, less work has been conducted on how the HNE contributes to the development of early numeracy skills, though the focus on the home has increased in recent years (LeFevre et al., 2009; Manolitsis et al., 2013; Niklas & Schneider, 2014; Thompson, Napoli, & Purpura, 2017). It is important to further investigate the HNE to understand the role of math language knowledge on the relation between numeracy-related activities and children's early numeracy skills, which contributes to their readiness to learn in school.

Home Numeracy Environment

Structure of the HNE. The HNE consists of parent-child engagement in direct numeracy activities (e.g., counting, reading number storybooks) and indirect numeracy activities (e.g., measuring ingredients when cooking, talking about money), which support young children's numeracy development (LeFevre et al., 2009; LeFevre, Polyzoi, Skwarchuk, Fast, & Sowinski, 2010; Manolitsis et al., 2013; Skwarchuk, Sowinski, & LeFevre, 2014; Thompson et al., 2017). Direct numeracy activities are ones in which children are explicitly taught quantitative skills (Skwarchuk et al., 2014). These tasks include identifying names of written numbers, counting objects, sorting objects by color, shape or size, counting down from the number 10, printing numbers, using number activity books, reading number story books, identifying numbers on signs when traveling, and reciting numbers in order. Indirect numeracy activities consist of a broader range of everyday tasks in which children are implicitly taught quantitative skills. Indirect numeracy activities include talking about money, measuring ingredients while cooking, using timers, making collections or groups of similar items, using calendars, playing card games, using clocks and discussing time, and using numbers to refer to temperatures, time, and dates (Skwarchuk et al., 2014).

The association between the HNE and early numeracy skills. Studies suggest that direct numeracy activities are a more consistent predictor of early numeracy skills than indirect numeracy activities (LeFevre et al., 2010; Manolitsis et al., 2013; Skwarchuk et al., 2014; Thompson et al., 2017), though there are a few exceptions (LeFevre et al., 2009; Skwarchuk et al., 2014). Notably, in one study, children had higher cardinal number knowledge at age 3 when their parents counted and labeled large sets of objects (e.g., direct numeracy activities; Gunderson & Levine, 2011). Additionally, Skwarchuk et al. (2014) found that direct and indirect numeracy activities were related to different aspects of kindergarteners' numeracy skills, such that direct activities predicted children's symbolic number system knowledge while indirect activities predicted children's non-symbolic arithmetic. However, LeFevre et al. (2009) found that indirect activities were related to children's numeracy knowledge but direct activities were not. In contrast, other studies generally have found that indirect activities were not related to children's numeracy skills, while direct activities were related (LeFevre et al., 2010; Manolitsis et al., 2013; Thompson et al., 2017). These mixed findings may be due to studies focusing on numeracy skills for different age groups and researchers using different HNE items. Additionally, the nature of indirect and direct numeracy activities differ and child characteristics may play a particular role in the extent to which the indirect HNE predicts numeracy skills.

Indirect activities may be more language-based than direct activities, considering that indirect activities may involve more dialogue about topics outside of numbers or counting and do not focus on explicit teaching of numeracy skills (Skwarchuk et al., 2014). Understanding math language may be important for learning from both indirect and direct activities; however, it may be more beneficial for learning from indirect activities because they are less structured and may be incidentally focused on teaching numeracy. Investigating children's math language knowledge as a moderator may help inform the reason why the indirect HNE has been found to be a less consistent predictor of numeracy skills than the direct HNE. The extent to which indirect activities predict children's numeracy skills may depend on children's knowledge of math language; whereas, direct numeracy activities may be consistently predictive of numeracy skills, regardless of children's initial math language ability, because direct activities are explicit and may be less language based (Skwarchuk et al., 2014).

The HNE has been shown to be related to early numeracy skills; however, prior work has not evaluated if that relation is moderated by children's knowledge of math language. Research on the HNE rarely investigates if the relation between numeracy activities and children's numeracy skills depends on characteristics of children, though a recent study suggests that benefits of the HNE depends on preschoolers' age (Thompson et al., 2017). Thompson et al. (2017) found that basic HNE activities (e.g., counting objects) were associated with numeracy skills for 3-year-olds and complex HNE activities (e.g., simple sums) were associated with numeracy skills for 4-year-olds. In order to better understand how the HNE contributes to children's early math skills, it is important to consider children's math language knowledge as a moderator of the relation between both aspects on the HNE (i.e., direct/indirect) and children's numeracy development, which may inform parents on how to facilitate their children's numeracy development. Due to differences in the nature of indirect and direct numeracy activities, it is important to look at these relations separate for both the indirect and direct HNE.

Indirect numeracy activities may rely more on the use of math language to discuss numeracy-related concepts, whereas engagement in direct numeracy activities may focus more on the use of numbers and counting to teach numeracy skills. When parents engage in indirect numeracy activities with their children, parents may build a back and forth dialogue to effectively facilitate children's exploration, problem solving, and content-specific math language needed to understand and develop early numeracy skills (NRC, 2009). Indirect numeracy activities typically are unstructured spontaneous interactions (LeFevre et al., 2010; Manolitsis et al., 2013; Skwarchuk et al., 2014; Thompson et al., 2017), such as talking about money when shopping, that may involve the use of math language (e.g., a parent talking about how an item costs “too much” money). Direct numeracy activities are used to explicitly teach numeracy (Gunderson & Levine, 2011; LeFevre et al., 2010; Manolitsis et al., 2013; Skwarchuk et al., 2014; Thompson et al., 2017), but may not involve as much back and forth dialogue about numeracy-related concepts. For example, direct numeracy activities may take a more structured format, such as counting out a simple set of objects or practice writing numbers that would not require dialogue involving the use of math language. As such, children may learn more numeracy from indirect activities when they have higher initial math language knowledge. In contrast, direct numeracy activities may be predictive of early numeracy regardless of children’s level of math language knowledge because of the explicit focus on teaching numeracy.

Theoretical Framework

According to Vygotsky (1978), children learn from the environment around them, specifically when parents or caregivers facilitate and encourage children's development of specific skills. Early math skills (e.g., numeracy, math language) are learned within sociocultural contexts where children are actively engaging in numeracy related activities with caregivers or parents (John-Steiner & Mahn, 1996; Miller, 2011). Vygotsky’s sociocultural approach can be used to understand children’s development of numeracy skills and math language in the context of the HNE, where parents engage in numeracy-related interactions with children (Elliott & Bachman, 2018; Kleemans et al., 2012; Niklas & Schneider, 2014).

Vygotsky’s sociocultural approach posits that children learn to use psychological tools, such as language systems, by interacting with parents or caregivers in sociocultural contexts (Miller, 2011). As children develop, they use psychological tools to control, organize, and change their thought or behavior (Miller, 2011). According to Vygotsky (1978), language is the most important psychological tool because comprehending and producing language are processes that transform thinking. Based on this theory, it is hypothesized that children's level of math language may moderate their readiness to learn new numeracy concepts from indirect HNE activities that they engage in. Specifically, the zone of proximal development can be used to explain how children's acquisition of numeracy skills from HNE activities may depend on their math language knowledge (Miller, 2011; Vygotsky, 1978). The zone of proximal development explains that the problem solving that children can accomplish with support depends on their readiness to learn new concepts when engaging in activities with a parent or caregiver (Vygotsky, 1978). Indirect numeracy activities that parents and children engage in together almost always involve the use of language to describe, question, and problem solve (NRC, 2009). The amount of numeracy a child learns from indirect numeracy activities may, therefore, depend on their level of math language. For example, when engaging in an indirect numeracy activity such as baking, parents' use of math language may be embedded within the dialogue (e.g., a parent might say that they need "more" or "less" of an ingredient). The parent's use of math-specific language may help direct their child's thinking, and as their child gains an understanding of the word, he/she may begin to make associations between math language terms, quantities of items, and printed numbers, which will assist in solving independent numeracy tasks. However, a typical direct numeracy activity, like counting objects, relies primarily on the use of numeral names rather than math language. In accordance with Vygotsky's sociocultural theory and the zone of proximal development, math language may be an important psychological tool that is necessary to learn numeracy from indirect HNE activities.

Current Study

The present study was designed to investigate if preschoolers' level of math language knowledge moderates the relation between indirect and direct HNE activities and early numeracy skills. The extent to which the HNE predicts early numeracy skills may depend on preschoolers' knowledge of content-specific math language, particularly in relation to how indirect activities are predictive of numeracy development. Children's level of math language knowledge may contribute to how much they learn from the dialogue surrounding indirect numeracy activities but may not contribute to how children learn from direct numeracy activities. However, research studies have not considered preschoolers' math language knowledge when investigating the relation between the HNE and numeracy skills. The aim of the current study, guided by Vygotsky's sociocultural approach, is to investigate the relation between the HNE and numeracy skills while considering preschoolers' math language as a moderator.

There is one primary hypothesis in this study (H), with two sub-hypotheses.

H: The relation between HNE and numeracy outcomes.

Based on research findings that both the HNE and math language predict numeracy skills (Niklas & Schneider, 2014; Purpura, Napoli, et al., 2017) and the idea, adapted from Vygotsky's theory, that children's acquisition of numeracy skills from the indirect HNE may depend on their math language knowledge and readiness to learn new numeracy concepts, it is hypothesized that:

(Ha) Though there will not be a direct relation between indirect HNE and numeracy outcomes, math language will moderate the relation between the indirect HNE and early numeracy skills, such that children with high math language will benefit from a high indirect HNE but children with low math language will not.

(Hb) Though there will be a direct relation between direct HNE and numeracy outcomes, children's math language is not expected to moderate the relation between the direct HNE and early numeracy.

Method

Participants

Participants were recruited from twelve early childhood centers in a Midwestern region of the United States. Letters explaining the study, consent forms, and questionnaires were sent home to all parents of 3- to 5-year-old preschool children attending these centers. Parents of 146 students completed the background questionnaire and gave permission for their children to participate. Of those children, 21 were excluded from analyses for one of the following reasons: the child was in kindergarten (some preschools were located within elementary schools; $n = 11$), the child left school or moved before testing began ($n = 3$), the child refused to participate in testing ($n = 4$), or the child had language or developmental delays that prevented accurate testing ($n = 3$). Children excluded from analyses did not significantly differ from those to be included in regard to income, ethnicity, or parental education (all $ps > .05$).

The 125 preschoolers to be included in the analyses are 54% female, 70% Caucasian, 9% Asian, and 21% other or multiracial, which is approximately representative of the local demographics. Children ranged in age from 3.01 to 5.17 years (M = 4.09, SD = 0.57) at time of parental consent. Parents' highest education ranged from having earned a GED to holding a graduate degree; 23% of parents had some college or less, 32% had an Associate's or Bachelor's degree, and 45% had a graduate degree. Of the 125 participants, 11 were missing Time 2 numeracy scores and 4 were missing HNE measures. To address missing data from parent questionnaires and child assessments, FIML was used in StataSE 15, that way all available information is used to estimate the models.

Measures

Early Numeracy. The Preschool Early Numeracy Skills Screener – Brief Version (PENS-B; Purpura, Reid, Eiland, & Baroody, 2015) was used to evaluate preschoolers' numeracy skills. The PENS-B is a 24-item measure which takes approximately five minutes to administer and assesses the broad numeracy skills that children are exposed to in preschool and kindergarten. The PENS-B does not include the use of manipulatives, and only the assessment manual is needed to administer the test. Specific assessment areas include set comparison, numeral comparison, one-to-one correspondence, number order, numeral identification, ordinality, and number combinations. Children received one point for each correct answer. Although all 24 items were administered, a ceiling rule was applied during analyses such that children did not receive points for any correct responses after three consecutive incorrect responses (Purpura et al., 2015). The PENS-B has high internal consistency ($\alpha = .92$). Children were assessed on early numeracy in both the fall and the spring.

Math Language. The math language subtest is a researcher-created measure of mathematics-specific language. The measure has 16 items assessing comparative language (e.g., more, less, take away) and spatial language (e.g., near, far, before). Children were awarded one point for each correct response. In prior work (Purpura & Logan, 2015), these items were selected from a larger battery (i.e., broader range) of items using an item response theory framework. The selected items had a range of difficulty parameters and strong discrimination parameters. The specific words included on this measure were intended to be broadly representative of the quantitative and spatial language associated with mathematics. Quantitative words included take away, a little bit, most, more, fewest, and less. Spatial words included: nearest, under, first, far, below, front, middle, end, last, and before. All items were designed to be completed without exact quantitative skills and in a non-numeracy context. For example, the quantitative questions were asked in different ways: (a) comparing dots with such a gross difference that children would be able to respond correctly regardless of numeracy ability as long as they knew the meaning of the language terms (e.g., 10 vs. 2) and (b) using a picture of mostly full and mostly empty glasses when asking "Which glass has the most water?" or "Which glass has less water?" This math language task had an internal consistency of .80 for this sample. Children were assessed on math language in the fall and spring.

General language. The Test of Preschool Early Literacy (TOPEL; Lonigan, Wagner, Torgesen, & Rashotte, 2007) was used to evaluate preschoolers' general language ability using the definitional vocabulary subtest. This subtest includes 35 items with two questions per item for a total of 70 possible points. Children were asked to provide definitions and explanations of words (e.g., ''What is this? What is it for?"). Children were awarded 1 point for correctly identifying the picture and 1 point for providing a correct description of the picture. The definitional vocabulary assessment was stopped when the child responded incorrectly to both questions in an item for 3 consecutive items (i.e., six total questions). This task had an internal consistency of .94.

Home Numeracy Environment. Parents were asked to complete a researcher-created background information questionnaire. The questionnaire was based on previous research of LeFevre and colleagues (2009). Parents provided socioeconomic status information, such as educational achievement, income, and characteristics of the family and home environment. Parents also reported the frequency of practicing 9 indirect numeracy activities and 7 direct numeracy activities in the home with their children by responding to the prompt "In the past month, how often did you and your child engage in the following activities?" with six options ranging from "never" (0) to "multiple times per day" (5). Indirect numeracy activities included using calendars and dates, measuring ingredients when cooking, playing board games with die or spinner, using terms more/less, engaging in timed activities, making/sorting collections, sort things by size, color or shape, having your child wear a watch, and talking about money when shopping. Direct numeracy activities included counting objects, printing numbers, reading number storybooks, using number activity books, counting down from 10, learning simple sums, and identifying names of written numbers. The indirect HNE had an internal consistency of .77 and the direct HNE had an internal consistency of .72. Parent surveys of HNE were collected prior to preschoolers' first assessment.

Analytical Strategy

Data were analyzed using StataSE 15. Two hierarchical multiple regressions were conducted to test the hypothesis (see Figure 1). Early numeracy ability at Time 2 was used as the dependent variable for both regressions. In both models, age, gender, parental education, and general language ability were used as control variables due to established associations with math language and early numeracy (Chang, Sandhofer, & Brown, 2011; Jeon, Buettner, & Hur, 2014; Purpura & Reid, 2016; Sarama & Clements, 2009). Early numeracy ability at Time 1 was also included as a control variable due to Time 1 numeracy being associated with numeracy ability at Time 2 (Napoli & Purpura, 2018). Additionally, predictor and interaction term variables were mean-centered in order to aid in their interpretability. Specific analyses for the hypotheses are presented below.

Hypothesis

a) Although a main effect of indirect HNE was not expected, it was expected that math language would moderate the relation between indirect HNE and numeracy skills.

Analytic plan a. First, to test for main effects, children's numeracy ability at Time 2 was regressed on the indirect HNE (Time 1), preschoolers' math language (Time 1), preschoolers' numeracy (Time 1), and covariates. Then, an interaction of the indirect HNE and math language (Time 1) was added to the model. If there was a significant interaction of math language and indirect HNE predicting numeracy ability, simple slope analysis was conducted to determine the nature of the moderation effect.

b) It was expected that there would be a main effect of direct HNE, but it was not expected that math language would moderate the relation between direct HNE and numeracy skills.

Analytic plan b. First, children's numeracy ability at Time 2 was regressed on the direct HNE (Time 1), preschoolers' math language (Time 1), preschoolers' numeracy (Time 1), and covariates. Then, an interaction of the direct HNE and math language (Time 1) was added to the model. An interaction between direct HNE and math language was not expected to be significant.

Analytic plan combined analyses. If there were significant main effects or interactions in 1a and 1b, a robustness check was planned where indirect and direct HNE were included in the same model.

Results

Descriptive Statistics

Means, standard deviations, ranges, skewness, and kurtosis for covariates, child outcomes, and home numeracy environment scores are presented in Table 1. Correlations between covariates, math language, direct HNE, indirect HNE, and numeracy are presented in Table 2. Children's age was correlated with vocabulary, numeracy, and math language skills, as well as the direct and indirect HNE. Parents' highest level of education was correlated with preschoolers' numeracy and math language. Children's vocabulary scores were correlated with numeracy, math language, direct HNE, and indirect HNE. Numeracy and math language, as well as the direct HNE and indirect HNE were also strongly correlated.

Primary Analyses

Hypothesis a. There was a main effect of Time 1 covariates, math language ($\beta = 0.36$, $p < .001$) and numeracy ($\beta = 0.35$, $p < .001$), predicting preschoolers' numeracy at Time 2 (Table 3, Models 1&2). There was neither a main effect of indirect HNE on numeracy skills ($\beta = 0.11$, $p = .081$) nor did math language moderate the relation between the indirect HNE and numeracy skills ($\beta = -0.01$, $p = .969$). As expected, there was no main effect of indirect HNE predicting numeracy skills. However, the hypothesis that math language would moderate the relation between indirect HNE and numeracy skills was not supported.

Hypothesis b. There was a main effect of Time 1 covariates, math language ($\beta = 0.35$, $p < .001$) and numeracy ($\beta = 0.37$, $p < .001$), predicting preschoolers' numeracy at Time 2; however, there was no main effect of the direct HNE predicting preschoolers' numeracy at Time 2 ($\beta = 0.11$, $p = .113$; Table 3, Models 3&4). In addition, math language did not moderate the relation between the direct HNE and numeracy skills ($\beta = -0.03$, $p = .636$). The hypothesis that there would be a main effect of direct HNE was not supported. Moreover, as expected math language did not moderate the relation between direct HNE and numeracy skills.

Combined analyses. As there were no main effects of the HNE nor interactions, the robustness check was not needed.

Supplemental Analyses

Numeracy Outcome. As the interaction models were not significant, additional analyses were conducted to better understand the relation between the indirect HNE, direct HNE, math language, and numeracy skills. The finding that the direct HNE was not predictive of later numeracy was surprising considering that previous studies have found the direct HNE is a significant predictor of numeracy skills for this preschool age group (LeFevre et al., 2010; Manolitsis et al., 2013; Skwarchuk et al., 2014; Thompson et al., 2017). Prior work looking at the HNE has not included math language in the model; therefore, regression analyses were conducted to investigate associations between the direct HNE and numeracy skills when math language was not included in the model. When math language was removed from the model, both the indirect HNE ($\beta = 0.15$, $p = .028$) and the direct HNE ($\beta = 0.16$, $p = .015$) were significantly predictive of later numeracy skills in separate models (Table 4, Models 1&2), suggesting that the HNE is predictive of numeracy but when children's math language knowledge is included in the model it accounts for the variance in later numeracy that would have been accounted for by components of the HNE.

Concurrent Outcomes. Using the same variables of interest, regression analyses were conducted predicting Time 1 numeracy and Time 1 math language rather than longitudinal outcomes (Tables 5 & 6). Results indicated that the indirect HNE was not associated with concurrent numeracy ($\beta = 0.09$, $p = .131$) and the direct HNE was not associated with concurrent numeracy skills ($\beta = 0.03$, $p = .642$). Additionally, the indirect HNE was not associated with concurrent math language ($\beta = 0.10$, $p = .131$); however, the association between direct HNE and concurrent math language was marginally significant ($\beta = 0.12$, $p = .054$. Given that (1) math language knowledge at Time 1 was a significant predictor of later numeracy skills in the analyses for the primary hypothesis, (2) the HNE was a significant predictor of Time 2 numeracy skills when Time 1 math language was removed from the model, and (3) direct HNE was related to Time 1 math language, the relation between the HNE and later numeracy skills may be mediated by math language knowledge.

Mediation analysis. Prior work in this area has demonstrated that the direct HNE predicts early numeracy skills, however, such work has not yet fully explained this mechanism. It is important to consider the direct HNE's relation to math language, which is predictive of numeracy development (Purpura & Logan, 2015; Toll & Van Luit, 2014). Preschooler's knowledge of content-specific math language may explain the relation between the direct HNE and early numeracy skills. Although direct numeracy activities are not thought of as language-based activities, results of concurrent analyses suggest that children may be indirectly exposed to math language terms while engaging in direct numeracy activities with parents. Children may acquire math language from parents during direct HNE activities and use this language knowledge to evaluate and complete numeracy tasks independently.

Mediation analysis was used to examine if math language mediates the relation between the direct HNE and later numeracy skills, while controlling for time 1 numeracy and covariates. A math language average score was created to be used in the following mediation analysis to test whether the direct HNE is associated with the preschooler's average level of math language between Time 1 and Time 2. It is recommended to use three separate time points or waves of data when running mediation analyses with longitudinal data (Cole & Maxwell, 2003). The math language average score was used because it provided a better estimate of children's math language knowledge between Time 1 and Time 2 (NICHD ECCRN & Duncan, 2003). As Figure 2 illustrates, results indicated that the direct HNE was a significant predictor of math language, β

= 0.15, *SE* = .062, *p* = .015, and that math language was a significant predictor of numeracy skills, β = 0.40, *SE* = .091, $p < .001$. The effect of the direct HNE on numeracy skills attenuated after controlling for the mediator, math language, β = 0.10, *SE* = .054, *p* = .054, consistent with mediation. The indirect effect was tested using a bootstrap estimation approach with 1000 samples (Shrout & Bolger, 2002). These results indicated the indirect coefficient was significant, *b* = 0.46, *SE* = .230, *p* = .041, 95% CI= .011, .913.

Discussion

This study examined the relation between the HNE and children's numeracy development, and whether children's math language skills moderated that relation. Contrary to the hypotheses, results from the present study found no evidence that prior math language knowledge moderates the relations between the indirect HNE and later numeracy. Further, no direct effects were found for the relation between either HNE component (direct and indirect) and later numeracy. However, the lack of significant relation between the direct HNE and later numeracy skills was likely due to the inclusion of math language knowledge in the model, which prior research has not included. It is possible that subjective measures of parent-reported frequency of direct HNE engagement does not account for enough variance in children's numeracy outcomes when compared to objective measures of children math language knowledge. The results of supplementary analyses suggested that the direct HNE was related to children's initial math language knowledge, which in turn was related to later numeracy skills. Additionally, both the direct and indirect HNE were predictive of later numeracy skills when prior math language knowledge was excluded from the model. Results of supplemental analyses also indicated that children's math language skills fully mediated the relation between the direct HNE and numeracy skills. These findings suggest that the direct HNE is associated with both numeracy skills and math language knowledge and that math language may be a mechanism that explains the relation between the direct HNE and early numeracy.

Main Effects

Direct HNE predicting numeracy. It was hypothesized that the direct HNE would predict children's subsequent numeracy skills. Contrary to expectations, there was no main effect of the direct HNE on later numeracy skills. Although research suggests that there is a stable relation between the direct HNE and preschoolers' numeracy outcomes (LeFevre et al., 2010; Manolitsis et al., 2013; Skwarchuk et al., 2014; Thompson et al., 2017), there have been a few inconsistent findings across studies. For example, LeFevre et al. (2009) found that the direct numeracy activities were not related to children's numeracy skills, which is consistent with the findings of this study. Their findings suggested that the direct HNE may not have been related to numeracy knowledge because parents reported low levels of engagement in numeracy activities (LeFevre et al., 2009). Moreover, the present study, along with others, only account for the frequency of parent-child engagement in the HNE and not for the quality of parent-child interactions during direct numeracy activities (LeFevre et al., 2009). It is possible that this subjective measure of direct HNE activities does not fully capture the mechanism that explains and illustrates the importance of the direct HNE for children's acquisition of numeracy skills.

To further probe this null effect, supplementary analyses were conducted to compare the relation between the direct HNE and child numeracy skills with and without accounting for children's math language at Time 1 in the model. Results from the supplementary analyses suggests that when prior math language is not accounted for in the model, the direct HNE is a significant predictor of later numeracy skills. This finding is consistent with findings from previous studies on the direct HNE that did not control for children's math language knowledge

(LeFevre et al., 2010; Manolitsis et al., 2013; Skwarchuk et al., 2014; Thompson et al., 2017). However, it may be that when frequency of HNE activities and math language knowledge are measured at the same time point, math language becomes a predictor of later numeracy skills because it is an objective measure of children's knowledge that is more closely related to their ability to complete numeracy tasks.

Interactions

Math language moderating the relation between HNE and numeracy. It was hypothesized that math language would moderate the relation between the indirect HNE and numeracy skills, but findings did not support this moderation effect. There are a number of potential explanations for why math language did not moderate the relation between the indirect HNE and child numeracy skills. First, previous research suggests that parents may not draw children's attention to numeracy-related concepts during unstructured, activities that are not explicitly intended to teach children math concepts (Thompson et al., 2017). Indirect activities may be language based, but if children are not hearing or using math language then their learning of numeracy from indirect activities may not depend on their level of math language knowledge. For example, if parents do not focus on helping children understand numeracy-related concepts during indirect HNE activities, then they are likely not using much math language during these interactions. Thus, even if children hear a high amount of parent language during indirect activities, if they are not exposed to math language during these activities, then it would be unlikely that their own math language knowledge would moderate the relation between indirect HNE activities and their numeracy skills.

Another potential reason why there was no moderating effect of math language on the association between the indirect HNE and later numeracy is that children may not engage in indirect HNE activities very often. Indeed, other studies suggest that parents typically report engaging children in indirect activities less frequently than in direct activities (LeFevre et al., 2010; Manolitsis et al., 2013; Thompson et al., 2017), which is consistent with the frequency of indirect HNE activities in this study. Although indirect activities, such as measuring ingredients, provide opportunities to discuss numeracy concepts and use math language (e.g., a parent explaining to child that they need to purchase more eggs because they have fewer than a recipe calls for), parents may not initiate conversations surrounding numeracy often enough for children to use their math language knowledge to acquire numeracy skills.

Results from supplementary analyses suggest that there may be a relation between *direct* numeracy activities and concurrent math language knowledge. Parents may use high levels of math language when explaining numeracy concepts to children and it is possible that they use direct numeracy activities more often than indirect numeracy activities to explain these numeracy concepts. It is possible that parents use more math language while engaging children in direct numeracy activities than in indirect activities, and this heightened use of parent math language during direct activities may be associated with children's understanding of math language. Although direct numeracy activities are thought of as activities where children are explicitly taught numeracy skills (Skwarchuk et al., 2014), parents may use more numeracy-focused language to contribute to young children's understanding during these activities. Future work is needed to explore whether the type of HNE activities, direct or indirect, lead to differences in math language use by parents and children, and whether these differences in language use explain later child numeracy skills.

Math Language as a Mediator

Supplemental analyses investigated whether children's math language across time-points (i.e., average score of Time 1 and 2 math language knowledge) mediated the relation between the frequency of child engagement in direct HNE activities and later numeracy skills. Results of these exploratory analyses demonstrated that average math language knowledge mediated the relation between the frequency of engaging in direct HNE activities and later child numeracy skills. This suggests that the relation between engagement in direct numeracy activities and later numeracy performance is explained by young children's math language knowledge. This finding is in line with previous studies that find math language acts as a mediator of the relation between preschoolers' general language and numeracy skills (Toll & Van Luit, 2014b; Purpura, Logan, Hassinger-Das, & Napoli, 2017). It appears that when parents engage their children in direct numeracy activities, they may expose their children to math language terms that can help children develop numeracy skills. For example, during a direct HNE activity such as printing numbers, parents may initiate a dialogue with children explaining how some numbers represent quantities that are "bigger", "smaller", "more", or "less" than the quantities of other numbers. Further research is needed to understand exactly how having math language knowledge facilitates children's numeracy learning within the context of direct numeracy activities.

Limitations and Future Directions

Although the results of the current study yield important insights regarding the relations between the HNE, early numeracy skills, and math language knowledge, limitations of the study and areas for future research should be noted. One limitation is that this study design was correlational in nature, and therefore, causal implications cannot be inferred. Though causal implications cannot be made, results provide evidence that children's math language knowledge may support their acquisition of numeracy skills from direct numeracy activities that they experience with parents. Prior research provides evidence for a causal relation between math language knowledge and numeracy skills (Purpura et al., 2017); however, additional research is needed to evaluate whether or not there is a causal relation between the direct HNE and math language knowledge.

A second limitation is that parent report was used to measure frequency of parent and child engagement in the direct and indirect HNE, which may have resulted in reporter bias. That is, it is possible that parents may have been influenced by social desirability while completing the survey or may not have remembered how often they engaged with their child in home numeracy activities in the month prior to the study visit. Future research should use real time methods, including time diary approaches, to assess parent report of engagement in daily activities with children so that parents can report on daily activities in the moment rather than trying to remember prior engagement over a month-long period. Additionally, this study focuses on the frequency of home numeracy activities rather than a measure of quality, such as the use of math language during these activities. Future research on the HNE is needed to understand the type of parent involvement that is needed to influence children's acquisition of numeracy skills. More work is needed to examine the use of math language during parent-child practices at home and how that relates to children's development of numeracy skills and math language knowledge. Further studies should include observational methods to measure the content specific language environment that surrounds engagement in home numeracy activities (Levine et al., 2010).

Another limitation is that the sample was relatively homogeneous. Seventy percent of the sample was Caucasian; therefore, results may not generalize to other racial and ethnic groups.

Though recent research shows that the HNE is a predictor of monolingual Spanish speaking preschooler's numeracy skills (Susperreguy, Douglas, Xu, Molina-Rojas, & LeFevre, 2018), more work is needed to understand the relations between the HNE, early numeracy, and math language knowledge for young children from more diverse ethnic and linguistic backgrounds.

Although this study focused on investigating the unique relations between different aspects of the HNE (i.e., indirect and direct activities) and preschooler's math outcomes, correlations between study variables suggest that there is significant overlap between the indirect HNE and the direct HNE (see Table 2). The strong correlation between the indirect HNE and the direct HNE may indicate that, generally, when parents frequently engage their children in indirect numeracy activities they also frequently engage them in direct numeracy activities. The two HNE factors share significant variance which may be why neither the indirect HNE nor the direct HNE were predictive of later numeracy skills when added in the same model (the model that did not account for prior math language knowledge). Similarly, the direct HNE was no longer related to later math language knowledge when the indirect HNE was added to the model. Prior research has used a bifactor model to show that the direct and indirect HNE are distinct constructs (Hart, Ganley, & Purpura, 2016); however, this work focused on the home numeracy environments for children who varied widely in age, and did not link HNE constructs to direct assessments of numeracy skills. Investigating the effect of the indirect HNE and direct HNE together may be a better way to understand the relation between parent-child numeracy activities and preschooler's math outcomes. One previous study that included both indirect and direct numeracy activities in their HNE measure found that this broader measure of the HNE was a significant predictor of preschooler's numeracy development (Niklas, Cohrssen, & Tayler, 2016). More research is needed to understand how specific aspects of the HNE and broad measures of the HNE are related to preschooler's numeracy and math language outcomes.

Conclusion

The findings from this study fill an important gap in the HNE literature. Although math language knowledge did not moderate the relation between the HNE and numeracy skills, math language was found to be a mediator of the relation between direct numeracy activities and numeracy skills. This finding extends findings from previous work indicating significant relations between the direct HNE and numeracy skills. Particularly, this study's results provide evidence that the direct HNE is related to math language skills, and suggest that math language mediates the relation between the direct HNE and numeracy skills. These findings provide rationale for future research to investigate the use of math language during direct numeracy activities to better understand its relations to early math outcomes. Better insight on parents' math language use during home numeracy practices is necessary to understand how preschoolers' math language and numeracy learning may be fostered.

Table 1. *Descriptive Statistics of Covariates, Child Outcomes, and Home Numeracy Environment*

	N	*M*	*SD*	Range	Min.	Max.	Skew	Kurtosis
Age	125	4.18	.58	2.14	3.12	5.26	-.03	-.98
Parent Education	124	6.94	1.67	6.00	3.00	9.00	-.37	-.94
Vocabulary T1	125	50.82	12.35	58.00	8.00	66.00	-1.32	1.44
Numeracy T1	125	10.17	5.85	23.00	.00	23.00	.24	-.83
Numeracy T2	114	13.55	5.95	24.00	.00	24.00	-.16	-.98
Math Lang. T1	125	11.25	3.85	14.00	2.00	16.00	-.77	-.28
Math Lang. T2	114	13.03	3.01	13.00	3.00	16.00	-1.17	.84
Direct HNE	121	2.24	.79	4.29	.14	4.43	-.05	.29
Indirect HNE	121	1.80	.80	3.38	.25	3.63	.01	-.69

Table 2. *Correlations between Covariates, Child Outcomes, and Hume Numeracy Environment*

	1.	2.	3.	4.	5.	6.	7.	8.	9.
1. Age	--								
2. Gender	-.21*	--							
3. Parent Education	-.07	-.06	--						
4. Vocabulary T1	.57*	-.00	.11	--					
5. Numeracy T1	.60*	-.00	.35*	.53*	--				
6. Numeracy T2	.55*	-.02	.19*	.51*	.69*	--			
7. Math Lang. T1	.51*	.03	.31*	.62*	.67*	.68*	--		
8. Math Lang. T2	.45*	-.03	.30*	.53*	.59*	.65*	.76*	--	
9. Direct HNE	.27*	.01	.01	.21*	.22*	.35*	.29*	.26*	--
10. Indirect HNE	.19*	-.08	-.06	.20*	.19*	.31*	.21*	.15	.64*

† $p \leq .10$, * $p < .05$, ** $p < .01$

Table 3. *Regression Analyses Predicting Numeracy Skills (Time 2)*

	Model 1		Model 2		Model 3		Model 4		Model 5	
Predictor	*β*	*SE*	*β*	*SE*	*β*	*SE*	*β*	*SE*	*β*	*SE*
Indirect HNE	.11	.06	.11	.06	--	--	--	--	.08	.08
Direct HNE	--	--	--	--	.11	.07	.11	.07	.06	.08
Math Lang. T1	.36**	.09	.36**	.10	.35**	.09	.33**	.10	.35**	.09
Numeracy T1	.35**	.09	.35**	.10	.37**	.09	.38**	.09	.36**	.09
Vocabulary T1	-.01	.09	-.01	.09	.01	.09	.01	.09	-.00	.09
Age	.14	.09	.14	.09	.11	.09	.10	.09	.13	.09
Parent Education	.02	.07	.02	.07	.01	.07	.01	.07	.02	.07
Gender	.02	.06	.02	.06	.00	.06	.00	.06	.01	.06
Indirect HNE * Math Lang. T1	--	--	-.01	.07	--	--	--	--	--	--
Direct HNE * Math Lang. T1	--	--	--	--	--	--	-.03	.07	--	--

† $p \leq .10$, * $p < .05$, ** $p < .01$

Table 4. *HNE Domains Predicting Numeracy Skills (Time 2)*

	Model 1		Model 2		Model 3	
Predictor	*β*	*SE*	*β*	*SE*	*β*	*SE*
Indirect HNE	.15*	.07	--	--	.08	.08
Direct HNE	--	--	.16*	.07	.12	.09
Numeracy T1	.46**	.09	.49**	.09	.48**	.09
Vocabulary T1	.14	.08	.15	.08	.14	.08
Age	.18	.10	.12	.10	.14	.10
Parent Education	.06	.07	.04	.07	.05	.07
Gender	.04	.07	.02	.07	.03	.07

$^{\dagger}\ p \leq .10$, $^{*}\ p < .05$, $^{**}\ p < .01$

Table 5. *HNE Domains Predicting Numeracy Skills (Time 1)*

Predictor	Model 1		Model 2		Model 3	
	β	*SE*	*β*	*SE*	*β*	*SE*
Indirect HNE	--	--	.09	.06	.12	.07
Direct HNE	.03	.06	--	--	-.05	.07
Vocabulary T1	.17*	.07	.15*	.07	.15*	.08
Age	.56**	.07	.56**	.07	.57**	.07
Parent Education	.38**	.06	.38**	.06	.39**	.06
Gender	.13*	.06	.14*	.06	.15*	.06

† $p \leq .05$, * $p < .05$, ** $p < .01$

Table 6. *HNE Domains Predicting Math Language (Time 1)*

Predictor	Model 1 β	Model 1 *SE*	Model 2 β	Model 2 *SE*	Model 3 β	Model 3 *SE*
Indirect HNE	--	--	.09	.06	.03	.08
Direct HNE	.12†	.06	--	--	.10	.08
Vocabulary T1	.40**	.07	.40**	.07	.40**	.07
Age	.29**	.08	.31**	.08	.29**	.08
Parent Education	.28**	.06	.29**	.06	.29**	.06
Gender	.10	.06	.12	.06	.11	.06

† p =.05, * $p < .05$, ** $p < .01$

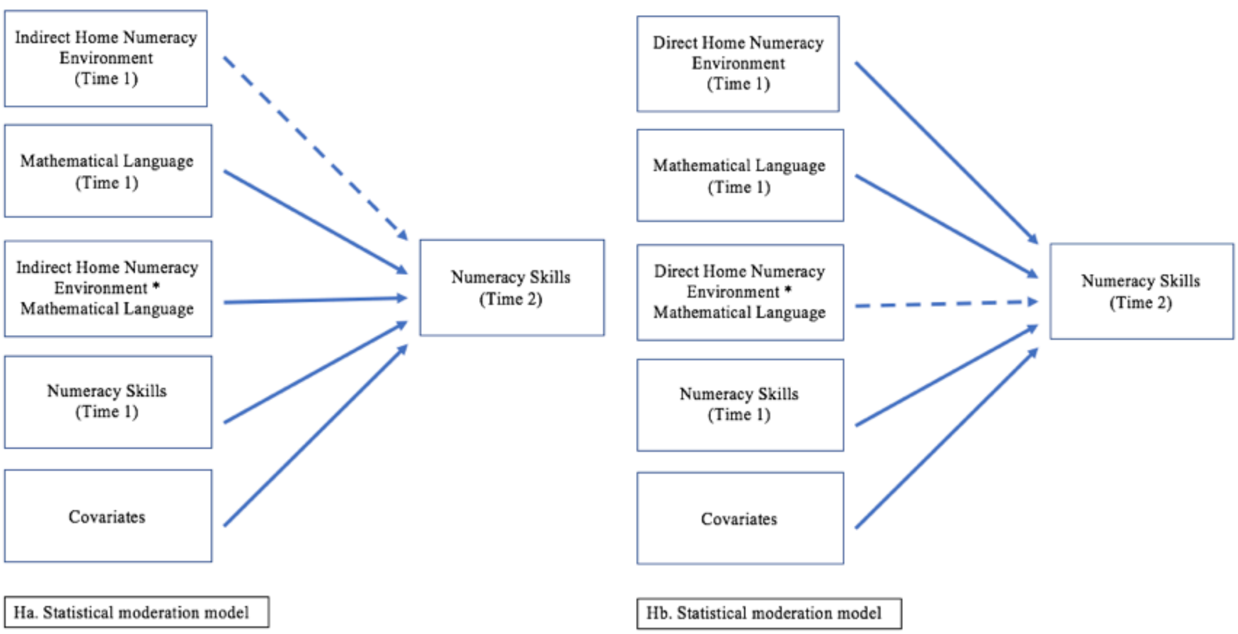

Figure 1. *Moderation models being tested. The dotted arrows are not expected to be significant.*

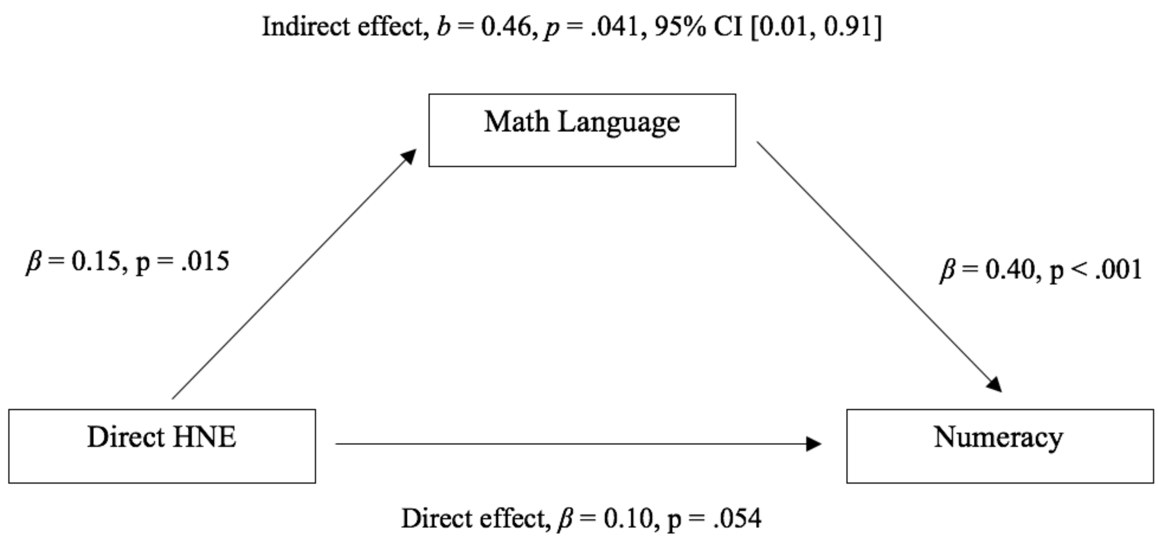

Figure 2. *Model of the direct HNE as a predictor of numeracy skills, mediated by math language knowledge.*

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