# IMPLEMENTING *COMMON CORE STATE STANDARDS FOR MATHEMATICS*: FOCUS ON PROBLEM SOLVING

by

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*Dedication*

*For my family*

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My study of perseverance has extended beyond the dissertation study itself; completing this dissertation while working full time and having many things in my life happen has been a personal journey of perseverance too. From my mother's death, to a few of my own major health setbacks, to getting married, to a miscarriage, to a wonderful daughter being born, life has thrown many curve balls at me through this journey. It has been a testament to my own perseverance and determination to complete this Ph. D. Because of this, I want to first acknowledge my own perseverance and will to keep going. I've finally made it.

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| | |
|---|---|
| ADP: | American Diploma Project |
| AMTE: | Association of Mathematics Teacher Educators |
| ASSM: | Association of State Supervisors of Mathematics |
| AYP: | Adequate yearly progress |
| CCSSI: | Common Core State Standards Initiative |
| CCSSM: | Common Core State Standards for Mathematics |
| CCSSO: | Council of Chief State School Officers |
| CFCF: | Critical Friend Conversation Form |
| CPM: | College Preparatory Mathematics |
| ECA: | End of Course Assessment |
| GCF: | Greatest common factor |
| ICTM: | Indiana Council of Teacher of Mathematics |
| IMP: | Interactive Mathematics Program |
| IRB: | The Institutional Review Board |
| MEA: | Model Eliciting Activities |
| NAEP: | National Assessment of Educational Progress |
| NCLB: | No Child Left Behind Act |
| NCSM: | National Council of Supervisors of Mathematics |
| NCTM: | National Council of Teachers of Mathematics |
| NGA: | National Governor's Association Center for the Best Practices |
| NRC: | National Research Council |

NSF: National Science Foundation

PSC: Problem Solving Components

PSSM: Principles and Standards for School Mathematics

SMC: Standards of Mathematical Content

SMP: Standard of Mathematical Practice

TRF: Teacher Reflection Form

# ABSTRACT

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Utilizing action research as the methodology, this study was developed with the ultimate goal of describing and reflecting on my implementation of one aspect of the *Common Core State Standards for Mathematics (CCSSM)* in an algebra classroom. This implementation focused on the Problem-Solving Standard of Mathematical Practice (SMP) as described in *CCSSM* (Making sense of problems and persevere in solving them). The research question that guided my work was the following: How is the *Common Core State Standards for Mathematics* (*CCSSM*) Problem-Solving Mathematical Standard enacted in an algebra class while using a *Standards*-based curriculum to teach a quadratics unit?

I explored this by focusing on the following sub-questions:

- Q1. What opportunities to enact the components of the Problem-Solving Mathematical Standard are provided by the written curriculum?

- Q2. In what way does the teacher's implementation of the quadratics unit diminish or enhance the opportunities to enact the components of the Problem-Solving Mathematical Standard provided by the written curriculum?

- Q3. In what ways does the teacher's enactment of problem-solving opportunities change over the course of the unit?

Reviewing the literature related to the relevant learning theories (sociocultural theory, the situated perspective, and communities of practice), I outlined the history of *CCSSM*, National Council of Teachers of Mathematics *(*NCTM), National Research Council (NRC), and the *No Child Left Behind Act of 2001*. Exploring the details of *CCSSM*'s Standards of Mathematical Content (SMCs) and Standards of Mathematical Practice (SMPs), I discussed problem solving, the Problem Solving Components (PSCs) listed in the Problem-Solving SMP of *CCSSM*, teaching through problem solving, and *Standards*-based curricula, such as *College Preparatory Mathematics (CPM)* which is the algebra curricula I chose for this study.

There are many definitions of the construct problem solving. *CCSSM* describes this construct in unique ways specifically related to student engagement. The challenge for teachers is to not only make sense of *CCSSM*'s definition of problem solving and its components, but also to enact it in the classroom so that mathematical understanding is enhanced. For this reason, studies revealing how classroom teachers implemented *CCSSM*, especially in terms of problem solving, are necessary.

The Critical Theoretic/Action Research Paradigm is often utilized by researchers trying to improve their own practice; thus, I opted for an action research methodology because it could be conducted by the practitioner. These methods of data collection and analysis were employed in order to capture the nature of changes made in the classroom involving my teaching practice. I chose action research because this study met the key tenets of research in action, namely, a collaborative partnership concurrent with action, and a problem-solving approach.

While I knew how I wanted to change my classroom teaching style, implementing the change was harder than anticipated. From the onset, I never thought of myself as an absolute classroom authority, because I always maintained a relaxed classroom atmosphere where students were made to feel comfortable. However, this study showed me that students did view my presence as the authority and looked to me for correct answers, for approval, and/or for reassurance that they were on the right track. My own insecurities of not knowing how to respond to students in a way to get them to interact more with their group and stop looking to me for answers, while not being comfortable forcing students to talk in front of their peers, complicated this study. While it was easy to anticipate how I would handle situations in the classroom, it was hard to change in the moment.

The research revealed the following salient findings: while the written curriculum contained numerous opportunities for students to engage with the Focal PSCs, the teacher plays a crucial role in enacting the written curriculum. Through the teacher's enactment of this curriculum, opportunities for students to engage with the Focal PSCs can be taken away, enacted as written, or enhanced all by the teacher. Additionally, change was gradual and difficult due to the complexities of teaching. Reflection and constant adapting are crucial when it comes to changing my practice.

As a classroom teacher, I value the importance of the changes that need to be made in the classroom to align with *CCSSM*. I feel that by being both a teacher and a researcher, my work can bridge the gap between research and classroom practice.

# CHAPTER 1: INTRODUCTION

The purpose of this dissertation is to describe the implementation of a critical aspect of successful teaching and learning of mathematics in today's highly demanding academic environment, namely, the problem-solving practices outlined in the *Common Core State Standards for Mathematics (CCSSM)* Standard of Mathematical Practice (SMP) (National Governors Association [NGA] for Best Practices & Council of Chief State School Officers [CCSSO], 2010). As will be shown, application of the SMP redirects the task of improving the proficiency of the mathematics teaching, and learning process, to include both teacher and student instead of the teacher-dominant model widely accepted and used by many school districts.

For more than two decades, The National Council of Teachers of Mathematics (NCTM) has endorsed problem solving in the mathematics classroom. NCTM has a series of standards publications (e.g., NCTM, 1980, 1989, 2000) suggesting that mathematics instruction should move away from the traditional format of teacher lecture followed by independent student work time and move toward a classroom environment that allows more time for student problem solving to take place. Additionally, the National Research Council (NRC) (1989) promoted problem solving in the mathematics classroom in *Everybody Counts* and emphasized that opportunities for learning important and engaging mathematics should be provided for *all* students in *Adding It Up* (NRC, 2001).

With the twenty-first century came the technological era, driving the need for an intellectual society literate in quantitative critical thinking skills. NCTM (1989, 2000) proposed that algebra was important for both postsecondary education and the workforce. Specifically, NCTM (2008) defined algebra as “a way of thinking and a set of concepts and skills that enable students to generalize, model, and analyze mathematical situations.” The *Common Core State Standards Initiative* (*Common Core State Standards Initiative* [*CCSSI*], 2010) stated that mathematical understanding includes the, “ability to justify, in a way appropriate to the student’s mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from” (p. 4). In other words, quantitative critical thinking skills should be learned through conceptual understanding because studies have shown that students retain the mathematics more and understand more conceptually when the mathematics is learned in this manner.

The need for a society that is literate in quantitative critical thinking and problem-solving skills fuels the argument of "algebra for all" (NCTM, 2000; NRC, 2001). In the United States, mastery of algebraic content has become a “gate-keeper” to post-secondary education and career opportunities and a “civil right” for all students (Moses & Cobb, 2001). Mathematics is a gatekeeper because many career options are currently available only to students who have taken advanced mathematics courses. Additionally, institutions of higher education have increased the number of mathematics courses required for admission. Thus, mathematics is a civil right for all students to provide equality for all students. To address this issue, many states require all students to enroll in and successfully complete an algebra course as a requirement for attaining a high school diploma (Reys, Dingman, Nevels, & Teuscher, 2007).

Unfortunately, the solution of mandating completion of an algebra course had some drawbacks. For example, after nationwide implementation of *No Child Left Behind*

(*NCLB, 2001*) and its push for “algebra for all” became a reality, many individual states began to generate their own mathematical standards. Variation in the mathematical content of these standards and accompanying curriculum was extensive with few commonalities (Smith & Tarr, 2011). As a result, student algebraic knowledge and preparedness for college and career varied greatly from state to state. *CCSSM* emerged as a means of solving this problem of variation in student preparedness (Hirsch & Reys, 2009).

*CCSSM* offered a common set of mathematical standards for all states. While *CCSSM* was initially adopted as the K-12 mathematics curriculum in 44 states and the District of Columbia, this number fluctuates as several states have opted to rescind their adoption or have adopted their own variation of *CCSSM*. Reasons for inaction, recension, or variation on the *CCSSM* offer differ. Some felt *CCSSM* was an attempt at a national curriculum, which they opposed for various reasons including the belief that curriculum should be decided at a local level, or that curriculum should be differentiated and adapted to meet the specific needs of the students in a particular school. Other states, like Minnesota, felt its own standards were more rigorous and, because it is typically considered a top-rated state in terms of education, there was no need to change math standards which had just been revamped (Weber, 2012). Still, others felt that the costs imposed by *CCSSM* were too great.

Given the attention in *CCSSM* to mathematical practices, conceptual understanding, and real-life mathematical contexts, problem solving is arguably at the heart of *CCSSM*. Teaching algebra using problem-solving methodology requires a shift from traditional teacher-centered techniques to a *Standards*-based approach of mathematics teaching and learning (consistent with NCTM's vision and standards (Principles and Standards, NCTM)) as noted in programs such as *Core-Plus Mathematics Project* (Core-Plus Mathematics Project) (developed at Western Michigan University), *Connected Mathematics Project* (What is CMP) (developed at Michigan State University), and *College Preparatory Mathematics* (*CPM*) ("CPM Educational Program") (an Eisenhower grant-funded mathematics project). These programs share a common formulation, namely that lessons contain problems for at least three different purposes: (1) to introduce students to unfamiliar material, (2) to review mathematical content from prior courses, and (3) to highlight connections among mathematical topics and the so-called "real" world (Heibert, 1999). Using this approach, students co-construct knowledge by "inventing" solutions to problems and "discovering" the mathematical content without being lectured at by the teacher (Stein & Smith, 2010). The ensuing results yield student exposure to the same mathematical content as in a traditional teacher-centered classroom, but in new and more engaging ways.

According to NCTM (2000), problem solving can be considered as a process of engaging in a task for which the solution method is not known in advance. In order to find a solution, students may draw on their knowledge, and through this process, they will often develop new mathematical understandings (p. 52). Although the concept of problem solving has been studied by mathematics education researchers, the majority of the research has focused on students' mathematical understanding (e.g., Cai, 2000; Cai, 2003; Gallagher, De Lisi, Holst, McGillicuddy-De Lisi, Morely & Cahalan, 2000; Iversen & Larson, 2006; Pugalee, 2001; Wong, Lawson & Keeves, 2002). Few research studies about problem solving have been directed to both classroom practices of teachers and the students' interaction with the subject of mathematics (e.g., Ho & Hedberg, 2005). Because of the relative novelty of *CCSSM*, I was unable to locate studies focused on the implementation of the Problem-Solving Standard for Mathematical Practice (SMP) in *CCSSM*, although studies that explore math teachers' practices in general do exist and have widely been studied (i.e. Cheung & Slavin, 2013; Hattie, 2008; Rakes et. al., 2010; Seidel & Shavelson, 2007).

Problem solving, one of eight SMPs in the *CCSSM* pedagogy structure, is both a relevant and essential element for ensuring the success of students who choose either the college path or the career path. Many definitions of the construct of problem solving exist. *CCSSM* breaks down this construct in unique ways depending on the manner in which student engagement is carried out. According to *CCSSM*, mathematically proficient students will engage in problem- solving activities as outlined in Table 1.

Table 1. Problem-Solving Components of Mathematically Proficient Students (NGA & CCSSO, 2010)

| Component Number | Problem-Solving Component (PSC) |
|---|---|
| 1 | Explain to themselves the meaning of a problem |
| 2 | Look for entry points to a problem's solution |
| 3 | Analyze the givens, constraints, relationships, and goals |
| 4 | Make conjectures about the form and meaning of the solution |
| 5 | Plan a solution pathway |
| 6 | Consider analogous problems and try special cases and simpler forms |
| 7 | Monitor and evaluate their progress and change course if necessary |
| 8 | Explain correspondences between representations |
| 9 | Check their answers and ask themselves does this make sense |
| 10 | Understand the approaches of others |

The challenge for teachers is to not only make sense of *CCSSM*'s definition of problem solving and its components, but to apply these in the classroom so that mathematical understanding is enhanced. Because of the importance of enhancing student mathematical understanding, studies which investigate how classroom teachers implement *CCSSM*, especially in terms of problem solving, are necessary. As a high school teacher, responsible for teaching algebraic content included in *CCSSM*, I studied my own classroom practices as I implemented a *Standards*-based curriculum with emphasis on problem solving. For the purpose of this study, I served in dual capacities; I was the teacher of the class that learned algebra through problem solving, and I was the researcher for the study, a methodology commonly referred to as “action research.” By studying my own teaching practices, and utilizing the action research approach, my goal was to make observations and document notable changes made to my teaching that ultimately lead to increased student engagement in and learning when using the problem-solving methodology selected for the study. I hypothesized that both student and teacher empowerment would result as students discovered and found new meaning for the need to develop understanding and proficiency in mathematics. Additionally, I hypothesized that the teacher would most likely be both encouraged and empowered by the resulting improvements in student learning performance. Because of the systematic documentation, and analysis of changes made in my teaching practice, I sincerely hoped and confidently expected that this study would serve to help my colleagues and make a worthwhile contribution to on-going research in mathematics education.

## Purpose of the Study and Research Questions

As noted, the purpose of this study was to use action research methodologies to implement the problem-solving aspect of *CCSSM* into an algebra classroom. One underlying research question will serve to guide this effort. The question is:

> How is the *Common Core State Standards for Mathematics* (*CCSSM*) Problem-Solving Mathematical Standard enacted in an algebra class while using a *Standards*-based curriculum to teach a quadratics unit?

I explored this by focusing on the following sub-questions:

- Q1. What opportunities to enact the components of the Problem-Solving Mathematical Standard are provided by the written curriculum?
- Q2. In what way does the teacher's implementation of the quadratics unit diminish or enhance the opportunities to enact the components of the Problem-Solving Mathematical Standard provided by the written curriculum?
- Q3. In what ways does the teacher's enactment of problem-solving opportunities change over the course of the unit?

Because this study serves as one example of how a teacher began to implement *CCSSM* in terms of the Problem-Solving SMP, it should be pointed out that my research questions address what seemingly can be interpreted as the larger problem that exists: how can a classroom teacher implement problem solving based curriculum? Using the *Standards*-based curriculum helped address the notion of students co-constructing knowledge through a problem-solving approach. It should also be pointed out that my literature review prior to setting up my study did not yield any studies directed at the implementation of the Problem-Solving SMP. By addressing the implementation of the Problem-Solving SMP, this dissertation adds to the existing field of research regarding *CCSSM* implementation.

## Significance of the Study

Implementing *CCSSM* in an algebra classroom is still considered a new approach, as *CCSSM* was not mandated to be tested in the high school curriculum until the 2014-2015 academic year (CCSSI, 2010). Although there is minimal research available regarding the implementation of *CCSSM* in a high school algebra class, research that focused on problem solving in the classroom is abundant and provides insight into a multitude of research approaches. This study contributes to the field of mathematics education in two unique ways: (1) it provides evidence of how a teacher implements *CCSSM* in an algebra classroom through a focus on problem solving and (2) how and why a teacher makes decisions to change her teaching practices as a problem-solving approach to implementing *CCSSM* is adopted. My goal is to provide both the mathematics research community and high school mathematics teachers with details of one such implementation. The focus of future research could be directed at teachers who are addressing the problem-solving component of these standards in different courses and at different grade levels. Practicing classroom teachers may be interested in this research as they can seek to explore actual outcomes of *CCSSM* implementation in the classroom when focusing on problem solving and what changes occurred after implementing the changes. This action research with the teacher serving as both the teacher and the researcher may be the motivation necessary for a classroom teacher to begin implementation. Finally, the authors of *CCSSM* stand to gain valuable insight from first-hand experience of how their intended curriculum is currently being implemented in an algebra classroom. This type of knowledge can provide worthwhile feedback to the *CCSSM* authors because it serves to address important issues including how teachers are taking the intended curriculum and applying it in the classroom, what changes, clarifications, or further explanations may be needed.

## A Personal Narrative

My personal experiences have helped shape this study and, undoubtedly, influenced the role I played as both researcher of this study and as a teacher in a high school algebra classroom. Throughout the course of this study, I took appropriate measures to journal all thoughts and experiences related to my research questions. These journal entries contained my reflections on both my role as a teacher and as a researcher. As a result, these reflections were intended to support the development of my own narrative to disclose to the reader personal experiences in this novel (for me) approach for teaching algebra.

As a classroom teacher, I value the importance of changes needed to align with the *CCSSM* initiative. Considering, from my own teaching experience, meaningful changes in the teaching arena rarely happen instantly, change in my teaching practice will be gradual. I expect this gradual change will allow the students to adapt to the new standards, as well as the new teaching practices. Finally, as a researcher, I am interested in documenting my experience and findings and disseminating them to other teachers and researchers. I feel that by being both the teacher and researcher, my action research approach can bridge what could otherwise appear as an elusive gap between research and classroom practices.

**Summary**

After familiarizing myself with the development of the *CCSSM* mandate for classroom delivery of mathematics, action research methodologies were described for this study for the purpose of implementing the Problem-Solving SMP of *CCSSM* into an algebra classroom.  Relevant research questions were developed to identify the logistics and the impact of implementing the *CCSSM* Problem-Solving SMP.  In large part, the study is significant because it provides evidence showing how a teacher implements *CCSSM* in an algebra classroom with the focus being on problem solving.  My own personal narrative factored into the significance of the study because I was both the teacher implementing a new approach, and the researcher studying its implementation.

# CHAPTER 2: LITERATURE REVIEW

This chapter provides a review of the literature related to the study's theoretical and conceptual frameworks. My research question and problem are situated within the constructivist paradigm. My inquiry aim is one of understanding (Denzin & Lincoln, 2011); I am connecting "action to praxis and build[ing] on antifoundational arguments, while encouraging experimental and multivoiced texts" (Denzin & Lincoln, 2011, p. 92). This chapter consists of two discrete sections. The first section outlines structures used to frame the study: sociocultural theory, situated perspective, and communities of practice. The second section contains a literature review detailing the history of the development of *CCSSM*, the construct of problem solving, as related to curriculum development, and the methodology of action research. Specifically, it includes research findings related to *Common Core State Standards for Mathematics* (*CCSSM*), *Standards*-based mathematics curricula and action research methodology.

## Relevant Learning Theories

According to Vygotsky (1978), learning is sociocultural, and in this study, I created a sociocultural setting where students and teacher concomitantly built knowledge through sharing and communicating ideas. The sociocultural setting in this case was my classroom.

### Sociocultural Theory

Vygotsky (1978) proposed that there is more to learning than what is in the mind. He focused on the interactions between individuals, specifically the interactions among classmates. According to his theory, individual development cannot be understood without reference to the social and cultural context within which it is embedded; higher-order thinking develops out of social processes. He postulated that mental tools (tools beyond memorization of facts and procedures) extend our mental abilities, which in turn enable us to solve problems and create solutions (Vygotsky, 1978). For example, when children expand their mental tools, they do not apply a memorized algorithm, taught to them with a certain set of rote examples; instead, they are able to go beyond this type of mental processing and solve problems that are rich in context as well as open-ended in nature. Sociocultural theory is grounded in the idea that culture determines the "what" and "how" a person learns. The culture being referred to is the school and classroom culture. Furthermore, sociocultural theory emphasizes the roles that participation in social interactions and activities, organized culturally, play in influencing psychological development and is more concerned with learning as an act of enculturation rather than the mental representations of an individual (Scott & Palincsar, 2013).

Classroom environments shape students' social and individual development. According to Vygotsky, "social relations or relations among people genetically underlie all higher functions and their relationships" (Vygotsky, 1981, p. 163). Furthermore, "higher forms of mental activity are derived from social and cultural contexts and are shared by members of those contexts because these mental processes are adaptive" (Berk & Winsler, 1995, p. 12). Therefore, in classrooms where the norm is problem solving, students will be better equipped to solve problems because they will have developed the higher mental activities needed to do so in collaboration with their peers.

According to Inglis & Foster (2018), when the theoretical framework of a research study is sociocultural theory, it is important to observe classroom interactions. As will be shown later, I approached this study with a sociocultural perspective in order to examine how I, the teacher, facilitated an environment for students to learn interactively. While I did not observe the classroom interactions of students to study the learners, instead, I observed students' interactions as a result of my facilitation. According to the sociocultural perspective, "besides presenting necessary information, extended opportunities for discussion and problem solving in the context of shared activities are essential for learning and development" (Berk & Winsler, 1995, p. 113). This emphasizes the importance of social activities for students to learn in mathematics. In this study, classroom learning will be student-centered, which means that students will be interacting with each other and with the mathematics content, while simultaneously discussing and solving an assigned problem. Students will have expectations beyond memorizing a set of facts and skills; they will be provided opportunities to be actively engaged with the mathematics through collaborative learning that is focused on the use of native resources and tools of the students' collective minds to solve the problem. They will learn to solve problems by focusing on the use of mathematical information with a specific goal in mind. Through social interactions, students will develop problem-solving skills instead of passively receiving information through lecture-words as would be provided in a teacher-centered classroom.

**Situated Perspective**

The situated perspective emphasizes that, "much of what is learned is specific to the situation in which it is learned" (Anderson, Reder, & Simon, 1996, p. 5), accentuating the mismatch between typical school situations and real-world situations like the workplace. For example, "there are vast differences between the ways high school physics students participate in and give meaning to their activity and the way professional physicists do" (Lave & Wenger, 1991, p. 99). This mismatch is highlighted in *CCSSM* as one of its stated goals is to prepare students to successfully enter either institutions of higher learning or the workplace. The *CCSSM* initiatives align with the situated learning perspective in that both the institutions of higher learning and the workplace emphasize that more attention needs to be given to the relationship between what is taught in the classroom and what is needed outside of the classroom. Lave and Wenger (1991) propose that learning should be viewed as a social process where knowledge is co-constructed, suggesting that learning occurs when situated in specific context and embedded within a particular social environment. Furthermore, students must play an active role to help themselves gain knowledge and understanding (Kucuk, 2018). In this study, my goal is to provide students with problems situated in real contexts, in order to facilitate their application to situations outside the classroom. According to Lave & Wenger (1991), situated learning affords an individual the opportunity to learn by socialization, visualization, and imitation with an emphasis on learning in an authentic context. For this reason, problems presented to students must be in a realistic context. Solving problems in an authentic context affords students the opportunity to learn by socialization; having students use manipulatives and concentrate on the multiple representations of the problem's mathematics further ignites students' learning. The researchers further mentioned that learning begins by trying to solve problems as students are engaged in social interactions with classmates and engaged in peer collaboration.

This means that the mere act of collaborating with peers about the authentic problem leads to learning mathematics. Therefore, when learning is problem centered, in a community of practice, comprised of novice and expert members, attempting to solve problems ensures that learning occurs.

## Communities of Practice

The idea of communities of practice has found its way into the everyday language of most educators (Wenger, 2010). Communities of practice may take on a variety of forms, but they all share a basic structure. A community of practice is a "unique combination of three fundamental elements: a domain of knowledge, which defines a set of issues; a community of people who care about this domain; and the shared practice that they are developing to be effective in their domain" (Wenger, McDermott, & Snyder, 2002, p. 27). In this study, an Algebra I class served as the domain; the shared area of interest consisted of students improving their own problem-solving skills and earning a passing classroom grade, as well as me improving my practice of facilitating the problem-solving activities. The community component necessitated that members interacted and engaged in communal activities, supported and encouraged one another, and shared information. The structure of the *Standards*-based curriculum encouraged these relationships as students and teacher worked together to solve problems. Finally, I expected that the shared collection of resources over time would lead to what others refer to as collaborative problem solving (De Boeck & Scalise, 2019).

Generally speaking, a community of practice is defined as, "a collection of people who engage on an ongoing basis in some common endeavor" (Eckert, 2006, p. 683). A community of practice develops certain ways of doing, viewing, and talking about things.

There are two conditions that are crucial in the development of a successful community of practice. These are: shared experiences over time and a commitment to shared understanding. Where these ideals exist, members of the community of practice will engage in mutual sense-making related to their shared interest (Eckert, 2006). In terms of this study, the community of practice was my algebra classroom, the members of the community were my students and me (their teacher). We developed certain ways of doing mathematics (i.e. *Standards*-based learning, group work), along with tackling common perceptions of viewing mathematics from a perspective of discovery instead of a learning from a traditional mathematics lecture. We discussed the mathematics, both in small groups and as a whole class, through respectful exchanges that were both meant to be meaningful and pertinent. In summary, the community of practice consisted of the students in my Algebra I classroom, using the *Standards*-based curriculum, structured in a way that required students to interact in order to solve problems in a collaborative manner.

A common trait of communities of practice is one that features a learning curriculum rather than a teaching curriculum. The difference between the two characteristics is that in a learning curriculum everyday practices are viewed from the perspective of the learners. Whereas, in a teaching-curriculum setting, newcomers are instructed on how to do things and only get an external view of the knowledge (Lave & Wenger, 1991). This difference can be seen in the classroom setting. Lecture-based classrooms are a teaching curriculum because they model the typical same routine every day; class begins by reviewing the previous night's homework and is followed by direct instruction in which the teacher, as the authority figure, tells students the information about the new material. Finally, class concludes by having students practice the processes just explained. In contrast, *Standards*-based classrooms are typically those of a learning curriculum. In this type of classroom, students are actively engaged in the mathematics, working collaboratively to solve authentic problems on their own, with the facilitation of the class from the teacher. This study features a shift in learning mathematics from the teaching curriculum mode where students are passive learners to the learning curriculum mode where students are engaged in authentic problem-solving activities.

Furthermore, in a classroom that is a learning curriculum, learning, as stated by Pryko, Dorfler, and Edes (2017), "entails change in one's identity, as well as the (re-) negotiation of meaning of experience" (p. 391). This study required both the students and me (the teacher) to change our traditional identities; namely, that the teacher was no longer the "giver" of knowledge, and the students were no longer the "receivers" of said knowledge. Instead, the teacher and students formed a community of practice and discovered the mathematics through complex, engaging problems. This re-negotiation of identity roles was critical to the mathematical learning experience and the formation of communities of practice where students were no longer passively listening to my lectures about the mathematics they should know, but instead, they experienced mathematics through problem solving, learned to become better problem solvers, and learned the mathematics at the same time. Students had to learn not to rely solely on me as the authority of the classroom and I had to learn to allow students to work collaboratively, without me, so that their learning experiences were maximized.

## Common Core State Standards for Mathematics

Before examining *CCSSM,* I present a brief overview of the activities involving the development and implementation of educational standards. NCTM (NCTM, 1980, 1989, 1991, 1995, 2000) and the National Research Council (NRC) (NRC, 1989, 2001) formulated a vision for school mathematics. However, states, until *CCSSM*, were left on their own to make the vision a reality (Kendall, 2011) and individual states' interpretation of this vision often varied significantly (Reys, 2006). The *No Child Left Behind Act (NCLB)* (2001) increased schools' accountability for their students' mathematics learning (Dingman, Teuscher, Newton & Kasmer, 2012) by requiring all states to have standards and aligned assessments. Consequently, *CCSSM* was developed to provide greater focus and coherence for school mathematics in the United States (NGA & CCSSO, 2010) and to make mathematics more challenging for all students (Confrey, 2007; Confrey & Krupa, 2012), with the goal of adequately preparing students to enter college or the workforce. The sections that follow describe the development of *CCSSM* and the specific influences of NCTM and NRC that have shaped *CCSSM*.

### National Council of Teachers of Mathematics

Since the 1980s, NCTM has made recommendations for improving the teaching and learning of mathematics in the United States. Its first such publication was *An Agenda for Action: Recommendations for School Mathematics of the 1980's* (NCTM, 1980) which presented a "responsible and knowledgeable viewpoint of the directions mathematics programs should be taking in the 1980s" (p. i). Its release was considered NCTM's "best-considered advice to society concerning future direction for educational programs" (p. i) and represented NCTM's efforts to improve mathematics education for

American students. *An Agenda for Action* suggested that mathematics should shift to a problem-solving approach.

Almost a decade later, NCTM published *Curriculum and Evaluation Standards for School Mathematics* (hereafter, *Curriculum Standards*) (NCTM, 1989). This document further developed the ideas advanced in *An Agenda for Action,* by stating that learning should be an active process rather than one of memorization and practice; these standards were grounded in the assumption that the final product of education should be a student who, when faced with an unconventional problem, can logically formulate a hypothesis and successfully use mathematical methods to solve the problem. *Curriculum Standards* further urged practitioners to shift their focus towards problem solving and away from lectures.

In order for standards to remain viable, "the goals and visions they embody must periodically be examined, evaluated, tested by practitioners, and revised" (NCTM, 2000, p. x). Although consistent with the *Curriculum Standards* (NCTM, 1989) *Principles and Standards for School Mathematics* (hereafter, *PSSM*), (NCTM, 2000) was based on a decade of research and practice that was perceived by critics to be missing from the previous version (Schoenfeld, 2002). This research promoted problem solving in the mathematics classroom, and a shift to a problem centered classroom. By including examples of students' work and classroom episodes, this publication presented a comprehensive vision for mathematics education where, "all students have access to high-quality, engaging mathematics instruction" (NCTM, 2000, p. 3). NCTM's *PSSM* offered "descriptions of what mathematics instruction should enable students to know and do" (p. 28) as a result of participation in a quality K-12 mathematics program.

**National Research Council**

Released in the spring of 1989, *Everybody Counts: A Report to the Nation of the Future of Mathematics Education* scrutinized mathematics education in the United States from kindergarten through graduate school. This document identified the perceived areas for improvement of the system at the time and called for an implementation of a national strategy aiding states and localities in improving the quality of mathematics education (NCTM, 1989). *Everybody Counts* suggested that the study of mathematics could further the development of critical habits of the mind, "to distinguish evidence from anecdote, to recognize nonsense, to understand chance, and to value proof" (NRC, 1989, p. 8). Persevering through problem solving and shifting away from lecture-based classrooms would improve the U.S. system of education. A critical shift away from thinking that 'not all students can do math' to one of 'everyone can do math' was called for and the public was reminded that "children respond to expectations of their parents and teachers" (p. 11). In response, teachers were encouraged to embolden students to work hard and persevere because "hard work pays off" (p. 11).

Likewise, in later work by the NRC, the Mathematics Learning Study Committee, established in 1998 and chaired by Jeremy Kilpatrick, published *Adding It Up: Helping Children Learn Mathematics* (NRC, 2001). Driven by the premise that all citizens must know basic mathematics to fully participate in society, this report emphasized that mathematics should no longer be restricted to just a select few American students who have what is perceived as an ability to succeed. Geared for K-8 students, but applicable across K-12, this report addressed the question held by many mathematics educators of what must be done to boost the mathematical performance of all students. As a result of their analyses, Kilpatrick and his committee chose *mathematical proficiency* to represent what they proposed as necessary competencies for all students of mathematics. Mathematical proficiency was presented as a compilation of five interwoven and interdependent strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. These strands could not be evaluated or assessed as simply present or absent but must rather be developed over time. Due to the five components of mathematical proficiency being interwoven, students who had the opportunity to engage with all strands, often in integrated ways, became competent with each strand individually and thus became mathematically proficient. Mathematically proficient students would persevere in problem solving as well as learn mathematics through authentic problems.

**No Child Left Behind Act of 2001**

*NCLB* was introduced by the United States Department of Education in 2001 (*NCLB*, 2001). The initiative supported the implementation of high-quality state standards and assessment systems and became the national focus in education because, “nearly 70 percent of inner city fourth graders are unable to read at a basic level on national reading tests” while “high school seniors trail students in Cyprus and South Africa on international math tests” and “nearly a third of our college freshmen find they must take a remedial course before they are able to even begin regular college level courses” (*NCLB*, 2001, p. 4). According to *NCLB*, if each state had a system of standards along with ways to measure progress, and if states were held accountable under this policy, then, “states can ensure that no child lacks the basic skills needed to succeed in our increasingly competitive, global economy” (Department of Education, 2009, p. 1).

States were required to act as, "*NCLB* legislation accelerated state efforts to write and adopt standards on a grade-by-grade basis as well as strengthened the focus on assessment" (Dingman, Teuscher, Newton & Kasmer, 2012, p. 2). Mandates from *NCLB* required that all students demonstrate proficiency which allowed individual states to set independent proficiency standards.

Moreover, *NCLB* required states to introduce school accountability systems that applied to all public schools and students in the state. These accountability systems required annual testing of public school students in reading and mathematics in grades 3 through 8 (and at least once in grades 10 through 12) and rating of school performance, both overall and for key subgroups, with regard to whether they were making adequate yearly progress (AYP) toward the state's proficiency goals. *NCLB* required that states introduce sanctions and rewards for every school based on their AYP status with severe repercussions for persistently low-performing schools that received Title I aid.

As a result of *NCLB*, states developed and implemented challenging academic content standards and administered annual assessments aligned to the standards. In essence, states were required to measure the achievement of students against the state standards. Some states implemented their own versions of curriculum standards and methods of annually assessing these standards. Although NCTM's standards (e.g., 1989, 2000) influenced the content of state standards (Reys, 2006), states generally worked independently of each other to create their own standards. Suggestions in states' standards guides were not always consistent with the recommendations set forth by the NRC and NCTM, as no law mandated such alignment and states typically included more standards than could realistically be addressed in the available instructional time

(Kendall, 2011). In 1995 and 1997, The Council of Chief State School Officers (CCSSO) published reports summarizing the content and quality of state curriculum standards. The consensus was that, although remarkably different, state standards generally pushed for greater emphasis on higher level mathematics for all students (Reys, 2006), which is what *NCLB* was trying to ensure.

In an effort to make college and career readiness a priority in the United States, *Achieve* launched the American Diploma Project (ADP) in 2005. This project was directed at improving postsecondary preparation by aligning high school standards, graduation requirements, and assessment and accountability systems with the demands of college and careers (Achieve, 2012). *Achieve* then partnered with the National Governors Association (NGA) and the CCSSO to develop internationally benchmarked, college and career ready, standards. Thus, in 2009 the Common Core State Standards Initiative was introduced. In the development of *CCSSM*, NCTM's *PSSM* and NRC's *Adding it Up* played an essential role in developing the Standards of Mathematical Practice (SMPs) (e.g., Make Sense of Problems and Persevere in Solving Them; Reason Abstractly and Quantitatively; and Construct Viable Arguments and Critique the Reasoning of Others). According to a *CCSSM* document "these practices rest on important 'processes and proficiencies' with longstanding importance in mathematics education" (NGA & CCSSO, 2010). After drafts of these standards underwent critique and revision, NGA, in conjunction with CCSSO published *CCSSM* in June 2010. With this publication, *CCSSM* became the next step in the evolution of mathematics teaching and learning in the United States.

**Support and Opposition to the Implementation of *Common Core State Standards of Mathematics***

It is reasonable to expect that with any major educational change comes controversy. Soon after the release of *CCSSM*, NCTM, the National Council of Supervisors of Mathematics (NCSM), the Association of State Supervisors of Mathematics (ASSM), and the Association of Mathematics Teacher Educators (AMTE) issued a joint statement expressing their support for *CCSSM* stating that *CCSSM* is a "welcome(d) milestone" to the standards movement that began decades earlier (NCTM, NCSM, ASSM, & AMTE, 2010, p. 1). Support also came from many voices in the private sector and institutes of higher education, according to Michael Cohen, president of *Achieve* (NCTM, NCSM, ASSM, & AMTE, 2010).

However, not everyone expressed support of the new mathematics standards. Sol Garfunkel, a member of the writing team for *CCSSM*, argued that high-stakes tests based on the standards would dampen the mathematics curriculum for a decade and continue to widen the achievement gap (Garfunkel, 2012). According to Ujifusa (2012), opponents of *CCSSM* viewed the document as a pipeline for the private sector to access taxpayer dollars and suggested that the government coerced states by linking the federal Race to the Top grants to the adoption of *CCSSM.* Heck, Weiss, & Pasley (2011) also highlighted potential areas of concern regarding *CSSM*, especially if *CCSSM* is only implemented superficially (e.g., the current curriculum is claimed to be aligned with *CCSSM*, but no changes to the classroom, such as implementing problem solving, have been made). Melton (2011) offered recommendations to overcome these challenges. Some believed that it was most important for *CCSSM* to remain a living document (Garfunkel, Hirsch, Reys, Marrongelle, & Sztajn, 2011), which would be continuously reviewed and updated by research as it was made available. As such, the standards would not become an outdated document but would remain consistent and up to date with current research in the educational field. As of 2019, the support and opposition of the *CCSSM* standards was still present. While some argue against *CCSSM*, especially in the early grades (Carlsson-Paige, 2015), Clements, Fuson, and Sarama (2019) maintained that *CCSSM* is developmentally appropriate and were based on research.

**Overview of Common Core State Standards of Mathematics**

According to Phil Daro, one of the authors of *CCSSM*, "the goal of the standards is to answer the question 'What is the math I want students to walk away with?'" (Confrey & Krupa, 2010, p. 3), with the answer to the question being stated as follows: the aim of *CCSSM* is to improve students' mathematical outcomes through clarity and specificity – a coherent set of standards (NGA & CCSSO, 2010). Unlike the mathematical content of some states' standards typically described as 'a mile wide and an inch deep,' *CCSSM* assigns fewer mathematical topics at each grade level, with the expectation of considerably greater depth in coverage while emphasizing coherence through the connection of ideas across and within topics (NGA & CCSSO, 2010). According to Confrey and Krupa (2010), *CCSSM* is grounded in the following key assumptions: (a) 100% mastery of the preceding year's standards, (b) standards are high points, not finish lines or curriculum, and (c) the grain size for effective change should be at the chapter or unit level rather than at the lesson level. These characteristics begin to streamline the content and expectations of all mathematics students in states who have adopted *CCSSM*. With less review of previously taught material each year, teachers can spend more time on each topic, thus giving students a more in-depth understanding and thus mastery of the material.

**Standards of mathematical content.** One major part of *CCSSM* is the Standards of Mathematical Content (SMCs). They are described as being, “fewer, higher, clearer” (Kendall, 2011, p. 25) than existing state standards. This means that there are less standards at each grade level, the standards are asking students to use more high-level thinking skills and the standards are clearer to teachers, relieving the confusion of previous state standards. For each grade level, an overview page is provided which contains domains (larger groups of related standards) and clusters (groups of related standards) for the purpose of briefly familiarizing teachers with content expectations, while reminding them of the Standards for Mathematical Practice.

Subsequent details follow for each domain, cluster, and specific standards (definition of what students should understand and be able to do). At the high school level, standards are listed in conceptual categories: (a) number and quantity, (b) algebra, (c) functions, (d) modeling, (e) geometry, and (f) statistics and probability (NGA & CCSSO, 2010). Together, the domains, clusters, and standards provide teachers with what they should teach while leaving the “how” up to the individual teacher’s discretion (Kendall, 2011). Varying by grade level, the SMCs are a “balanced combination of procedure and understanding” (NGA & CCSSO, 2010, p. 8). The high school standards are not grouped by traditional course paths such as Algebra I, Geometry, and Algebra II; instead, the flexible grouping of Number and Quantity, Algebra, Functions, Modeling, Geometry, and Statistics and Probability allows for a variety of non-traditional paths, such as integrated curriculum pathways, to be implemented. However, many states have augmented the standards by grouping them into traditional course paths or adding standards (Reys et al., 2013), while other states, such as Indiana, have made *CCSSM* their own by making their own standards (Reys et al., 2013).

**Standards for mathematical practice.** The SMPs describe, “what it means to do mathematics and describe skills, dispositions, and understandings of mathematics that students should have” (Koestler, Felton, Bieda, & Otten, 2013, p. IX). Consistent across K-12 grade levels, SMPs are largely based on the NCTM process standards which were outlined in *PSSM* and the strands of mathematical proficiency described in *Adding it Up* (NGA & CCSSO, 2010).

Similarities between NCTM’s *PSSM* (2000), NRC’s *Adding it Up,* and *CCSSM* are evident. For example, consider the problem-solving SMP. In the process standards (NCTM, 2000), problem solving is described as, “engaging in a task for which the solution method is not known in advance. In order to find a solution, students must draw on their knowledge, and through this process, they will often develop new mathematical understandings” (p. 52). In *CCSSM,* students are asked to, “make sense of problems and persevere in solving them” (NGA & CCSSO, 2010, p. 6). They are expected to do this by, “explaining to themselves the meaning of a problem,” “look(ing) for entry points to its solution,” and, “planning a solution pathway” (NGA & CCSSO, 2010, p. 6).

Furthermore, *Adding It Up* (NRC, 2001) calls for students to develop a *productive disposition* as mathematical problem-solvers. A *productive disposition* is defined as, “the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that the steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics” (p. 131). These attributes require students to be active participants, making sense of the problems posed to them, and be given opportunities to invent solution strategies by building on prior knowledge (Koestler, Felton, Bieda & Otten, 2013). Likewise, *CCSSM*'s Problem-Solving SMP asks students to "make sense of problems and persevere in solving them" (NGA & CCSSO, 2010, p. 6).

Additionally, the teacher plays a crucial role in supporting the students' problem-solving effort by creating classrooms where students, "explore, take risks, share failure and successes, and question one another…[so that] they will be more likely to pose problems and to persist with challenging problems" (NCTM, 2000, p. 52). This notion is consistent with *CCSSM*'s description of problem solving, namely, that students should, "understand the approaches of others," "make conjectures," and, "monitor and evaluate their progress and change course if necessary" (NGA & CCSSO, 2010, p. 6).

One can ask: Just how central is problem solving to *CCSSM*? McCallum (2011) suggested that the SMPs of, "make sense of problems and persevere in solving them" and, "attend to precision" span all eight SMPs. These measures are illustrated in Figure 1, which shows CCSS Mathematical Practices. This figure shows, that while learners are working on the other six SMPs, they should always be focusing on attending to precision and making sense of problems and persevering in solving them. These last 2 are what *CCSSM* calls overarching habits of mind, meaning that they should be continually practiced throughout the other mathematical practices as well, because changing habits of mind means changing the way a learner approaches a problem, problem solving, and mathematics in general.

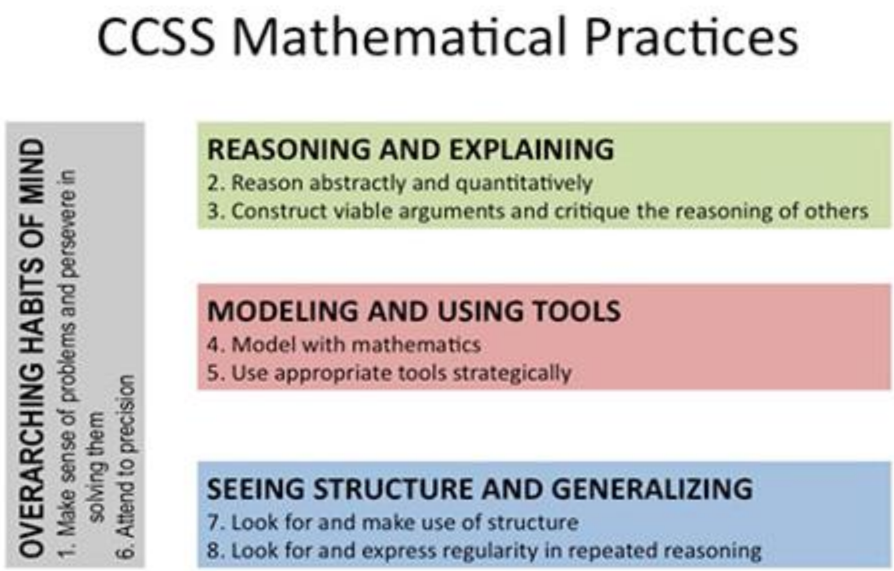



Figure 1. Grouping the standards for Mathematical Practice (McCallum, 2011).

NCTM's (2000) five process standards, previously described, continue to spiral back to both problem solving and persevering while solving. For example, the *reasoning and proof* process standard emphasizes the importance of encouraging students to make conjectures that will assist in their problem-solving attempts. For this to happen in a classroom, students should have opportunities to make, explore, analyze, and justify their conjectures. According to the *representations* process standard, students should create and use representations; select, apply, and alternate between representations, and use representations to model phenomena (NCTM, 2000). These two process standards are combined in *CCSSM*, which states that students benefit from using manipulatives or drawing pictures to help understand and solve problems (NGA & CCSSO, 2010).

Moreover, *PSSM*'s *communication* process standard emphasizes the importance of making solution strategies present in the classroom. There are several ways of doing this. For example, students can be asked to make sense of the problem-solving strategies of others, justify their own solutions to others, and mathematically convince peers about different points of view. In the Problem-Solving SMP, *CCSSM* stresses the need for students to understand others' solution strategies, as well as the mathematical connections between the different strategies (NGA & CCSSO, 2010).

## Problem Solving

### Defining Problem Solving Throughout History

The definition of problem solving, as adopted and used in the context of this study, has evolved over the past fifty years. In 1945, Polya stated that solving problems is a practical skill, like swimming. In order to learn to do it, one has to observe and imitate what others do and finally, "learn to do problems by doing them" (p. 5). This suggests that students learn to problem solve by practicing problem solving. He also suggested that, "it is easy to keep on going when we think that the solution is just around the corner; but it is hard to persevere when we do not see any way out of the difficulty" (Polya, 1945, p. 93). Polya went on to say that the student learns to persevere through not being successful, to appreciate small advances, and to wait for the essential idea. If a student has no opportunity to familiarize himself with the varying emotions of the struggle for the solution in school, his mathematical education has failed in the most vital point (Polya, 1945). This acquired perseverance, not merely the act of problem solving, is an important attribute of *CCSSM.*

In 1953, Henderson and Pingry posited that problem solving required a goal, a blocking of the goal for the individual (meaning the individual cannot readily solve the problem) and an acceptance of the goal (the individual figures out how to solve the problem). They too focused on the "perseverance" of problem solving, emphasizing the blocking of the goal, and then later the acceptance of the goal. Later, Reitman (1965) characterized a “problem” as a situational circumstance in which someone has, “been given a description of something but does not yet have anything that satisfies the description” (p. 126), and a “problem solver” as someone recognizing and accepting a goal without an immediate way to reach the goal. Reitman focused on the fact that a problem must not be readily solvable, but that perseverance in solving that problem is crucial. Schoenfeld (2014) attributes problem solving to situations in which students do not have a pre-packaged method of solution. Based on these conclusions, the act of problem solving should focus on a problem that cannot be easily solved while emphasizing that perseverance is required in solving the problem.

Lester (1985), like Polya, stressed that, “the ultimate goal of instruction in mathematical problem solving is to enable students to think for themselves” (p. 41). NCTM’s (2000) description of problem solving incorporated essential elements of the above-mentioned views. They maintained that, “problem solving means engaging in a task for which the solution method is not known in advance. In order to find a solution, students may draw on their knowledge, and through this process, they will often develop new mathematical understandings” (NCTM, 2000, p. 52), or, otherwise expressed, students will develop, extend, and enrich their understanding through problem solving. For the purposes of this study, I used the *CCSSM* version, namely, “Make sense of problems and persevere in solving them” (NGA & CCSSO, 2010). In addition, I included the ten components presented in Table 1 (p. 18). The *CCSSM* definition encompasses the “blocking” of the goal, the perseverance, and the act of solving problems to persevere and become better at solving them. It also emphasizes the personal actions of students drawing on their previous knowledge, developing new meaning, and understanding the approaches of others, which implies that the problem-solving process is different for each individual.

**Teaching Through Problem Solving**

According to NCTM (1989, 1991), teaching through problem solving can help all students learn key concepts and skills required by different states and districts. Such an approach begins with an appropriate problem related to the required content. The problem used must allow for multiple solution approaches. In a sense, this problem serves as a vehicle for mathematical exploration in addition to forming the organizational focus and stimulus for student learning. The resulting learning environment is active because students explore problem situations and often “invent” their own solution strategies with teacher guidance (Cai, 2003).

Another benefit of teaching mathematics through problem solving may be found in the work of Kloosterman and Stage (1992). They suggested that increasing a student’s motivation to learn to solve mathematical problems will in turn increase the likelihood that s/he will become a proficient mathematical problem solver. Given this proposed correlation between problem solving and the motivation to learn mathematics, classroom teachers must also tackle the quandary of motivating students to learn mathematics through problem solving. In order to do this, less emphasis must be placed on precise computation, as students who believe computation is the key to mathematical learning may be less motivated to be proficient problem solvers. Instead, teachers must increase students’ belief that mathematics is useful for the purpose of finding a solution to a real problem under study. The view of mathematics as a tool for finding a solution may increase motivation to learn mathematics which then increases mathematical problem-solving ability (Kloosterman & Stage, 1992). Polya (1945) first mentioned this when he said:

> If you cannot solve the proposed problem do not let this failure afflict you too much but try to find consolation with some easier success, try to solve first some related problem; then you may find courage to attack your original problem again. (p. 114)

Likewise, Schoenfeld (1985) stated that problems are well posed when a solution path is not immediately obvious. These research findings indicate that perseverance in problem solving is essential for success.

In this study, appropriate problems were selected by using the *CPM* curriculum. The students actively worked in groups to solve the assigned problems. They used prior knowledge to develop their own solution strategies and worked in collaboration with their peers as me (as their teacher). My guidance consisted of posing questions to help the students rethink their questions or reconsider the decisions they made. Motivating students to learn in this manner was challenging at first because of the novelty of the method; however, the students were open to the new problem-solving approach because they were working in groups and had the assistance and guidance of everyone else in the group, including myself. In order to motivate students, I did not grade the class work for correctness, but rather conceived the group work as learning tools for finding solutions to the real problem at hand.

## Studying Problem Solving

Over the years, a variety of quantitative and qualitative research methodologies have been used to study problem solving. Tools for collecting qualitative data, such as student and teacher questionnaires, student and teacher interviews, and classroom videotaping have been used in many qualitative studies related to student mathematical problem solving. For example, in Francisco & Maher's (2005) study, problem-solving sessions and individual and small-group student interviews were videotaped. These sessions were subsequently coded to identify examples of mathematical reasoning that emphasized sense making and justifying ideas, and to help researchers provide insights leading to the promotion of mathematical reasoning in problem solving. Another qualitative method that has been used involved examining student documentation of their effort during the actual problem-solving process. For example, Pugalee (2001) asked ninth grade algebra students to write what they did as they solved problems. This written work was used to determine whether students' written descriptions of their problem-solving methods showed evidence of the presence and type of metacognitive (the monitoring of one's mental activities) behavior. Findings suggested that a metacognitive framework was evident in the students' writings about their problem-solving processes. Pugalee also claimed that results showed the importance of implementing the use of writing as an integral part of the mathematics curriculum in general, and problem solving in particular.

In many of the problem-solving studies, researchers focused on the students. For example, Wong, Lawson and Keeves (2002) trained a group of ninth grade mathematics students to use a self-explanation procedure while studying a new theorem in geometry. In this procedure, students used prepared questions after reading the assigned material as prompts to carry out specific ways of transforming what they were studying or writing, or to develop their own explanations of the mathematical material under study. Findings indicated that the group of students that was taught this self-explanation technique showed more frequent use of knowledge access, knowledge generation, management, and elaboration activities than the group that was not taught the technique. Additionally, these students scored higher on the problem-solving test, especially on items that required students to extend their application of the newly-acquired knowledge to problem types that were substantially different from those presented in the original study material. The researchers also suggested that the usefulness of this simple self-explanation procedure could potentially be a powerful technique for acquiring knowledge during the study because it served as a powerful predictor of problem-solving performance.

Iversen and Larson (2006) investigated Model Eliciting Activities (MEAs) as a means to measure student performance on a complex real-world task. According to Clarke & Lesh (2000), MEAs tend to focus on problem-solving situations that feature a smaller number of "big ideas" involving higher order understanding and abilities. Iversen & Larson (2006) found that the high MEA achievers considered all possible data relations (all of the relationships between the data), whereas the low MEA achievers mostly considered the available data intuitively. While the use of MEA's was an addition to the mathematics curriculum being taught in the classroom of participants in the aforementioned studies, an alternate approach to studying mathematical reasoning used by researchers was to consider student mathematical thinking through the use of existing curriculum. For example, instead of trying to change something that students do, Pugalee (2001), along with Gallagher, De Lisi, Holst, McGillicuddy-De Lisi, Morely and Cahalan (2000), investigated students' current work. As discussed earlier, Pugalee (2001) investigated whether students' written descriptions about their mathematical problem-solving processes showed evidence of a metacognitive framework and, if so, which types of behaviors were evident.

According to Pugalee, the relationship between conceptual knowledge and metacognitive knowledge is especially important during problem solving because metacognition helps students use appropriate information and strategies during the problem-solving process. Gallagher, et al. (2000) investigated strategy flexibility in mathematical problem solving. They studied the problem solving of juniors and seniors in high school, asking them to solve problems taken from released versions of the SAT-M. They found that only "high-ability" students used intuitive strategies to solve unconventional problems (items presented infrequently in textbooks and either require an unusual use of a familiar algorithm or can be readily solved by the use of logical estimation or insight). Although the boys outperformed girls overall, the observed gender difference was greater in items requiring the use of spatial skills, shortcuts, or multiple solution paths to solve the problem. These findings are worthwhile and were considered in my study because, as a teacher of the *Standards*-based curriculum (*CPM*), I needed to allow flexibility in the way students solve the problems. From lessons learned, just because I solved the problem one way does not mean that my solution is the only way or the "proper" way to solve the problem. Allowing students to draw upon their own prior knowledge, was important, yet challenging. Additionally, my classroom consisted of a mixed ability group. Thus, the study about MEA's focusing on problem solving was helpful because not every student was intuitively considering all the data. Having this prior knowledge was helpful. Finally, Pugalee's study was informative because determining the appropriate information and strategies to use in problem solving was often a difficult task for students.

In some studies, student problem-solving skills were compared with that of others. For example, Cai (2003) studied Singaporean fourth, fifth, and sixth grade students' mathematical problem-solving skills using four mathematically rich tasks. The level of student thinking and reasoning was captured through five lenses: (a) examination of their solution strategies, (b) display of mathematical domain knowledge, (c) representation of solution processes, (d) justification of the mathematical reasoning, and (e) ability to pose new problems based on a problem situation. This study suggested that most students select appropriate solution strategies to solve tasks and choose appropriate representations to communicate their solution processes. The nature of this finding is relevant to my study because, while my initial instinct may be to help students find an appropriate solution strategy as well as the words to communicate their solution process to others, I need to refrain from doing so and not let teacher lust impede the learning, as Tyminski (2009) stated. It is my firm conviction that, given enough time, support, and resources, most students will be able to come up with an appropriate solution strategy on their own.

Yet another aspect of problem solving that needs to be examined involves teachers as study participants. Ho and Hedberg (2005) studied the practices of three fifth-grade teachers in three Singapore schools. After viewing their teaching methodology, researchers invited the teachers to a workshop that explored Polya's (1945) four phases of problem solving: (1) understanding, (2) planning, (3) executing, and (4) looking back. Following the workshop, the teachers were observed again. A study was conducted to examine the pre- and post-workshop participation sessions with emphasis on the teaching of mathematical problem solving and changes in the teachers' classroom practices and how student learning of problem-solving skills was impacted by the changes in the teachers' classroom practices. The results indicated that, prior to the Polya training, all three teachers shared some common methods of teaching, but they differed in the amount of time spent on the various phases. After the intervention, one teacher increased the amount of time spent on solving problems and implemented group work, while another teacher afforded students more classroom time to reflect back on their work than had been provided prior to the intervention. This study indicated that teachers are different in their enactment of attempting to change their teaching in a manner that is consistent with problem solving. Even though the three teachers attended the same training, they each focused upon different aspects of the training to implement in their classrooms.

Henningsen & Stein (1997) discussed how classroom-based factors can affect a student's engagement with mathematical tasks that were set up to encourage high-level cognitive demand. Their findings suggested that a large number of support factors must be present to ensure successful maintenance of high-level cognitive demand. Specifically, the prime influences associated with maintaining student engagement at a high level of doing mathematics are: (a) building tasks based on students' prior knowledge, (b) scaffolding lessons and tasks, (c) appropriating an adequate amount of time to complete the exercises, (d) modeling of high-level performance, and (e) requiring explanation of their solutions and meaning to their work. The study did mention that the factors most frequently found to influence the decline of the assigned tasks were inappropriateness of the task, classroom management, and inappropriate amounts of time to complete the exercises.

Additionally, Henningsen and Stein (1997) conducted research on cognitive demand and found that student engagement was successfully maintained at a high level when a large number of support factors were present. These research findings influenced my teaching strategy to purposefully create a classroom atmosphere in which mathematical tasks build on previous knowledge, and when possible, to select tasks that have a high level of cognitive demand potential. As the classroom facilitator, I followed the *CPM*'s provided to guide me in scaffolding the students in their mathematical explorations while keeping the cognitive demand at the appropriate level. I actively encouraged students to provide explanations and meaning for their mathematics work. By striving to enact these two directives, I kept the mathematical tasks at a high level of cognitive demand and helped the students establish mathematical connections to prior knowledge.

Burmeister, Elliott, Weber, Whalen, Sprader and White (2018) discussed how teaching through problem solving requires a paradigm shift on the part of the teacher. The challenge for teachers is to identify quality tasks. These tasks are more than just exercises in which students repeat and practice processes their teacher demonstrated for them in a lesson. Instead, these tasks are problems that require students to use higher-order thinking, the solution to the problem is not immediately known, and students must invent the mathematics to solve the problem at hand, building upon prior knowledge and skills learned in previous lessons. Once teachers find quality problem-solving tasks, they will see that these "provide a motivating way for students to engage in deep mathematics" (p. 7). According to Burmeister et al., the keys to successful teaching through problem solving involves collaborating with other educators and building a collection of quality tasks. Thus, the careful consideration and selection of high-quality tasks is a critical activity and will play an important role in this study. Palmer, Johansson, and Karlsson (2018) suggested that in order for teachers to teach through problem solving, they must step out of their own comfort zones and try something new, something like an activity that may be summed up as "letting go of control and saying less." The capacity to change classroom delivery methods will challenge all teachers, including myself. However, as will be shown later, I managed to step out of my comfort-zone and started to make the changes. Buchbinder, Chazan and Capozzoli (2019) suggested that the many obligations of a teacher, including choice of textbooks, can fuel a teacher's resistance to changing the way in which they teach. For this study, I selected a text that contained a large number of high-quality tasks, especially one developed for teaching a unit about quadratic functions.

Changing teaching methods to incorporate the *CCSSM* Problem Solving SMP can introduce challenges. One of these challenges is related to Boole's notion of "teacher lust." Tyminski (2009) warned that teacher lust, a state in which the teacher may remove an opportunity for students to engage in mathematics for themselves, can be problematic for the classroom teacher. He stated, "teachers, however, harbor a natural desire to impose their own understandings upon their students, even though this approach may be in opposition to their education goals" (Tyminski, 2009, p. 296). Teacher lust can potentially hinder opportunities for student learning because teachers give into the urge to tell students exactly what to do or control the direction of student thinking. For example, when a teacher poses a question to a student and then, almost without pausing, provides an explanation in which the question is answered, the student no longer has to reflect upon or respond to the question; this is an example of classroom teacher lust. Tyminski (2009) proposed two forms of teacher lust, namely, enacted and experienced: "Enacted teacher lust is an observable teacher action that may remove an opportunity for students to think about or engage in mathematics for themselves" (p. 307). Some examples of enacted teacher lust include directing and/or limiting student solution strategies and pathways or providing information in a manner that reduces the cognitive level of the task. Experienced teacher lust, on the other hand, is "the impulse to act in the manner described above" (Tyminski, 2009, p. 307). Experienced teacher lust often happens unconsciously, as teachers are unaware of this tendency and simply proceed into a state of enacted teacher lust without reflection upon the outcome of this action. For the purpose of this research, I considered Boole's construct of teacher lust and held back any impulse to interrupt and help the students in order that all students had equal access to their classmates' commentaries and exchanges. Finally, I made an effort to promote student collegiality by encouraging them to consider each other as a mathematical expert capable of a meaningful contribution even though, on occasion, that may have turn out to be an incorrect answer.

## Standards-Based Curricula

Concerned with mathematics performance, reported by the *National Assessment of Educational Progress* (*NAEP*) and consistently low performance on international assessments since 1970, the National Science Foundation (NSF) supported the development of 13 research-based K-12 mathematics curricula (Reys & Reys, 2007).

The high school portion of these releases included *Core-Plus Mathematics Project*, *Math Connections*, *Interactive Mathematics Program* (*IMP*) and *Mathematics: Modeling Our World*. These curricula were field-tested before becoming commercially available and were supported by research, unlike the releases of many textbooks that are made without formal assessment and field tests. At the same time, other organizations funded similar mathematics curriculum projects. For example, *CPM*, *College Preparatory Mathematics* was funded through an Eisenhower Grant. In this study I focused on using *CPM* because it was still broken down into the traditional math sequence (i.e. Algebra I, Geometry, Algebra II). While it is a *Standards*-based curriculum, *CPM* still follows roughly the same topics in the same order as the curriculum the rest of my mathematics department follows.

**Collaborative Learning Groups**

For the study, students will be working in collaborative learning groups, as is common with a *Standards*-based curriculum. Collaborative learning (a.k.a. cooperative learning) is a long-standing education idea that is still relevant to education. Collaborative learning refers to a set of instructional methods in which students work in small, mixed-ability learning groups with emphasis on learning that is "student" directed. Students in the groups share a dual responsibility namely, self-learning the material, and helping other members of their group learn. Establishing classroom norms of cooperation, support, and student-directed learning are essential for collaborative learning to ensure success. Teachers often find that creating a collaborative classroom for themselves in a workshop is valuable preparation for establishing these same norms in their own classrooms (Sharan & Sharan, 1987). Collaborative learning values students' contributions to a process of "knowledge making." This type of experience teaches students that they have mathematical authority (Beckman, 1990). When students are working together toward a common goal, academic work becomes something valued by peers, unlike traditional classrooms where one student working hard can be seen as a "nerd" or "teacher's pet." Research indicates that students working in small groups learn more of what is taught and retain it longer than when taught through a teacher-centered, lecture format (e.g., Beckman, 1990; Slavin, 1987). Furthermore, *CCSSM* calls for, "college and career readiness" and some would say that working in collaborative learning groups is preparation for the reality of the work force (Beckman, 1990).

## Quadratic Equations

Vaiyavutjamai and Clements (2006) pointed out that little attention has been paid to the teaching and learning of quadratic equations in mathematics education literature, as evidenced by a lack of research on the subject. Zhu and Simon (1987) found that students can learn to factor quadratic expressions without direct instruction through, "carefully chosen sequences of worked-out examples and problems" (p. 137). Bosse and Nandakumar (2005) reported on the techniques in which students engage while solving quadratic equations. According to Bosse and Nandakumar (2005), students only utilize the methods of Quadratic Formula or Completing the Square after they have exhausted the possibility of factoring the Quadratic formula, when in actuality, the probability of a randomly chosen quadratic equation to be factorable is very small. Kotsopoulos (2007) discussed the difficulties in student understanding and solving of quadratic equations. Commonly, students struggle with solving quadratic equations, especially when they are expected to solve them in different ways. While Kotsopoulos (2007) attributed the struggle with factoring to failure to recall main multiplication facts, Tall, de Lima, and Healy (2014) maintain that the lack of understanding of quadratic equations can be traced to a lack of understanding of linear equations. According to their study, when dealing with quadratic equations, students simply used formulas with little success or understanding. On the other hand, Vaiyavutjamai and Clements (2006) suggested that if teacher-centered instruction focused strongly on the manipulation of symbols (and not the meaning of symbols) student performance involving the solution of quadratic equations would increase. However, their conceptual understanding would still remain low. On the other hand, Ellis (2011) found that when students work with an open-ended problem with multiple entry points, thus having opportunities to visualize a concrete representation of the problem situation and the opportunity to work collaboratively in groups, their generalization processes were enhanced and reinforced. This cyclical interaction process promoted the development and refinement of generalizations, which were dynamic, socially situated processes that could evolve through collaboration.

## Action Research

### Working Definition of Action Research

Speaking generally, "action research is grounded in a qualitative research paradigm whose purpose is to gain greater clarity and understanding of a question, problem or issue….action research commences with a question, problem or issue that is rather broadly defined" (Stringer, 2007, p. 19). Furthermore, Kemmis (1985), stated that action research links the applied concepts of reflection and action together. Kemmis described action research as a continuous cycle of planning, acting, observing, and reflecting. According to these definitions, action research can be applied in many different situations. However, speaking more specifically, Mertler (2012) stated that action research is largely about examining one's own practice. Linking this practice to schools, Elliott (1985), claimed that the purpose of action research is to improve the quality of teaching and learning, as well as the environment in which teachers and students work. Action research is intended to support teachers and groups of teachers, in coping with the challenges and problems of practice and carrying through innovations in a reflective way. (Altrichter, Posch & Somekh, 1993)

Emphasizing the importance of teacher reflection, Capobianco (2004) defined action research as, "a form of systematic, self-reflective inquiry undertaken by teachers to improve their own practices and understanding of these practices" (p. 2). Stenhouse (1984) suggested that this, "systematic enquiry [be] made public" (p. 6). The work of Miller and Pine (1990) served to create a definition that merged the ideas of Stenhouse, Capobianco, and Kemmis by defining action research as a process using journaling, critical friends, and dissemination to examine teaching and learning that occurred in their own classrooms. Additionally, Roychoudhury (1995) suggested that action research, "is a form of inquiry that is committed to improving some practical situation and is conceived and carried out by *insiders*" (p. 138). Although the cited definitions of action research suggest different meanings, most researchers seem to agree that at the core of these definitions is a concept advocating that teachers are striving to improve their teaching practices using a systematic process. For the purposes of this study, action research is defined as the practice of a classroom teacher acting in a combined capacity of teacher-researcher, who uses a systematic approach to change her teaching practice in a manner that will lead to improvement of student achievement.

**Teacher-Researcher**

Teachers may act as researchers by collecting classroom data with the intention of making a change to their teaching practice. Jaworski (1998) suggested that many teachers construct activities, reflect on these activities, and use the feedback from their reflections in their practice daily. However, formalizing the research process by systematic documentation is the principal characteristic that identifies the role of a teacher-researcher. Feldman (1994) explained that the teacher-researcher merges student and teacher ideas and experiences together and designates as important those, “decisions she makes and the actions she takes as a result of those decisions” (p. 97). This definition suggests that a teacher-researcher has a unique way of thinking about his/her classroom. Specifically, the teacher-researcher is positioned to provide a unique insider’s perspective on how students and teachers work together to construct knowledge and influence curriculum.

Phrases such as *teachers as researchers* and *teacher-research* are used synonymously when action research is considered. Here, *teacher-researcher* is defined as a teacher engaged in action research in the classroom. This designation is intended to clarify the difference between an outside researcher entering a teacher’s classroom to conduct research and a classroom teacher who chooses to systematically reflect on her practice and document the changes s/he makes to improve the teaching process. Unlike outside researchers whose principal goal is to identify classroom modifications teachers should implement, a teacher-researcher generates questions and implements the changes to her practice. Identifying needed changes and results to be expected from the changes, as is typical in research, empowers the teacher in her role, transforms her into a teacher-researcher and alters the way she elaborates on her teaching practice.

**Characteristics of Action Research**

Currently, educational policies are typically based upon the results of large quantitative studies. However, action research studies conducted by classroom teachers are typically rooted in concepts such as professional development. Results of action research studies provide examples of student work, teacher work, etc., which add to the quantitative studies. Yet, it is the sample student and teacher work that other professionals find beneficial for understanding and changing their own practice (Jaworski, 1998).

Because dissemination of action research is seen as a form of professional development, many action research studies conducted in the context of mathematics education are focused on professional development (e.g., Brulles, Saunders & Cohn, 2010; Jaworski, 1998; Keazer, 2012; Noffke, 1994; Tinto, Shelly, & Zarach, 1994) and influence how professional development is approached by teachers, administrators, and professional developers. For example, Tinto, Shelly, and Zarach (1994) found that teachers found their voices while engaging in research on their teaching. They stated that through research, teachers felt empowered and became leaders within the local mathematics teacher community through dissemination. Additionally, Noffke (1994) stated that action research helps teacher to grow in their professions through dissemination. And Jaworski (1998) credited Kemmis (1985) with stating that action research is a means to grow and change professionally as an educator.

Teacher reflection has been given a place of prominence in many action research studies involving mathematics education (e.g., Adams, 2008; Bonner, 2006; Jaworski, 1998; Keazer, 2012; Raymond & Leinenbach, 2000). For example, Raymond and Leinenbach (2000) focused on teacher reflection in mathematics. They stated that one challenge of any action research study is to distinguish it from reflective teaching. The difference between reflective teaching and action research is that action research requires a systematic process to bring about change, and it concludes with dissemination about the study. On the other hand, reflective teaching is more informal and requires the teacher to reflect on her practices and make changes based solely upon her reflections. The difference is that action research is research-based and situated in current knowledge in the field; therefore, any changes made in the classroom are not based solely on the teacher's observations. These studies all focus on reflection in mathematics classrooms and claim that reflection is a catalyst for change to occur. Because action research is largely about examining one's own practice, reflection is at the core of action research. One researcher defines reflection as, "the act of critically exploring what you are doing, why you decided to do it, and what its effects have been" (Mertler, 2012). Another research source suggested that reflective teaching is the systematic process of using educational theory, research, experience, and the analysis of a lesson's effectiveness in order to develop instruction that assesses student learning (Parsons & Brown, 2002). In Brown's (2002) dissertation study, she investigated teachers' perceptions of the influences of action research (i.e. reflection) on their teaching practices and their impact. Brown (2002) reported that action research and reflective teaching are forms of staff development, which is one of the most important factors leading to teacher improvement.

She found that action research's defined structure aided teachers in more systematic and conscious data collection, analysis and reflection. Specifically, one teacher in the study talked about how her planning was impacted by post reflection practices such as spending dedicated time each day to reflect on the day's lesson to note in a plan book what went well and what went wrong. Spending time reflecting on her instruction allowed her to rethink, make new connections, and thus improve previous instructional practices. All six teachers in the study reported that reflection was an important piece that supported changes in their teaching practices now in terms of planning, focusing, and assessing. Similarly, Paryani's (2019) dissertation study found that teachers viewed a new evaluation system, implemented in an action research study, as time for reflection, ongoing support and continuous professional development. In this study, teachers reported that spending time reflecting on the day's lesson allowed for personal observations that were "tailor-made" to their individual needs as a teacher.
In summary, reflection is a critical component in many types and uses of action research.

Another theme identified in action research studies involving mathematics education is teacher change (e.g., Adams, 2008; Jaworski, 1998; Raymond & Leinenbach, 2000; Timmerman, 2003; Tinto et al., 1994). Teacher change is usually gradual and is seen as an outcome of action research, because reflection often leads to changes in teacher beliefs, which in turn leads to changes in a teacher's classroom practices to realign their teaching with their beliefs (Edwards & Hensien, 1999). According to Nelson (1993), three approaches for promoting change in teacher's beliefs are worth considering. These include introducing disequilibrium in order to encourage teachers to reconstruct their ideas about mathematics education, encouraging teachers to implement research-based knowledge, and working through the "growing pains" of instituting the new beliefs in the classroom.

Teacher change in the mathematics classroom would be much more challenging without collaboration and support provided by teachers, university researchers, and critical friends (e.g., Bonner, 2006; Edwards & Hensien, 1999; Jaworski, 1998; Keazer, 2012; Raymond & Leinenbach, 2000). One cannot help but be mindful of the fact that in each mathematics collaborative action research study, teachers cite the importance of the support, feedback, guidance, and different perspectives (Edwards & Hensien, 1999). In a typical cycle, feedback and collaboration have led to teacher reflection upon delivery methods, which in turn have led to teacher changes. This reflection-leading-to-change product has served as a catalyst for professional development as mathematics teachers wrote about their experience, shared their ideas, and disseminated their work. Edwards and Hensien (1999) also observed that early on, in each study, teachers viewed university researchers as the "experts" but as the studies progressed, planning and feedback among the teacher-researchers became collaborative and valued as a basis for improvement.

## Model of Action Research

Many models exist to represent systematic change in classroom teaching practice when using action research. One type of model is built around a cyclical notion, suggesting a "spiral" (e.g., Bachman, 2001; Calhoun, 1994; Lewin 1946; Piggot-Irvine, 2006; Stringer, 2007). Each model begins with a problem or topic which undergoes observation or monitoring of current practice. The collection and synthesis of data follows. This leads to an action which serves as the catalyst for the next cycle. This process repeats with stages such as look, think and act. As well as fact finding, plan, take action, evaluate, and amend the plan. Each action in the cycle is subsequently refined and repeated. Another type of model emphasizes stages or steps in each cycle (Altrichter et al., 1993; Hendricks, 2009; Mertler, 2012; Riel, 2007). For example, planning, taking action, collecting evidence, and reflecting are all stages in Riel's (2007) model, whereas acting, evaluating, and reflecting are descriptive phrases used in Hendrick's model. These models are similar as can be evidenced by their shared elements.

**Benefits of Action Research**

Benefits for teachers involved in the action research process are well documented. First, in order to realize meaningful improvements in classroom practice, teachers have to take charge of the research process (Beckett, Mcintosh, Byrd & McKinney, 2011; Tinto et al., 1994). This action will in turn bridge the gap between researchers and teachers (Davis, 2007; Jaworski, 1998), and provide teachers with their own teaching-voice (Jordan, Perry, & Bevins, 2011; Tinto et al., 1994). Because, when teachers become researchers in their own classrooms, these teacher-researchers offer a unique and necessary insight that an outside researcher will rarely, if ever, be able to fully envision. For example, Jaworski (1998) examined six secondary mathematics teachers who wanted to improve the learning of mathematics in their classrooms. She found that teachers who explored critical questions in their own classrooms were able to make significant changes in their teaching. Similar benefits of action research were evidenced in statements that identified action research as a process that helped to empower teachers as educators and solidified their positions as agents of change within their schools (Goswami & Stillman, 1987; Price & Valli, 2005; Susman & Evered, 1978; Tinto et al., 1994), thereby allowing teachers to discover their own voices as a result of studying their own teaching. Still, other benefits include providing teachers with a means to explore and recognize potential growth and changes within her own classroom. It allowed teachers to identify problems – political, practical, and personal – and gave them the opportunity to try and solve those problems (Capobianco, 2004). Because action research requires a systematic inquiry based on a constant gathering of evidence, teacher-researchers can see first-hand just how much they have progressed, an accomplishment that can lead to personal and professional satisfaction.

Finally, action research is useful to the school administration because it promises progress in professionalization (Calhoun, 1993). Through the dissemination of their work, teacher-researchers not only improve their own classroom practices but acquire measures and experience for potential improvement in other classrooms throughout the school corporation. When principals and district officials are aware of teacher-researcher activity, they can then use the outcomes of this activity as proof of improvement to more readily initiate changes in other classrooms. With publication and dissemination, the potential to influence classrooms in the state and nation is substantially increased.

## Summary

In this chapter, relevant literature was outlined and reviewed. To recap, the learning theories utilized in this study are sociocultural theory, situated learning, and communities of practice. In the next chapters, I will outline my study, studying myself, as the teacher of the classroom in an attempt to change my teaching style to align more with *CCSSM* and to focus on problem solving in my classroom. I will be studying my changes through the methodology of action research, which is described in the next chapter.  Chapter Four outlines the findings of my research, and. Chapter Five concludes with a discussion of the findings as well as implications for future research.

# CHAPTER 3: METHODOLOGY

This study emerged from a desire to improve the problem-solving skills of my students while also enhancing my ability to facilitate problem-solving activities. I employed action research methods of data collection and analysis in order to capture the nature of modifications made in the classroom involving both my students and my teaching practice, in order to answer the following research question:

How is the *Common Core State Standards for Mathematics* (*CCSSM*) Problem-Solving Mathematical Standard enacted in an algebra class, while using a *Standards*-based curriculum to teach a quadratics unit?

This question was further explored by focusing on the following sub-questions:

- Q1. What opportunities are there to enact the components of the Problem-Solving Mathematical Standard provided by the written curriculum?
- Q2. In what ways does the teacher's implementation of the quadratics unit diminish or enhance the opportunities to enact the components of the Problem-Solving Mathematical Standard provided by the written curriculum?
- Q3. In what ways does the teacher's enactment of problem-solving opportunities change over the course of the unit?

In order to address these questions, I documented all adjustments made to my classroom teaching practices, which provided opportunities for students to solve problems in a manner that aligned with the Problem-Solving Components (PSCs) of *CCSSM*. Since this was an action research study, I, the teacher, served as the unit of analysis. Therefore, the actions of the students were used only as they provided information relevant for improving my practice. This study has the potential to benefit high school and college algebra teachers in at least two ways: (1) it provides classroom examples of students engaged in tasks that align to the PSCs of *CCSSM* and (2) it documents changes to my teaching practice that occurred during one implementation of this mathematical practice. Since the release of *CCSSM*, in 2010 not much is known about specific aspects of its implementation.

This study represented a departure from traditional, teacher-centered classrooms for both the student participants and me. As a learner of mathematics, I participated in traditional, teacher-center classrooms, so naturally, as I began my teaching career, I used this familiar style. Like me, many mathematics teachers experienced similar learning situations, and as a result, may not be aware of alternate teaching approaches. In addition, practicing mathematics teachers may not understand the necessity to change, or the potential to positive impact on student learning. The student participants in this study also learned mathematics in teacher-centered classrooms where they were typically shown systematic steps to use in order to solve specific types of problems. However, to be a successful problem solver, according to the *CCSSM* PSCs, and to potentially become a more quantitatively literate citizen, the students' classroom experiences had to change. At the time of this action research study, few studies existed that focused on teacher efforts to enact *Standards*-based teaching in secondary mathematics classrooms, where those studies connect problem solving pedagogies to *CCSSM.* My study is an attempt to begin to fill this void; it aligns the PSCs of *CCSSM* to opportunities provided by a *Standards*-based curriculum and the classroom teacher.

In this chapter, I describe and justify methodological choices for my action research study using a three-stage approach. First, I discuss the basic tenets of action research and explain my rationale for using an action research methodology. Second, I describe the teaching environment used in the study, including information about the state, school, curriculum, students, and my role as the researcher. Finally, I detail the design of the study, including the phases, instruments, data collection, and analysis.

## Action Research

### Basic Tenets of Action Research

According to McNiff (1988), the paradigm of research in which we work determines what we look for, the way in which we understand and interpret what we observe, and how we solve emerging problems. She described the three major paradigms of educational research as: (1) The Empirical/Positivist Research Paradigm, (2) The Interpretive Research Paradigm, and (3) The Critical Theoretic/Action Research Paradigm.

The Empirical/Positivist Research Paradigm is used primarily with a quantitative approach to research. Working within this paradigm, quantitative researchers observe and describe reality. The purpose of the Empirical/Positivist Paradigm is to observe particular events and predict the outcome of future events using statistical information deduced from the study. The Interpretive Research Paradigm closely aligns with a sociological perspective. Interpretive researchers impose a framework unto which they must fit their practice. For example, case study research involves an outside researcher, who conducts research on an insider's practices. The Critical Theoretic/Action Research Paradigm is often utilized by researchers trying to improve their own practice. In fact, Carr and Kemmis (1986) emphasized this notion when they stated that, "action research is not about verification from the given event, but about intelligent action coming from wise judgments arising from immediate location of the event itself" (p. 64).

The primary goal of my research was to improve my own teaching practice and I opted for the action research methodology because, by nature, the research is conducted by the practitioner; I also operated within the Critical Theoretic/Action Research paradigm over the course of this study. The salient feature of this paradigm that appealed to me was that theory was put into practice instead of being disjoint from the actions that occurred in the classroom. This means the boundaries between theory and practice, "dissolve and fade away, because theory is lived in practice and practice becomes a form of living theory" (McNiff, 1988, p. 35).

According to Coghlan & Brannick (2010), the broad characteristics that define action research are: "(1) research *in* action, rather than research *about* action; (2) a collaborative democratic partnership; (3) concurrent with action; and (4) a sequence of events and an approach to problem solving" (p. 3). Research *in* action, rather than research *about* action requires participants who directly experience issues that need to be resolved to use a scientific approach to study the resolution of those issues. This process may be followed with a researcher working independently or, with a researcher partnering with an individual or group who want to improve practice. Whether the researcher is working alone (as both the researcher and participant), or in a partnership arrangement, the subject(s) of the study participate actively in a cyclical process. This is in direct contrast with traditional research, where participants are considered the subjects or objects of study. Another important qualitative element in action research is the way people are brought into the process of inquiry and action and how they participate and collaborate throughout the study. Concurrent with action means that the researchers and participants are working together, in all parts of the research process, to make the action more effective while building up a body of knowledge at the same time (Coughlan & Coghlan, 2002). Finally, action research is a structured sequence of events comprised of iterative cycles that include gathering data, analyzing data, planning action, taking action, evaluating action, and repeating the process. It is for these reasons that action research is popular in classrooms of practicing teachers, as teachers study themselves rather than being studied by a researcher or studying someone else.

**Methodological Choice of Action Research**

Discussing challenges and successes of research studies with others can help clarify situations, view a situation from another viewpoint, and/or provide support and guidance (Altrichter, et al., 1993; Carr & Kemmis, 1986; Kember et al., 1997; Mertler, 2012). These conversation partners are referred to in research as critical friends who, "have empathy for the teacher's research situation and relate closely to his or her concerns, but at the same time are able to provide rich and honest feedback" (Altrichter, et al., 1993, p. 61). Although I worked under the guidance of a university researcher who served as my advisor, this study was a collaborative partnership between me and my critical friend (a local teacher with a Ph.D. in Mathematics Education). To maximize the utility of our conversations, my critical friend developed a thorough understanding of the study. We engaged in discussions and she asked questions while avoiding anecdotes or criticism that might distract from a better understanding of the project (Altrichter, et al., 1993; Carr & Kemmis, 1986; Kember et al., 1997). Through this collaborative process my critical friend provided me, the teacher-researcher, with an alternative perspective through which to view the project. Among other things, the conversations served to facilitate the removal of any positive or negative emotions that could possibly cloud the data. For this study, I provided my critical friend with articles describing the role of a critical friend (e.g., Bambino, 2002; Costa & Kallick, 1993) along with my proposal. Also, during this study, my critical friend and I met weekly to discuss my implementation of *CCSSM*, focusing on problem solving via the *CPM* curriculum in my Algebra I classroom.

Consistent with Coghlan & Brannick (2010), my study was concurrent with action, regarding both the classroom and the curriculum. As a classroom teacher, I took three actions that made distinct changes to my practice. For my first action, I changed the method of instruction. I selected the topic of quadratics functions, engaged the students in discussions and facilitated problem solving instead of using a traditional lecture approach; therefore, my instruction transitioned from teacher-centered to student-centered. For my second action, I selected a *Standards*-based curriculum, *CPM*, which focused on problem solving, instead of using a traditional textbook. For my third action I examined each individual lesson in the curriculum and noted the presence of particular aspects of the Problem-Solving SMP. Daily reflection centered on both the classroom and the curriculum. I noted changes made to my teaching practice after delivering each lesson along with changes in students' engagement with problem solving.

The final characteristic of action research is that it "follows a sequence of events and an approach to problem solving" (Coghlan & Brannick, 2010, p.3). The study consisted of three sequential phases: pre-implementation, classroom implementation, and post-implementation. Throughout this sequence of events, my classroom practice evolved from a format considered to be traditional and teacher-centered to one that was *Standards*-based and student-centered. Action research itself is also considered an approach to problem solving because it is an application of the scientific method of experimentation to practical problems. Thus, it is applying the scientific method as a means to solving a problem in which the solution method is not readily available. The desired outcomes of action research are not only solutions, but also the catalyst for learning from the outcomes (both intended and unintended) and a contribution to scientific knowledge (Coghlan & Brannick, 2010). Specifically, instead of focusing on the outcomes or solutions, action research focuses on the entire process and the iterative learning cycle that complements the process of conducting action research. In my study, my goal was to solve the problem of how to implement *CCSSM* problem solving in my Algebra I classroom.

Sagor (2000) defined action research as, "a disciplined process of inquiry conducted *by* and *for* those taking the action. The primary reason for engaging in action research is to assist the "actor" in improving and/or refining his or her actions" (p. 3). My purpose was to improve my own classroom implementation of a *Standards*-based curriculum and to refine my teaching of Algebra I so that my teaching practice became consistent with the *CCSSM* standards, and in particular, the Problem-Solving Standard for Mathematical Practice (SMP). McNiff (1988) suggested that action research is, "a coherent approach to the everyday practice and problems of teachers in ordinary classrooms who are trying to understand, make sense of, and improve their professional lives" (p. 19). Since I served both as teacher and researcher for this study, using action research was a likely methodology choice because it allowed me the opportunity to document the change in my practice in a systematic way. According to Patrizio, Ballock & McNary (2011), studying our practice means studying ourselves. Vygotsky's sociocultural theory highlighted the importance of culture in individual development and the unit of analysis being the individual engaged in a group learning situation. In the case of my study, my unit of analysis was myself, the teacher, engaged in my classroom as my students learned to solve problems, and I learned to facilitate problem solving. Through analysis of recorded and audio-taped daily videos, backed by written records of changes to my own teaching practice, I studied my implementation of a *Standards*-based curriculum as a way to implement problem solving in my classroom as defined by *CCSSM*.

In my research, I investigated instances of student engagement with the Focal PSCs. In order to do this, I designed several tools to assist in data collection. The Teacher Reflection Form (TRF) (see Appendix A) assisted me in systematically documenting opportunities for students to engage in the Focal PSCs in each of the 14 lessons of the quadratics unit (this unit included portions of two chapters of *CPM*'s curriculum that addressed various aspects of quadratic functions). This student engagement with the Focal PSCs included the following forms of interaction: student-student, student-teacher, and student-curriculum. As a teacher-researcher, I simultaneously taught and collected data. This TRF form helped me consistently collect data from each daily lesson and focus on the opportunities I provided for engaging students with the Focal PSCs. The TRF was developed and revised using an iterative process during a pilot study in my classroom. At the beginning of the pilot study, I asked general questions such as, "What do I think students learned?" After the pilot study, I realized that this question was too broad and general. As a result, I addressed each Focal PSC separately when I revised the TRF in preparation for the implementation of the actual study. Additionally, during the pilot study I recorded my reflection on the challenges I faced, and what changes needed to be made to overcome these challenges. The original plan was for data collection to occur during my planning period each day after the completion of each lesson; however, due to time constraints, sometimes data collection happened at the end of the school day. There were additional instances of student engagement with the Focal PSCs that went undetected while teaching. These became evident when I analyzed the videotapes (and audio tapes when needed) of the lessons.

Finally, the methodology of action research is a self-reflective process that involves continually evaluating practices, solutions, and the researcher herself, with a view to improve the quality of the situation (McNiff, 1988). Reflection is a vehicle through which a connection is made between what is happening in the classroom and the steps taken to change it. Unlike more traditional forms of educational research, action research is carried out "on-the job" while focusing on the critical question that action researchers pose, "How do I improve what I am doing?" (Whitehead, 1993). This is the question that I attempted to answer about my own classroom practice. More specifically, "How did I develop strategies to facilitate students' problem-solving activities?" Considering the current attention given to the implementation of *CCSSM*, I implemented "on-the-job" research using action research.

**Model of Action Research**

For the purpose of this study, I used Altrichter, Posch and Somekh's (1993) model for documenting change in practice. The model's four stages included the following: (a) finding a starting point, (b) clarifying the situation, (c) developing action strategies and putting them into practice and (d) making teachers knowledge public. Stages (b) and (c) are cyclical in nature and can occur several times, each time clarifying the situation and improving the action steps (see Figure 2).

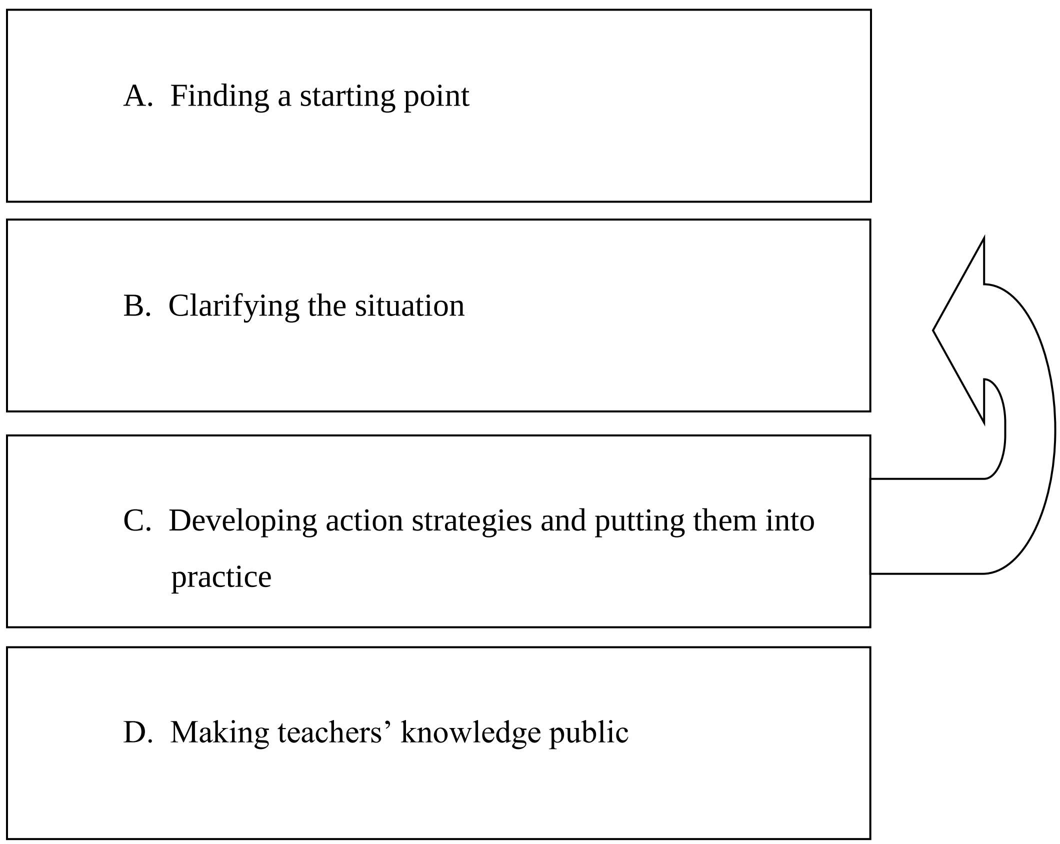



Figure 2. Model of Action Research. (Altrichter et al., 1993)

These stages outline a general framework of action research, which I applied to my high school mathematics classroom. For example, my starting point was the decision to implement *CCSSM* through problem solving. As a classroom teacher, I reflected on my practice in order to clarify the situation. This enabled me to determine which changes needed to be made prior to each class meeting. I developed action strategies and put these into practice. For example, I modified the manner in which individual students worked in each group and the questions I asked during facilitation. I then cycled back to the clarification stage, in which I determined what subsequent changes needed to be made; this procedure continued daily until the study was complete. In planning my study, I calculated a potential for 13 cycles of clarifying the situation and developing an action plan to occur over the course of the research project because there were 14 lessons. However, as I implemented the Standards-based unit, I learned that each lesson took multiple days to enact, thus the potential 13 cycles grew to 45 cycles, as I implemented the study over the course of nine weeks. Additionally, I learned that changes take time, and often it took me multiple days or weeks to implement the necessary changes. Throughout the course of this research, I continued to refine my implementation of the problem-solving based *CPM* curriculum.

Finally, I plan to share my ideas with my colleagues. This sharing will take place during professional development meeting at my school corporation level and department meetings in my high school. Outside of my school corporation, I plan to share my research findings through a series of professional development workshops with local teachers in partnership with the Wabash Valley Consortium, an organization that provides professional development to local teachers in Tippecanoe County, Indiana. I also plan to share my work with other teachers through state math conferences, such as Indiana Council of Teacher of Mathematics (ICTM), and nationally through NCTM. Sharing my research with other teachers will provide them with an example of how to put research into practice, thus bridging the gap between research and classroom practice. I can share a firsthand account of what it was like to change my traditional teaching style into a classroom that promotes problem solving through a *Standards*-based curriculum. In my professional development, I can offer teachers the challenges I faced, how I approached these challenges, and answer questions they may have about my newly acquired teaching experience.

**College Preparatory Mathematics**

According to Tom Sallee, Co-founder of *College Preparatory Mathematics* (*CPM*), “direct instruction alone is not enough” (*CPM*, 2013). Students need to approach mathematics in a different way in order for retention to occur. Because of this predominate view, *CPM*, a problem-centered curriculum, was created in the spring of 1989. *CPM* was developed by a group of 30 teacher-authors, three university mathematicians, and the director of the *Northern California Mathematics Project* (Kysh, 1991; 1995). The goal was to write mathematics course content that would help students learn problem solving strategies, understand concepts, master basic skills and procedures, and retain knowledge for a longer period of time (Dietiker et al., 2013). Kysh (1991), in her description of *CPM*, said that an original goal was to move the content of the first-year algebra course as far as possible in the directions recommended by NCTM’s curriculum standards, without changing the sequence of courses or putting students in jeopardy in a future course. Course topics that were emphasized included graphing, equation solving, and simplifying, as most of the work fell into these categories. However, topics that received less attention included the simplification of rational expressions and radicals. The following learning goals were established for first year algebra (now called *Core Connections*, Algebra): students will be able to (a) move away from rule-applying and towards a rule generating approach; (b) learn to use a scientific calculator effectively and efficiently; (c) continue to develop confidence as problem solvers (extend their strategies to include writing equations and relating them to graphs), and persevere in working on open-ended questions and investigation; (d) become more aware of their own thinking and describe their efforts orally and in writing; (e) develop the positive attitude that 'algebra is important'; (f) assume responsibility for their own learning. In order to leave time for problem solving and teaching for understanding, it was decided that most of the work typically done in the first three chapters of Algebra I textbooks would be omitted since it was review of the previous course (Kysh, 1995).

The curriculum was research-based, focusing on (a) problem-based lessons to develop concepts, (b) student study teams to foster mathematical discourse and peer support and (c) spaced practice over days and weeks (mastery over time) (Dietiker et. al, 2013). Problem-based lessons were intended to change the classroom from one of *telling* students how to do the mathematics, to a classroom where students are, "asked to solve problems designed to develop the method" (*CPM*, 2013). Collaboration allowed students to develop new ways of thinking about mathematics, increased student ability to communicate with one another about mathematics, and helped them to strengthen their understanding by having to explain their thinking to others (*CPM*, 2013). Spaced practice (i.e., reviewing previously taught concepts, and spacing practice of a particular concept throughout multiple lessons) was deemed an effective learning tool for long-term retention. Mixed practice, a natural complement to spaced practice, was added. In dealing with mixed homework, students need to recall how to solve a particular problem as well as identify the type of problem (*CPM*, 2013).

In 2013, the curriculum was revised and aligned with *CCSSM.* The course I focused my attention on, *Core Connections: Algebra,* was designed to prepare students for a rigorous college preparatory algebra course. "It helps students to develop multiple strategies to solve problems and to recognize the connections between concepts" (Dietiker et al., 2013). The lessons met all the Standards of Mathematical Content (SMCs) of *CCSSM* and embedded the SMPs of *CCSSM* into the curriculum (*CPM,* 2013; Dietiker et al., 2013). In the *CPM* curriculum, the big ideas, such as ratios and proportional relationships, the number system, expressions and equations, geometry, and statistics and probability, appear repeatedly and are interconnected throughout the course. For example, students learn to solve equations in Chapter 3. From this basic building block, they then solve systems of equations in Chapter 4, solve quadratic equations in Chapter 9 and solve complex equations in Chapter 10. In *CPM* courses, the authors ask students to explain their work and defend their thinking. The courses were designed to help students make connections between ideas, teaching students to persevere in problem solving.

Lessons are structured for students to collaborate with one another and work in teams. During class time, students work in groups to solve challenging problems, which introduce new material. While this is happening, the teacher provides guided scaffolding and support, thus helping students bridge their understanding of the standards. The homework consists of practicing the newly learned concept and, a "Review and Preview" section that reinforces previously introduced skills and concepts, while seeking to prepare students for new concepts and skills (*CPM*, 2013).

Each chapter is divided into sections that are organized around core topics. For example, Chapter 8 is broken into two sections; Section 1 introduces factoring and Section 2 addresses quadratic functions. Within each section, lessons include, “activities, challenging problems, investigations and practice problems” (*CPM*, 2013). The teacher notes include a “suggested lesson activity” section that provides ideas for lesson introduction, specific tips and strategies for lesson implementation, and suggestions for bringing the lesson to closure. To help students understand the core ideas, “Math Notes” boxes are strategically placed in student books, synthesizing the new core idea. Finally, “Learning Log Reflections” appear periodically at the end of lessons to allow students an opportunity to synthesize what they know, as well as identify areas that need additional explanation (*CPM*, 2013).

*CPM* provides, “learning strategies that are consistent with the *CCSS* Standards for Mathematical Practice” (*CPM*, 2013). *CPM* also provides daily opportunities for students to solve realistic, non-routine problems that are rich in mathematics.

> By having students make sense of the problem, rather than being told how to solve a particular kind of problem step by step, *CPM Core Connections: Algebra* helps students to develop deep conceptual understanding of the mathematics, procedural fluency, and perseverance. (*CPM*, 2013)

Additionally, the curriculum teaches problem-solving strategies that the students use continuously. Finally, the curriculum equips students with strategic competence and adaptive reasoning. Considering the similarities of philosophies, the *CPM* technique is closely linked to *CCSSM*’s Problem Solving Practice.

These problem-solving strategies in *Standards*-based curriculum like *CPM*, help students to understand the mathematics and perform better in mathematics. According to Boaler (2006), students at Railside High, an urban high school that utilized a *Standards*-based curriculum like *CPM,* performed better in mathematics than students at two nearby high schools that followed a traditional classroom style. At Railside, students learned more, enjoyed mathematics more, and progressed to higher levels of mathematics than students in the other two traditionally taught schools that were featured in the study. Railside students worked in groups to solve problems and asked each other questions in order to reach mathematical consensus, leading to student perseverance when solving problems. It was evident that mathematical communication was salient and valued in the classroom at Railside.

Brenner (1998) found that more English language learners earned passing grades in an algebra course when *CPM* was adopted in a high school. The teachers used a constructivist philosophy to guide their implementation of the *CPM* curriculum. Like at Railside, the students in Brenner's study were expected to discuss mathematics with both their peers and their teacher. Brenner found that students were more engaged in the *CPM* curriculum, despite being English language learners; this same group of students did not communicate effectively in the traditional classroom setting. Formal evaluations along with anecdotal evidence of the *CPM* program indicated that achievement of students from various ethnic/racial backgrounds was indeed enhanced (Brenner, 1998).

## Context of the Study

It was important to consider contextual factors as I conducted this action research study. These involved (a) the educational policies of the state, (b) the climate of the school, (c) the chosen curriculum and unit within the curriculum, and (d) my role in the study.

**The Educational Policies of the State**

In 2010, Indiana adopted *CCSSM;* however, in 2013, due to concerns about costs imposed by the program and having a national curriculum, Indiana rescinded their adoption of *CCSSM* and created its own version of college and career readiness standards. This served to "identify the clearest, most rigorous, and best aligned standards in Mathematics (and English/Language Arts) to ensure that Hoosier students will graduate meeting the definitions for college and career as defined in Indiana's processes" (Indiana Department of Education, 2014, p. 4). These new standards were reviewed and adopted on April 28, 2014. The SMPs were retained in the Indiana standards, under the title *Process Standards*; therefore, this study is still relevant to practicing algebra teachers who were mandated to teach the SMPs in their mathematics classrooms. For these teachers, my study serves as one practical example on how a teacher implemented the Problem-Solving SMP in her classroom and transitioned from using a traditional approach to a problem-solving approach.

**Prophet High School**

This action research study took place at Prophet High School (hereafter, PHS), a large, suburban high school in Indiana, serving students in grades 9-12 that is situated in a university town. At the time of this study, nearly 100% of the 1740 students were Caucasian with approximately 37% of the students qualifying for free or reduced lunch (Indiana Department of Education, 2013). PHS followed a traditional schedule that included a seven-period day and had a graduation rate of approximately 91% (Indiana Department of Education, 2013). Of the students who graduated at that time, 77% receive a CORE 40 diploma, a college/career readiness diploma in the state of Indiana (Indiana Department of Education, 2013). At the time of this study, the PHS passing rate for the Algebra I End of Course Assessment (ECA) administered state-wide, was 76%; the passing rate for the state of Indiana was roughly 69% (Indiana Department of Education, 2013).

This study was conducted in an Algebra I class of approximately 25 students, mostly freshmen. Having taught at PHS for nine years at the time, I was quite familiar with the school, students, curriculum, and the community. My reason for choosing the Algebra I classroom to conduct my study was twofold. First, I was able to conduct the research in my own classroom. This allowed me to document changes in practice more readily as I had taught Algebra I in a traditional manner for several years. Second, Algebra I is a required course for all Indiana students. At the time of this study, in order to graduate from high school, all students needed to earn a passing score (determined by the Indiana Department of Education) on the End of Course Assessment (designed under the direction of the Indiana Department of Education) for Algebra I.

## College Preparatory Mathematics in Practice

In my classroom, I had always used a traditional approach (teacher-centered classroom) to teaching Algebra I. However, it is widely accepted by the educational research community that student-centered classrooms provide enhanced opportunities for students to learn and retain mathematical ideas and that all students can benefit from problem-centered learning (e.g. Beckman, 1990; Boaler, 2006; Brenner, 1998; *CPM*, 2013; Slavin, 1987). As previously noted, the goal of this action research study, was to change my teaching practice to implement problem solving into my classroom.

I chose the *CPM* curriculum because the writers of *CPM* provided a curriculum for several mathematics courses that utilize a problem-centered approach to develop problem solving strategies, conceptual understanding, basic skills mastery, and long-term retention of mathematics (Dietiker et. al, 2013). Research has indicated that students working in cooperative learning groups often learn more effectively than working alone (e.g. Beckman, 1990; *CPM*, 2013; Sharan & Sharan, 1987; Slavin, 1987).

Additionally, *CPM* was developed by mathematics teachers with classroom teaching experience in conjunction with researchers who were knowledgeable about research regarding best mathematics teaching practices (*CPM*, 2013). I found that this partnership between teachers and researchers gave the *CPM* message greater credibility from both viewpoints. Although *CPM* is a *Standards*-based curriculum, it still follows the traditional sequence of Algebra I, Geometry, Algebra II, and Pre-Calculus typically found in U.S. high schools. Furthermore, I found *CPM* to be consistent with the goals of my study which encouraged students to persevere in problem solving and provide me with a problem-based curriculum.

Table 2 provides an outline of the lessons in the *CPM* quadratics unit along with the pacing of the lessons. Because my students and I were new to *Standards*-based learning and teaching, I quickly learned that most lessons took multiple days (2-3) to implement.

Table 2. Quadratics Unit Lesson Outline Suggested by CPM

| Day | Lesson | Lesson Title | Goal |
|---|---|---|---|
| 1-2 | 1 | Introduction to factoring quadratic expressions using algebra tiles | Students will learn to factor polynomials using algebra tiles |
| 3 | 2 | Factoring with generic rectangles | Students will learn to factor without algebra tiles, but by using a generic rectangle method |
| 4 | 3 | Factoring with special cases | Students will learn to factor special cases |
| 5 | 4 | Factoring completely | Students will learn to factor with a GCF (greatest common factor) and the generic rectangle method combined |
| 6-7 | 5 | Factoring shortcuts | Students will learn shortcuts to factoring |
| 8 | 6 | Multiple representations for quadratic functions | Students will learn to identify quadratic functions from a table, story problem, graph, or function |
| 9 | 7 | Zero product property | Students will learn to solve quadratics using the Zero Product Property |
| 10 | 8 | More ways to find x-intercepts | Students will learn other ways to find x-intercepts |
| 11 | 9 | Completing the quadratic web | Students will learn to make connections amongst multiple representations for quadratic expressions |
| 12-13 | 10 | Completing the square | Students will learn how to solve quadratic equations by completing the square |
| 14 | 11 | Solving quadratic equations | Students will learn to solve quadratic equations through multiple methods |
| 15 | 12 | Quadratic Formula | Students will learn to solve quadratic equations by using the Quadratic Formula |
| 16 | 13 | More solving quadratic equations | Students will learn to solve quadratic equations through multiple methods |
| 17 | 14 | Choosing a strategy | Students will learn tricks for determining which method is best to use for solving quadratic equations |

Solving quadratic equations is a conceptually difficult topic in high school mathematics curriculum (e.g., Chaysuwan, 1996; Vaiyavutjamai & Clements, 2006; Vaiyavutjamai, Ellerton, & Clements, 2005). In fact, Chaysuwan (1996) reported that directly after learning lessons on quadratic equations, 70% of student responses to standard quadratic equations tasks were incorrect. Furthermore, students often struggle with the following: (a) connections between algebraic, tabular, and graphical representations, (b) a view of graphs as whole objects, (c) correctly interpreting the role of parameters, and (d) generalizing incorrectly from linear functions (Ellis & Grinstead, 2008). Additionally, quadratic functions are a topic first introduced in Algebra I; since this is their first exposure to the topic, students typically find the topic difficult. For these reasons, I chose to provide my students the opportunity to learn about quadratic functions using a student-centered approach within the framework of a *Standards*-based curriculum.

## The Problem-Solving Components of *Common Core State Standards for Mathematics* Problem-Solving Standard of Mathematical Practice

Each lesson contained several components of the attribute *Make sense of problems and persevere in solving them* (Problem-Solving SMP). Table 3 presents the problem-solving components, *PSC*s, along with the corresponding PSC phrase I will use to reference each component. I chose these phrases from the main point of the PSC in order to make talking about each PSC more concise.

Table 3. *CCSSM Problem-Solving Components* (NGA & CCSSO, 2010)

| PSC | PSC Phrase |
|---|---|
| *Explain to themselves the meaning of a problem* | Meaning |
| *Look for entry points to a problem's solution* | Entry Points |
| *Analyze the givens, constraints, relationships and goals* | Analysis |
| *Make conjectures about the form and meaning of the solution* | Conjecture |
| *Plan a solution pathway* | Plan |
| *Consider analogous problems and try special cases and simpler forms* | Analogous Problems |
| *Monitor and evaluate their progress and change course if necessary* | Progress |
| *Explain correspondences between representations* | Representations |
| *Check their answers and ask themselves 'does this make sense'* | Answer Check |
| *Understand the approaches of others"* | Approaches of Others |

Table 4 provides examples of each PSC. Additional information regarding the authors' intended meaning was not provided in the written curriculum.

Table 4. Lesson Examples (Dietiker et al., 2013)

| PSC Phrase Meaning | Lesson | Example |
|---|---|---|
| Entry Points | 1 | *As you circulate, emphasize that students should look for special strategies to find the dimensions (p. 716).* |
| Analysis | 2 | *Kelly wants to find a shortcut to factor $2x^2+7x+6$. She knows that $2x^2$ and 6 go into the rectangle in the locations shown at right. She also remembers Casey's pattern for diagonals. Without actually factoring yet, what do you know about the missing two parts of the generic rectangle? (p. 727).* |
| Conjecture | 10 | *Help Jessica with a new problem. She needs to complete the square to write $y=x^2+4x+9$ in graphing form…. find the vertex and the x-intercepts. What happened? What does that mean? (p. 784).* |
| Plan | 1 | *Move teams on to problem 8-3, which has students focus on finding the dimensions of a completed generic rectangle (p. 716).* |
| Analogous Problems | 3 | *Problem 8-25 introduces students to quadratics that are not in standard form or that have terms missing. Remind teams that you expect to hear mathematical discussions for each case. Expect some questions from teams that are unsure about putting 0 into their Diamond Problem (p. 730).* |
| Progress | 3 | *Explain how to factor a quadratic expression. Be sure to offer examples to demonstrate your understanding. Include an explanation of how to deal with the special cases, such as when a term is missing or when the terms are not in standard order (p. 733).* |
| Representations | 2 | *First model how to factor with algebra tiles, and then look for connections within a generic rectangle (p. 726).* |
| Answer Check | 1 | *Does Casey's pattern always work? Verify that her pattern works for all of the 2-by-2 generic rectangles in problem 8-3. Then describe Casey's pattern for the diagonals of a 2-by-2 generic rectangle in your Learning Log (p. 719).* |
| Approaches of Others | 1 | *Work with your team to find the sum and the product for the following generic rectangles (p. 719).* |

These examples illustrate the ways in which the *CPM* quadratics unit is consistent with the components of the Problem-Solving SMP.

**Teacher as Researcher**

As both the teacher and researcher in this study, I must provide some information about my mathematics-teaching background. Before this study, I taught in a traditional, teacher-centered manner. I began each class period by asking students what questions they had about their previously assigned homework; after I answered their questions and worked some problems from the homework, I presented a lecture to teach the new mathematical content. Students took notes and wrote down examples. If time allowed, students began their homework.

Two personal experiences contributed to my teacher-centered style of teaching mathematics. First, I thought classroom management was easier. For example, I taught six mathematics classes each day, covering four different mathematics courses and only was provided one 50-minute planning period each day. During this time, I was required to grade, plan, communicate with parents, assist struggling students, document necessary items, and complete other tasks required of teachers. Because many items required timely attention, little time was available for lesson planning. Consequently, traditional lessons were easier to plan, since I was very knowledgeable in the content of the courses I taught and were often the method used as planning time was lacking. Second, when I began teaching at PHS, other mathematics teachers taught in the traditional manner. Because two of them served as my mentors, and I had only experienced teacher-centered classrooms as a successful student of mathematics, this seemed to me to be the “proper” way to teach. On a few occasions, I introduced discovery lessons, a concept which I was introduced to during my graduate studies, but for the most part, I was a traditional teacher-centered mathematics teacher.

In 2014, when the new Indiana state standards were implemented, assessment of students changed. They now were required to apply mathematics to novel situations, and there was diminished attention on the assessment to computation. Students were also asked to interact with mathematics and discover concepts on their own. My mathematics lessons needed to engage students in the eight SMPs too. As a result, I began to consider the need to create a classroom where students would *Make sense of problems and persevere in solving them, Reason abstractly and quantitatively, Construct viable arguments and critique the reasoning of others*, to name a few. It did not take me long to realize that these goals could not be achieved through a teacher-centered classroom; rather, it became clear that it was necessary to change my teaching practice.

At the conclusion of this study, I hoped to gain a better understanding of how to provide productive opportunities for students to become better problem solvers and the necessary changes to my teaching practice that needed to occur for these opportunities to be realized. It was personally challenging to change how I had taught for eight years and to learn new ways to teach. It was also difficult to allow students time to grapple with the mathematics at hand, while not interjecting to tell them a correct answer or an approach to solve the problem. I hoped to find a new way for students to learn mathematics, interacting with its content more, and learning for themselves how basic elements of the structure of mathematics are connected and how they work. I hoped my students would gain a new appreciation for mathematics and see it as interactive and interesting, rather than a collection of boring "rules" to be memorized. I struggled with acquainting students with their new roles in the mathematics classroom. I also hoped that they would evolve into primary "knowledge sources," rather than depending only on me to convey important mathematics concepts to them, as is common in teacher-centered classrooms.

Others have written about challenges with their attempts to teach in a student-centered classroom (e.g., Cady, 2006; Umbeck, 2011). For example, Cady (2006) shared her experiences with implementing *Standards*-based practices. Overwhelmed by the daily paperwork, meetings, and planning that is associated with teaching, Cady found it difficult to implement *Standards*-based practices. She discussed her attempts at promoting classroom discourse, noting that "the length and depth of student responses gradually increased" (p. 461). Because my students were not used to participating in mathematical discourse, this turned out to be a challenge in my classroom as well. Through the use of probing questions, and insisting that students explain and justify their responses, I hoped to overcome this challenge. Like Cady, I prepared a list of questions to help promote my students' mathematical thinking (see Table 5). The list she adapted from Driscoll (1996), and I used, was chosen because it covered a variety of situations that could arise in a mathematics classroom.

Table 5. Questions to Promote Mathematical Thinking (Cady, 2006, p. 461, adapted from Driscoll, 1996)

| **Organize information and thinking** | **Clarify responses** | **Keeping students focused** | **Promoting reflection** | **Exposing students' understanding** |
|---|---|---|---|---|
| What strategy could/did you use? | Why is this a reasonable answer? | What is the problem asking? | What other ways might work? | What would happen if …? |
| How could/did you organize your information? Your thinking? | How did you reach that conclusion? | How did you begin to think about this problem? | How does this relate to …? | What is the pattern or rule? |
| Could you organize your information another way? | Make a drawing to show that. | Tell me more about what you did. | What are some possibilities? | How did you think about the problem? |
| What information is needed? Not needed? | Explain how you did this part. | What else could you try? | How is this different from (same as) ______'s? | What predictions can you make? |
| What else do you know from the information given? | Does anyone have the same answer but a different way to explain it? | How can you check your answer? | Does that always work? | How could you prove that? |
| | | | What do you think about what _____ said? | What strategy was most helpful? Why? |

Like Cady, Umbeck (2011) discussed her struggles with changing classroom norms, "Groups were initially unproductive, did not know where to begin, and were apprehensive about the perceived lack of guidance they were receiving. They expressed frustration about the open-ended nature of the task and the lack of direction I was providing" (p. 91). She elaborated by stating that students, "were more comfortable with recognizable routines and looked at me to maintain them" (p. 91). I also struggled with this same issue, after all, a teacher-centered classroom typically aligns with the norms of affirmation of ideas by the teacher and the teacher as the "knower of all things." Similar to Umbeck, I experienced an internal struggle as my approach to teaching changed and I had to let go of some control of the exact direction of each class. To ensure this transition was as smooth as possible though, I made these changes to the classroom at the beginning of the semester.

Because I was both the teacher of the class and the researcher for the study, I took on the role of a participant observer. A participant observer is someone who observes a situation firsthand, engaging personally in the activities. Participant observation requires a form of observation that is distinctively different from observational routines common in experimental research or clinical practice where items or events to be observed are specifically defined. Observation in action research is more ethnographic, enabling an observer to build a picture of the lifeworld of those being observed and an understanding of the way they ordinarily go about their everyday activities (Stringer, 2007, p. 75). As a participant observer, I was immersed in the classroom, participating in the interactions among students, and observing their behavior and collaboration with one another.

I recognize that my role as researcher is subjective. According to Peshkin (1988), it is not enough to simply acknowledge the inherent subjectivity in the researcher's role, but researchers should also systematically identify their subjectivity throughout the course of their research. I identified my subjectivity by regularly reflecting on the values and objectives I personally brought to this research and how these affected the research study. For example, prior to the study I approved of my students learning in a traditional manner and I tended to not look very critically at my own teaching practice.

Because I aimed for balance, fairness, and completeness in my data collection (Patton, 2002), I adopted a stance of being true to reporting both confirming and disconfirming evidence. For example, when students struggled with how to proceed in problem solving, it was challenging for me to ask a question to help them think about the problem in another way instead of simply telling them how to do the problem. As both the teacher and the researcher, I was cognizant of Boole's notion of teacher lust and did my best to heed Tyminski's (2009) warning that teachers harbor a natural desire to impose their own understandings on their students, even if this is in opposition to their educational goals. For example, one strategy I used required me to keep the list of questions (in Table 5) with me during the implementation of the quadratics unit. When a student sought my input instead of asking a peer or if a student was unsure how to proceed in his problem solving, I used these questions as a guide for what to ask the students.

To prepare for this study, I first conducted a pilot study in a previous year's Algebra I classroom. For the pilot study I chose two lessons in the Exponential Functions Unit of *CPM* (we had already completed the quadratics unit) to implement in my classroom to test my use of the Table 5 questions, the effectiveness of my TRF, and to gain some insight into the challenges I could expect when I began this study the following semester. This approach enabled me to discover that, although I carried the questions around, I often struggled to match the appropriate question with the current situation. To help with this predicament, during my lesson planning each evening prior to implementation of the lesson, I tried to anticipate questions/situations and had a few appropriate questions picked out for use in the next day's lesson. Although not always perfect, this approach gave me a starting point for responding appropriately. Additionally, it was difficult to handle situations when students were not going in the mathematical direction I expected. I also learned that sometimes my facial expressions and tone of voice gave away my perceptions of the worthiness of some of the students' answers; they often responded to this by asking "is this right?" "where did we go wrong?" or "I need help on this." Thus, I had to be aware of my body language and tone while facilitating a problem-solving environment in my classroom.

## Research Design

In this study, I aimed to investigate my purposeful implementation of the *CCSSM* Problem-Solving SMP. As designed, there were three phases to my study: pre-implementation, classroom implementation, and post-implementation. These are outlined in Table 6, along with goals and the timeline for execution.

Table 6. Phases, Goals, and Timeline

| Phase | Title | Goals | Timeline |
|---|---|---|---|
| I | Pre-Implementation | • Identify PSCs in 14 lessons of *CPM* quadratics unit<br>• Identify three Focal PSCs for examination during Phase II<br>• Review curriculum and modify as needed, to ensure the majority of lessons include the Focal PSCs<br>• Develop questions to be used during implementation to enhance opportunities for the Focal PSC's. | March - July 2014 |
| II | Classroom Implementation | • Enact *CPM* quadratics unit<br>• Reflect daily about the lesson in my TRF (journal)<br>• Collect written student group work, individual student group work, videotapes of classroom, audio tapes of groups, TRF, audiotapes of critical friend meetings<br>• Develop strategies for facilitating problem solving in my classroom | January – March 2015 |
| III | Post-Implementation | • Identify all instances of Focal PSCs through data sources<br>• Investigate changes in my facilitation and students' responses across the 14 lessons<br>• Consider the implications of this study for my practice and for other teachers | January 2016 –May 2019 |

**Phase I: Pre-Implementation**

The first research sub-question defined for the study, *What opportunities to enact the components of the Problem-Solving Mathematical Standard are provided by the written* *curriculum?* was addressed during Phase I. I analyzed the *CPM* curriculum for opportunities for potential student engagement with each PSC; each of the 14 individual lessons were examined to identify these opportunities. This activity required coding, so I found a coding partner, a graduate student in mathematics education. She was familiar with the concept of *Standards*-based curriculum, as well as student-centered classrooms. She had taught classes at the undergraduate level where her undergraduate students were using this format of teaching.

Phase I took place in several stages. First, my coding partner and I noted the frequency of the *CCSSM* PSCs (see Table 7) in the 14 lessons of the quadratics unit by establishing and following a coding procedure.

Table 7. Frequency of Opportunity to Engage with PSCs in Quadratics Unit

| Component | Lesson | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | l |
| Meaning | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Entry Points | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 2 |
| *Analysis* | *7* | *10* | *3* | *4* | *10* | *9* | *20* | *63* |
| Conjecture | 1 | 1 | 0 | 1 | 0 | 2 | 2 | 7 |
| Plan | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 2 |
| Analogous Problems | 1 | 0 | 5 | 1 | 2 | 0 | 0 | 9 |
| Progress | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| *Representations* | *0* | *9* | *3* | *1* | *4* | *12* | *11* | *40* |
| Answer Check | 4 | 1 | 2 | 1 | 1 | 0 | 2 | 11 |
| *Approaches of Others* | *9* | *2* | *2* | *2* | *3* | *1* | *1* | *20* |
| Total | 24 | 25 | 16 | 10 | 20 | 24 | 36 | |

Then, my coding partner and I met, and we coded Lesson 1 together, discussed each instance of the PSCs until we reached agreement on the coding content. We compiled a codebook with justifications for our decisions along with examples and non-examples. Next, we coded Lesson 2 independently and then met and discussed any differences in our coding until we reached a mutually agreed-upon version of the coding, again expanding our codebook with entries to account for questionable cases. We coded Lesson 3 together. We independently coded Lessons 4 and 5; met to discuss coding differences until we agreed and continued to finalize our codebook. When the coding of all 14 lessons was completed, we worked to achieve consensus in all cases of disagreement. For example, one initial disagreement was during Lesson 2, Problem 8-14. This problem asked students to factor using algebra tiles and make connections within a generic rectangle. My coding partner had coded Part b. of Problem 8-14 as *Analysis* and *Representations* because from her viewpoint, students were analyzing the connections between the different representations. I coded this same problem as *Approaches of Others, Analysis* and *Representations* because I thought students had to understand the work of a fictitious student (Miguel). After some discussion about the definition of what it meant to understand the *Approaches of Others,* we agreed that they were not necessarily expected to understand Miguel's approach; instead, they were recreating the algebra tiles to factor and then creating a generic rectangle similar to his rectangle. However, we agreed that students were analyzing the generic rectangle and connecting representations between the algebra tiles and the generic rectangle.

After coding the first half of the lessons (the first seven of fourteen lessons), I tabulated the curricular opportunities for students to engage with each of the PSCs to determine which of the 10 PSCs occurred in the lessons, and how often. From these, I was able to identify three of the most prevalent Focal PSCs, namely, *Analysis*, *Representations,* and *Approaches of Others* (see Table 7). The remaining Lessons 8 through 14 were coded only for the three Focal PSCs using the developed coding book. Once all lessons were coded, we determined if the curriculum needed to be modified to include additional opportunities for the students to engage with the Focal PSCs. We decided that if at least 10 out of the 14 lessons provided curricular opportunities for students to practice the Focal PSCs, then the Focal PSC mandate was sufficiently present in the curriculum; upon examination, all three Focal PSCs were sufficiently evident in the curriculum, therefore, no curricular modifications were made.

## Problem Solving Components of the Problem-Solving Standard of Mathematical Practice

The PSCs of the Problem-Solving SMP provide opportunities for students to problem solve in a variety of ways. This study focused on the three most prevalent PSCs, the Focal PSCs, in the quadratics unit of the *CPM* curriculum. These are (1) analyze the givens, constraints, relationships and goals (hereafter, *Analysis*); (2) explain correspondences between representations (hereafter, *Representations*); and (3) understand the approaches of others (hereafter, *Approaches of Others*). The following paragraphs detail the NCTM, NRC, and *CCSSM* recommendations related to these Focal PSCs.

***Analysis* Focal problem-solving component**. Providing students with opportunities to analyze mathematics is key to developing mathematical understanding. According to *PSSM* (NCTM, 2000), “good problem solvers tend to naturally analyze situations carefully in mathematical terms and to pose problems based on situations they see” (p. 53). These students consider simpler cases first and then look at a more sophisticated analysis. Teachers can foster a student’s natural ability to pose problems by creating an environment in which the student feels comfortable taking risks, exploring, sharing failures and successes, and questioning one another. This idea of student analysis complements *CCSSM* SMP 4, *Model with Mathematics*, in that the students should be able to analyze relationships in practical situations using tools such as diagrams, tables, graphs, flowcharts, and formulas (NGA & CCSSO, 2010). In both SMP 1, *Makes Sense of Problems and Persevere in Solving Them*, and SMP 4, *Model with Mathematics*, students analyze the given information using different tools; ultimately, their analysis leads to a more meaningful understanding of the problem and hopefully, a correct solution.

***Representations* Focal problem-solving component.** Representation is an important process standard expected at all grade levels (NCTM, 2000) that leads to enhanced mathematical understanding and strategic competence (NRC, 2001). Students should be able to, “select, apply, and translate among mathematical representations to solve problems” (NCTM, 2000, p. 69). In doing so, students will develop the mathematical understanding that, “different representations often illuminate different aspects of a complex concept or relationship” (p. 69). For example, students could use algebra tiles to model and explain a problem. Algebra tiles and equation mats are tools that aid students in solving problems visually. For example, with the algebra tiles, students are able to visualize the process of factoring by seeing the tiles represented in a rectangle. The equation mat allows students to visually add or subtract tiles to a mat, visually seeing how to keep the equation balanced while solving. After using algebra tiles, students could utilize a table to further consider the same problem. Computers and calculator simulations could also be used to investigate mathematical properties such as the parameters of a quadratic function. Students often understand connections among concepts and representations before they can verbalize their understanding (NRC, 2001). Furthermore, "when students have acquired conceptual understanding in an area of mathematics, they see the connections among concepts and procedures and can give arguments to explain why some facts are consequences of others" (p. 119). When students develop strategic competence, they are able to "formulate mathematical problems, represent them, and solve them" (p. 124) as they generate mathematical representations of problems that "capture the core mathematical elements and ignore the irrelevant features" (p. 124) to understand a problem. Opportunities for students to produce and compare multiple representations, develop conceptual understanding, and demonstrate strategic competence are present in the *CPM* curriculum.

***Approaches of Others* Focal problem-solving component**. *PSSM*'s (NCTM, 2000) process standard of *Communication* stated that students should, "analyze and evaluate the mathematical thinking and strategies of others" (p. 62). Further, "mathematically proficient students . . . justify their conclusions, communicate them to others, and respond to the arguments of others" (NGA & CCSSO, 2010, pp. 6-7). Additionally, *CCSSM*'s SMP *Construct viable arguments and critique the reasoning of others* provides additional opportunities for students to not just understand that approaches of others, but to also critique their reasoning through justifications and examples. They need to distinguish correct reasoning from flawed reasoning (NGA & CCSSO, 2010). Working together with their classmates provides authentic opportunities for students to view, understand, explain, and evaluate different approaches to a problem's solution. In a typical teacher-centered environment, many students find it difficult to consider, evaluate, and build on a classmate's thinking. Mathematics teachers can provide a supporting environment to foster the development of this understanding. The *CPM* curriculum provides numerous opportunities for students to develop mathematical proficiency by understanding the approaches of others.

**Phase II: Classroom Implementation**

The second research sub-question, *In what ways does the teachers' implementation of the quadratics unit diminish or enhance the opportunities to enact the components of the Problem-Solving Mathematical Standard provided by the written curriculum?* was addressed during Phase II and analyzed in Phase III. The classroom videotapes (supported by the audiotapes) addressed this question.

During Phase II, I taught the *CPM* quadratics unit. All lessons were videotaped, and all classroom group work was audiotaped throughout the course of the study. The audio recordings were referenced as needed for clarification of conversations during data analysis. These recordings consisted primarily of conversations related to one or more of the Focal PSCs. I also collected samples of written student work that resulted from the small group interactions, homework, and individual class work. See Figure 3 for the types of data collected in Phase II. Teacher data was collected from two additional sources: Teacher Reflection Forms (TRF) (see Appendix A) and audio-taped critical friend conversations (see Appendix B for the Critical Friend Conversation Form (CFCF).

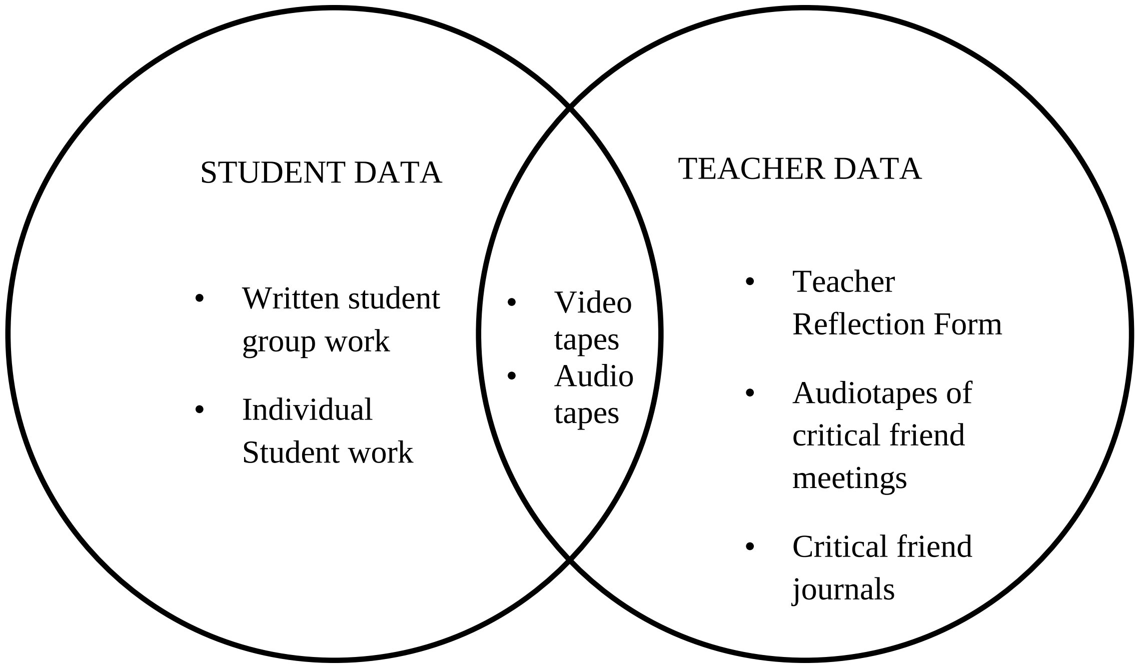



Figure 3. Types of data collected during Phase II.

The TRF served the purpose of a teacher journal (Jaworski, 1998) and was completed daily (after school) throughout the implementation phase of the study. I documented my observations and perceptions related to the students' classwork, homework, and small group discourse and additionally I documented my own reflections, perceived or otherwise, (Capobianco, 2004; Jaworski, 1998; Miller & Pine, 1990) regarding opportunities to engage with each Focal PSC and their availability to students during the lesson. If a Focal PSC was not evident, I reflected on why it was missing and how to address it in future lessons. I reflected on my facilitation and ways to improve my facilitation of these curricular opportunities based on an analysis of student responses to my questions.

Additionally, I audiotaped weekly critical friend meetings. The purpose of these meetings was to help address questions and challenges that arose during the implementation of the curriculum. Critical friend meetings are important since: "story-telling facilitates introspection because we have to order our experiences…it helps to clarify the situation further if the listeners can contribute actively to generating the story, for example, by posing questions, asking for additional information, and reflecting back to the narrator their provisional understanding of the situation." (Altrichter et al., 1993, p. 48)

As previously noted, my critical friend's participation in the meetings was not to solve my dilemmas, but instead to pose questions that would help me draw my own conclusions about ways to improve my facilitation of the students' problem-solving methods. The CFCF provided guidance to my critical friend on the types of questions to ask and provided a structure for our meetings. These questions were chosen because they helped me to reflect on my study without trying to "solve" my situation. Video clips were used to spark discussion with my critical friend about a particular lesson and the Focal PSCs. Specifically, since my critical friend and I met weekly for approximately one hour, I identified three short (about five minutes) video clips of instances of me facilitating each of the *Focal* PSCs (i.e., one video clip for each Focal PSC). My critical friend and I also journaled separately about the meetings.

**Reflecting on lesson planning.** Although lesson planning began in Phase I with the analysis of the *CPM* curriculum for opportunities for students to engage with each Focal PSC, the enactment of each lesson required additional lesson planning. For example, before every lesson was enacted in the classroom, I was mindful to spend sufficient time to familiarize myself with the lesson, the teacher notes, and suggestions provided by the *CPM* curriculum. Then I worked each assigned problem to ensure that I understood the expectations and also to identify potential places/topics where students may struggle. By spending this time before each lesson hypothesizing potential questions, I was better prepared to guide students and to form an appropriate response to the questions that was consistent with a *Standards*-based curricula approach to teaching. For each lesson that I planned, I compiled a summary of the lesson/questions along with my hypothesized questions and responses; this is also where I started to include Cady's questions that were appropriate for each problem that the students were working on in their respective groups. It is this summary that I decided to carry daily, rather than all of Cady's questions (although I kept that list on my desk for reference).

**Phase III: Post-Implementation**

Recall that the second research sub-question was addressed during Phase II but analyzed during Phase III. Due to unforeseen circumstances, my coding partner from Phase I was unable to assist me during Phase III and I had to find a new partner. My coding partner for this phase was a teacher experienced in teaching students using group work and problem-centered curriculum. This coding partner was ideal because she was both a classroom teacher and one who was very familiar with the teaching format that I was implementing in my classroom.

The goal of this coding was to document when each Focal PSC was present during the enactment of the curriculum. In establishing a codebook for the implementation, portions of the codebook for the written curriculum were used, but the codebook needed to be different as well to reflect the enactment. The following procedure was used in order to establish a coding procedure for analysis of the 45 enacted lessons First, my coding partner and I met and coded Lesson 1 together, discussing each instance of a Focal PSC occurrence until we reached agreement, thus developing a codebook with justifications for our decisions along with examples and non-examples. While coding, my coding partner and I looked for instances where a Focal PSC was evident. Each time one of these instances was noticed, a tally was recorded so that I had quantitative evidence of instances where Focal PSCs were enacted. Next, we coded Lesson 2 independently and discussed any differences in our coding until we reached agreement, again expanding our codebook to account for questionable cases. We then coded Lesson 3 together. We independently coded the remaining lessons and met to discuss differences until we agreed, continuing in much the same manner to finalize the codebook. In addition, we documented the exact wording which elicited the student response to acquire qualitative data. The group audio recordings served as a secondary source of data to address the research question. In instances where the videotaped audio could not be heard, the group audio recordings were consulted to hear the exact wording which elicited a student response about a Focal PSC. Through discussion, we came to consensus when initial agreement was not met. For example, when coding Lesson 12, Oscar asked "will the answers be the same for all of them?" And Fiona responded, "they should be." I originally coded this as *Approaches of Others* because of the exchange between Oscar and Fiona. My coding partner had not coded this, and after some discussion I understood why she did not code it. As she suggested, Fiona answered Oscar, but she did not explain anything to Oscar, so there was nothing for Oscar to understand about Fiona's approach. Instead, she simply told him a yes/no answer and the exchange ended.

Quantitative and qualitative analyses were performed at this point. Quantitatively, I examined the data to determine the number of instances when a Focal PSC was identified in a student response. Additionally, I qualitatively examined the instances where a Focal PSC was supposed to occur, or did occur, to determine if I enacted the curriculum as intended, enhanced the opportunity, or diminished the opportunity for students to engage with the Focal PSC. I also looked for potential situations indicating whether my questioning may have resulted in a Focal PSC response increase over time.

The third sub-question, *In what ways does the teacher's enactment of problem-solving opportunities change during the course of the unit?* was answered during Phase III as well. Recall that my critical friend asked me specific questions using the CFCF. Since the nature of the critical-friend conversation was open-ended, the analysis depended heavily on what was discussed in these conversations. Through these meetings, my weekly challenges and responses to the critical-friend questions were analyzed, looking for teacher change over the course of the study. Specifically, I intended to look at my responses to note changes in the detail that I used when talking about changes in my practice, and my attention to the Focal PSCs. However, the analysis took a different turn due to the critical friend conversations and observations made. As a result, I ended up focusing on changes in the following: (a) enactment of problem-solving opportunities, (b) group configurations, (c) questioning, and (d) facilitation.

Finally, when analyzing the data in Phase III, with the main source becoming the videotaped lessons, supported by the audio recordings, it became evident that the clearest way to write about each Focal PSC individually was with respect to how they individually addressed each sub-question. Therefore, individual analysis of each Focal PSC was addressed as it pertained to each sub-question

## Ethical Considerations

Just as with other research methods, action research requires researchers to follow a code of ethics. Since action research requires participation of the researcher, ethical considerations work in a special way. Of course, the same provisions for duty of care apply and participants involved have the same rights to safety and informed consent. In this study, the prescribed Institutional Review Board (IRB)'s ethical guidelines were followed, although there were no identifiable risks for participating in the study. Additionally, when action research is employed, it is important to ensure that all participants are informed of procedures to be used and that processes are transparent to all involved. These ethics criteria help define action research as a rigorous form of research. For my study, all participants were informed of the procedures and processes to be used on the first day of class.

## Rigor

The extent to which research reaches a standard of quality, validity, accuracy, and credibility is referred to as rigor (Mertler, 2012). Rigor in action research is different from traditional criteria for evaluating the rigor of experimental research. Action research reports on issues of trustworthiness such as credibility, transferability, dependability, and confirmability (Stringer, 2007). Trustworthiness infers that the outcomes of the research do not reflect the personal perspectives, biases, or worldview of the researcher. It means that the outcomes are not based solely on simplistic or superficial analyses, of what is being investigated. To ensure trustworthiness, a variety of checks designed to measure credibility, transferability, dependability, and confirmability were adopted.

For example, credibility refers to the plausibility and integrity of the study. When considering action research, the feeling of trust is gained through standard methods such as prolonged engagement, persistent observation, triangulation, member checking, participant debriefing, diverse case analysis, and referential adequacy. In my study, I was engaged with the students for an extended period of time because I was both their teacher and the researcher of this study. My TRF, along with the video and audio recordings, and the collection of student work, served to meet the persistent observation standard methods prerequisite. Triangulation was obtained through multiple data sources, and the use of two coders (my coding partner and me).

Transferability implies the possibility of applying the outcomes of the study to other contexts. Although this is different from the case involving traditional quantitative or experimental studies in which research can be generalized to contexts and groups other than those involved in the research, it does mean that others who familiarize themselves with the study can make judgments to determine whether the situation is sufficiently similar to their own and whether the outcomes can be applied. The outcomes of my study could potentially be applied to other mathematics classrooms. My study serves as one example of an initiative focused on activities that would be of interest to individuals considering the start of implementation of problem solving in the classroom.

Dependability refers to research procedures directed at generating trust that are clearly defined and open to scrutiny. An inquiry audit is a means to prove dependability based on a detailed description of the procedures that have been followed; it provides a basis for judging the extent to which the procedures are dependable. This dissertation served as my inquiry audit. The design of my study is extensively described in this dissertation and, therefore, it is open to scrutiny.

Finally, confirmability points to evidence that the procedures described actually took place. An audit trail, which includes artifacts such as data collected, field notes, tapes, journals and other items related to the study provide evidence of the outcomes. These artifacts "confirm the veracity of the study, providing another means for ensuring that the research is trustworthy" (Stringer, 2007, p. 59). One method of organizing these artifacts is to keep a journal. In action research, the teacher-researcher uses the journal as a tool to aid in recalling classroom information as a common collection place for research issues and a reflection tool. It is also a means for creating a community of researchers. In this study, my TRF served as my research journal. In addition, I collected written student group work and individual student work as relevant to the Focal PSCs. The video and audio recordings also helped me recall information, situations, and conversations that occurred in the classroom.

## Limitations

One limitation of this study was its relatively short duration. Based on the experience gained, one Algebra I unit's worth of problem solving is considered a minimum for beginning to implement *CCSSM* into a high school algebra class by focusing on problem solving. Also, because many high school students switch teachers at the end of the semester, the maximum duration of the study could only be one semester long. The actual duration of this study was nine weeks long. Working with the students for an entire year would allow for further development of their problem-solving skills. However, focusing on one unit afforded me the opportunity to look more in depth at the Focal PSCs. Another limitation of this study was the small sample size. I only taught one section of algebra, which limited my sample size to approximately 30 students. Another potential limitation is the fact that I served as both teacher and researcher for the study, which could introduce bias into the research results. Acting in the capacity of a combined teacher-researcher can potentially serve to limit the reader with only one viewpoint, namely, my viewpoint. There is no doubt that my observation lens and my interpretations may be different from that of another researcher. Clearly, my perspective, as pure and objective as I would like to maintain it, is unique to me.

## Summary

After reviewing the research question, the methodological choice for action research was discussed. I chose action research because I met the key tenets of research in action, namely, a collaborative partnership concurrent with action, and a problem-solving approach. The context of the study was detailed within a framework that included the state, school, classroom, and *CPM* curriculum. Finally, the design of the study was outlined using a three-phase format emphasizing pre-implementation, classroom implementation, and post-implementation activities.

# CHAPTER 4: FINDINGS

In this chapter, I review the research questions and then discuss the findings for each question. Recall that the overall research question for the study was: How is the *Common Core State Standards for Mathematics* (*CCSSM*) Problem-Solving Mathematical Standard enacted in an algebra class while using a *Standards*-based curriculum to teach a quadratics unit? The three sub-questions were:

- Q1. What opportunities to enact the components of the Problem-Solving Mathematical Standard are provided by the written curriculum?
- Q2. In what ways does the teachers' implementation of the quadratics unit diminish or enhance the opportunities to enact the components of the Problem-Solving Mathematical Standard provided by the written curriculum?
- Q3. In what ways does the teacher's enactment of problem-solving opportunities change over the course of the unit?

The sub-questions were addressed in different phases of the study. During Phase I of the study, Q1 (i.e., Investigation of the written curriculum) was investigated. In Phase II of the study (i.e., Enactment of the written curriculum), Q2 was enacted. Finally, in Phase III (i.e. Change across time) of the study, Q2 and Q3 were analyzed and discussed.

**Opportunities to Enact the Problem-Solving Components in the Written Curriculum**

Ten components (PSCs) of the Problem-Solving Standard of Mathematical Practice (SMP) were extracted from the description in *CCSSM*. Table 1 is repeated here for reference of these components.

Table 1. Problem-Solving Components of Mathematically Proficient Students (NGA & CCSSO, 2010)

| Component Number | Problem-Solving Component (PSC) |
|---|---|
| 1 | Explain to themselves the meaning of a problem |
| 2 | Look for entry points to a problem's solution |
| 3 | Analyze the givens, constraints, relationships, and goals |
| 4 | Make conjectures about the form and meaning of the solution |
| 5 | Plan a solution pathway |
| 6 | Consider analogous problems and try special cases and simpler forms |
| 7 | Monitor and evaluate their progress and change course if necessary |
| 8 | Explain correspondences between representations |
| 9 | Check their answers and ask themselves "Does this make sense?" |
| 10 | Understand the approaches of others |

PSC 3 (*Analysis*), PSC 8(*Representations*), and PSC 10 (*Approaches of Others*) emerged as the leading components in terms of the number of instances explicitly found in the written curriculum from Phase I of the study. The presence of these three Focal PSCs, *Analysis, Representations,* and *Approaches of Others* were coded throughout the unit and the number of opportunities provided by the written curriculum for each of the Focal PSCs is summarized in Table 8.

Table 8. Number of opportunities for the Focal PSCs in the written curriculum

| *Lesson* | *Analysis* | *Representations* | *Approaches of Others* | Total |
|---|---|---|---|---|
| 1 | 7 | 0 | 9 | 17 |
| 2 | 10 | 11 | 2 | 24 |
| 3 | 3 | 3 | 2 | 8 |
| 4 | 4 | 1 | 3 | 8 |
| 5 | 10 | 3 | 3 | 16 |
| 6 | 9 | 12 | 1 | 22 |
| 7 | 20 | 11 | 1 | 32 |
| 8 | 19 | 15 | 2 | 36 |
| 9 | DNE[a] | DNE[a] | DNE[a] | DNE[a] |
| 10 | 16 | 21 | 5 | 42 |
| 11 | 13 | 12 | 8 | 33 |
| 12 | 18 | 7 | 2 | 27 |
| 13 | 16 | 6 | 5 | 27 |
| 14 | 17 | 9 | 7 | 33 |
| Total | 164 | 111 | 50 | 325 |

[a.]DNE means did not enact. Lesson 9 was not enacted due to time constraints imposed by the leadership at the high school under study. The content of Lesson 9 was not omitted but was incorporated into Lessons 6 through 8.

The table, which provides the number of opportunities provided by the written curriculum for each of the Focal PSCs in every lesson of the quadratics unit, suggests that in each lesson, there were ample opportunities for the students to engage with the Focal PSCs, as specified in the written curriculum. I noticed that *Analysis* is the most prevalent Focal PSC in the written curriculum, whereas *Approaches of Others* was the least prevalent. This could be because *Analysis* is easier to write into a curriculum since students can analyze both alone and together, whereas *Approaches of Others* can only occur when students are working in groups, or as an entire class.

The written curriculum provided opportunities for students to solve problems using a multifaceted approach. In the *CPM* curriculum, students analyzed, looked at multiple representations of quadratic functions alone, in groups, and as an entire class. They had opportunities to understand the approaches of other students both in their respective groups and as an entire class. Table 8 indicates that each Focal PSC was represented in the written curriculum for each lesson, except for the *Representations* Focal PSC was absent in Lesson 1.

### ***Analysis* Focal PSC**

Recall that the *Analysis* Focal PSC requires that students analyze the givens, constraints, relationships and goals of a problem. This means that students must analyze everything about a problem, not just the prescribed goal. They must understand what information they have been given, along with the constraints of the problem. They must make connections about the relationships presented in the problem and understand what the problem is asking them to find. Too often, students just focus on the goal of the problem and overlook analyzing the other aspects.

From Table 9, it can be determined that students had multiple opportunities to engage with the four stated aspects of the *Analysis* Focal PSC; the *CPM* curriculum afforded students the opportunities to engage with each of these four aspects of *Analysis*, providing attention to each part of *Analysis*, not just the goal of the problem. Focusing on the four aspects of the *Analysis* Focal PSC in the written curriculum emphasizes to students the importance of each aspect when analyzing a problem.

Table 9. Analysis Focal PSC Examples (Dietiker et al., 2013)

| Aspect of Focal PSC | Lesson | Example | Explanation |
|---|---|---|---|
| Givens | 2 | *Kelly wants to find a shortcut to factor $2x^2+7x+6$. She knows that $2x^2$ and 6 go into the rectangle in the locations shown at right. She also remembers Casey's pattern for diagonals. Without actually factoring yet, what do you know about the missing two parts of the generic rectangle?* (p. 727). | In this example, students are not focused on the actual factoring of the problem yet. Instead, they are looking at what they are given – Casey's pattern for diagonals, and the generic rectangle for the quadratic expression. Before trying to factor the quadratic expression, students are asked what they know about the missing parts of the generic rectangle, part of the given information of the problem. |
| Constraints | 7 | *Ask, 'Can we use the Zero Product Property to solve this? Why or why not?' Then have teams factor the quadratic so that they have a product equal to zero: 0 = (x + 4)(2x - 3) Ask, "If (x + 4) times (2x - 3) equals 0, then what does the Zero Product Property tell us?" When a student volunteers that one of the expressions must equal zero, take the opportunity to show students how you want them to record this information.* (p. 757) | The constraints of this problem are whether or not the Zero Product Property can be used to solve this equation, and what using the Zero Product Property means in this particular problem. If the Zero Product Property cannot be used, how can this problem be solved? If it can be used, what does that tell us for this particular equation? What would the next step look like? These are all the questions that need to be answered given the constraints of the problem. |

Table 9 continued

| | | | |
|---|---|---|---|
| Relationships | 8 | *Students will use graphing calculators and the graphing form of quadratic equations to find the x-intercepts and vertex of a parabola. Students will use square roots to solve an equation*. (p. 764) | In this example, students are analyzing the relationship between what they see on the graphing calculator and the graphing form of their quadratic equation. They are asked to find the x-intercepts both graphically and mathematically from the graphing form, which requires students to analyze the relationship between the equation and what they see on the graphing calculator. |
| Goals | 5 | *Start the lesson by asking a volunteer to read the lesson introduction. Then read the directions for problem 8-45 together as a class. Be sure students understand the goal of this activity: to factor each quadratic and look for patterns in both the sum and the product forms of the quadratics. You may want to point out the quadratics from problem 8-46 that you posed on a side whiteboard and state, 'By the end of this lesson, you will be able to use the patterns you find to factor several of these quadratics quickly without a generic rectangle.'* (p. 742) | By having a student read the lesson introduction and stressing the goal of the activity, to factor each quadratic expression and look for patterns in both the sum and the product forms, students are provided with clear guidance and understand what the end result should look like. The goal is not simply to factor the quadratic expression and stop. They are to look for patterns as well. Stressing this to students helps to clarify complex instructions and ensure that everyone is completing the entire task at hand. |

The maximum number of coded opportunities for the *Analysis* Focal PSC appeared in Lesson 7. This lesson contained 20 instances in which students analyzed the givens, constraints, relationships, and goals of a problem. The focus of Lesson 7 was to make connections between quadratic equations and their graphs. For example, Problem

8-64 asked the students what information was needed to sketch a parabola. It went on to have students analyze how many points are needed to sketch a graph, and which points are the most useful when sketching a graph, such as the x-intercepts, the vertex and the y-intercept. Problem 8-65 in this same lesson extended the students' level of analysis by helping them determine if one can find the x-intercepts of the parabola from the equation. As the written curriculum of this quadratics unit progressed, the number of opportunities for student engagement with the *Analysis* Focal PSC increased. For example, during Lessons 1-6, the maximum number of opportunities for analysis during an individual lesson was 10; this occurred in Lesson 2, during which students were instructed how to factor quadratic expressions without algebra tiles. Problem 8-14 stated "to develop a method for factoring without algebra tiles, first model how to factor with algebra tiles and then look for connections within a generic rectangle" (Dietiker et al., 2013, p. 372). The written curriculum was developed in a manner that afforded students the opportunity to analyze factoring with algebra tiles and learn how to factor the quadratic expressions without tiles. Problem 8-15 went on to explain the importance of learning to factor without algebra tiles: "Factoring with a generic rectangle is especially convenient when algebra tiles are not available or when the number of necessary tiles becomes too large to manage" (Dietiker et al., 2013, p. 373). The mean number of opportunities for *Analysis* during Lessons 1-6 was 7.5 with a median of 8. However, beginning with Lesson 7, subsequent lessons contained no less than 13 opportunities, with a mean and median of 17. Lesson 7 contained 20 opportunities from the written curriculum for students to engage with the *Analysis* Focal PSC. In Lessons 1-5, students were learning the procedure to factor (e.g. factoring trinomials, binomials, perfect square trinomials), which may possibly explain why the *Analysis* Focal PSC was more prominent in subsequent lessons. Beginning with Lesson 6, the lesson focus switched to parabolas, thus moving away from the procedure of factoring quadratic expressions and directing attention towards the development of multiple representations of quadratic functions. While looking at multiple representations of quadratic functions, primarily equations and their corresponding graphs, and tables with their respective equations and graphs, the curriculum provided scaffolding to guide students in their analyses and problem solving, by directing their attention to the relationships and connections between various representations of quadratic functions. In Lesson 7, which contained the maximum of 20 opportunities from the written curriculum for students to engage with the *Analysis* Focal PSC, students learned that solving a quadratic equation by factoring and applying the Zero Product Property resulted in the graphic solution of the x-intercepts of a parabola.

Also, in Lesson 7, Problem 8-66 required students to analyze the Zero Product Property: "The equation you wrote in part (c) of problem 8-65 is called a quadratic equation. To solve it, you need to examine what you know about zero. Study the special properties of zero below” (Dietiker et al., 2013, p. 392). This problem described examining, a.k.a. analyzing, what students knew about zero and its special properties. This was done in part by having the students play a game. “I need two volunteers,” I said. “Cole and Jase, come see me.” “I’m going to give them each a secret number and then they are only going to share their number with each other, but they will tell you (the class) their product. You have to guess the numbers.” “Tell them your product.” Cole said “16.” Immediately students started guessing. Oscar exclaimed “4 and 4.” Ansel chimed in “2 and 8.” “Ok,” I said. “So, what did you know about their numbers?” “Uh, we just started naming factors" Tom said. "Ok great." I replied. "I need two more volunteers." Erin and Oscar stood up. I gave them their secret numbers and then said, "tell them your product." Oscar smiled and said "Zero." Immediately students started yelling "zero and 17" "zero and 10" "zero and some number," exclaimed James. "Ok, so you don't know the other number, do you? But you know one number –go ahead James." "One number is zero because zero times anything is zero," James explained. The goal of the game was to develop an understanding that, as long as one of the numbers is zero, the product of the numbers would always be zero. Students used this approach to gain an understanding of the special properties of zero and how these properties could be applied to solving quadratic equations. That is once a quadratic sum is written as a product, each factor can be set equal to zero to solve for x. This entire approach was written into the curriculum in such a manner that students learned in a fun and interactive way, rather than having the teacher, as the authority, say, "The Zero Product Property tells us that one of the factors must be zero, so set each factor equal to zero and solve for x." This interactive method enabled the students to better comprehend and remember the significance of the Zero Product Property through the problem-solving process.

**Representations Focal PSC**

The *Representations* Focal PSC was coded whenever students were provided curriculum-based opportunities to explain relationships among representations, such as a table, word problem, graph, function, or physical model. Examples illustrating relationships between different representations can be found in Table 10.

Table 10. Representations Focal PSC Examples (Dietiker et al., 2013)

| Representations | Lesson | Example |
|---|---|---|
| Graph compared to an equation | 7 | *They can sketch the graph of a quadratic rule quickly, using its intercepts. Students will also learn how to find the x-intercepts of a parabola by factoring the corresponding quadratic equation and applying the Zero Product Property.* (p. 756) |
| Graph compared to table | 6 | *Students will be given information about four water balloon launches, each in a different representation. They will need to analyze the information given and create a table and graph on the resource page for each launch.* (p. 749) |
| Function compared to physical model | 2 | *First model how to factor with algebra tiles, and then look for connections within a generic rectangle* (p. 726) |

The *Representations* PSC was the only Focal PSC to be absent from a lesson (Lesson 1) in the entire quadratics unit of the *CPM* curriculum. In Lesson 1, students were introduced to factoring quadratic expressions; however, they did not have opportunities to relate these expressions to another form of representation. While multiple representations were used in this lesson, there was not enough evidence to enable the coding of the *Representations* PSC by simply using multiple representations. This is, students had to have been given the opportunity to explain corresponding relationships between the different representations in order to code this PSC; thus, it was not coded.

While the authors of the *CPM* curriculum did not have students discuss or explain the similarities between the representations in the first lesson, it is important to remember that Lesson 1 was only an introduction to the unit. This unit was focused on building rectangles using algebra tiles as a process for factoring quadratic expressions. It required students to recall how to multiply expressions using algebra tiles and generic rectangles, a process that was introduced five chapters earlier. However, the technique involving the use of generic rectangles and algebra tiles to multiply expressions had not been used and reinforced in a sufficient number of class periods; thus, students needed a refresher. Ideally, all students would easily recall this process and this introductory lesson would not be needed; however, the authors of the *CPM* curriculum likely sensed the complex nature of the topic and included a review lesson to help the students develop a solid understanding of the concept of identifying *Representations* before moving forward with factoring. Thus, the next lesson built upon the processes that students practiced and developed in Lesson 1.

In Lesson 2, the curriculum provided 11 opportunities to explain correspondences between the representations presented in Lesson 1. The maximum number of coded opportunities for the *Representations* PSC appeared in Lesson 10. This lesson contained 21 instances in which students compared and contrasted representations of quadratic functions. The focus of this lesson involved completing the square, a method used to change a quadratic equation from standard form ($y = ax^2 + bx + c$) into graphing form ($y = a(x - h)^2 + k$), also known as vertex form. In the opening problem, 8-98, the curriculum provided the standard form of a quadratic equation and students were asked to use the *Zero Product Property* to find the vertex. Previously, students would find the x-intercepts (which were always found via examples that factored) and then, by managing to locate the axis of symmetry (via the midpoint of the x-intercepts) the midpoint x-value would then be substituted into the standard form of the quadratic equation to determine the corresponding maximum or minimum y-value of the vertex. In problem 8-98, the quadratic equation was not factorable, which meant that students could only use their graphing calculators to graph and estimate the values of the x-intercepts and the vertex. The next problem, 8-99, provided students with the graphing form of the same quadratic equation. It asked students to find the x-intercepts and vertex of the graph. In this example, the x-intercepts contained the square root of a natural number with a non-perfect square result. Students were then asked to compare and contrast the standard form and graphing form of quadratic equations to determine which was easier to use to find the vertex, x-intercepts, and y-intercept.

In Lesson 10, students were actively moving between multiple representations of quadratic equations. They completed the following tasks: (1) compared graphing form (e.g., vertex form: $(x - h)^2 = a(y - k)$) to standard form (e.g., $y = ax^2 + bx + c$), (2) used algebra tiles to represent a quadratic equation, and (3) changed from standard form into graphing form via the process of completing the square, while using algebra tiles, and an equation mat. The algebra tiles helped students to see that it was impossible to factor the problem into a square with algebra tiles unless one added or subtracted tiles. The equation mat helped students to grasp the concept of equality, meaning that if tiles were added or subtracted to make a square, one must do so in some fashion on the other side of the equation, to preserve equality. Additionally, in this lesson, by using algebra tiles, students learned to model a quadratic equation by constructing a square out of the algebra tiles. This was readily accomplished when the quadratic equation factored; when it did not factor, it was impossible to use tiles to form or complete a square. The process of completing the square referred to how many tiles students needed to add (or remove) to fill in the corner of their modeled quadratic equation, thus creating a square. By using an equation mat in Lesson 10, the students learned that the number of tiles added/subtracted in the process of completing the square must also be added/subtracted at the other side of the equation mat (where the y is) in order to keep the equation balanced. From there, students would factor the equation represented by the tiles in the square, and then write the equation in graphing form by isolating the y tile (see Figure 4 below). By working with multiple representations, students had opportunities to solve problems and to develop fluency in comparing and moving between representations.

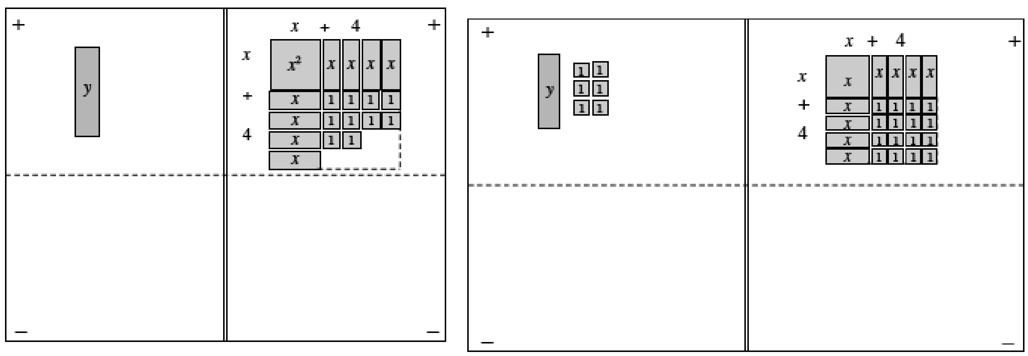



Figure 4. Completing the square with an algebra mat.

**Approaches of Others Focal PSC**

Although *CPM* promotes the daily use of collaborative learning, we only noted instances in which students were explicitly asked to use the *Approaches of Others* Focal PSC*;* this was coded whenever students were provided opportunities via the written curriculum to understand the approaches of others. The significance of understanding the approach that other students (different than any particular individual student) take in solving a problem, serves to imply that the problem-solving process is different for each individual. Clearly, students develop into better problem solvers by learning multiple ways to solve a problem and choosing the method that is the most efficient and applicable to the problem at hand. An example of the *Approaches of Others* PSC from Lesson 1 is the following: “Work with your team to find the sum and the product for the following generic rectangles” (Dietiker et al., 2013, p. 719). This problem asked students to work in their groups, discussing any strategies they discovered that could help determine the structure of the rectangle. The *Approaches of Others* Focal PSC appeared in 50 instances (approximately 15% of the Focal PSCs) noted across the 14 lessons, the fewest number of times of the three Focal PSCs. Three lessons (Lessons 1, 11 and 14) contained more than five opportunities for students to engage with this Focal PSC.

In Lesson 1, an introduction to factoring, students worked with their group to find the sum and product for the generic rectangles provided. Figure 5 presents problem 8-3 in which students were asked to work with their teams to find the sum and product for the given rectangles. It asked students to discuss strategies they used to determine the dimensions of the rectangles. Lesson 1 provided nine opportunities for students to engage with the *Approaches of Others* Focal PSC.

a.

| | |
|---|---|
| $2x$ | $5$ |
| $6x^2$ | $15x$ |

b.

| | |
|---|---|
| $-2y$ | $-6$ |
| $5xy$ | $15x$ |

c.

| | |
|---|---|
| $-9x$ | $-12$ |
| $12x^2$ | $16x$ |

Figure 5. Generic rectangle examples used to find the product and sum. (Dietiker et al., 2013, p. 719)

Lesson 11, containing eight instances involving *Approaches of Others* focused on solving quadratic equations by factoring and using the Zero Product Property, and subsequently solving the quadratic equations by completing the square. The goal of this lesson was for students to learn that even if a quadratic equation cannot be factored, one can still find the x-intercepts of its parabola using the “completing the square” approach.

"Complete the square and solve. Does your solution match your estimate from part (b)" (Dietiker et al., 2013, p. 418)? (See Figure 6 for the equation mat students used to complete the square.) This lesson provided multiple opportunities for students to gain an understanding of the approaches of others.

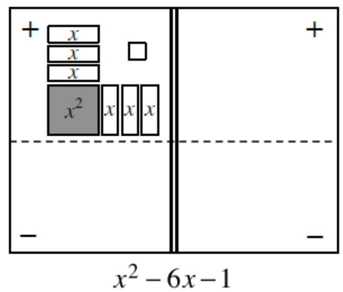



Figure 6. Equation mat students used to complete the square. (Dietiker et al., 2013, p. 418)

For example, a fictitious team was trying to solve a quadratic equation that could not be factored. One fictitious team member, Kira, concluded that the quadratic equation did not have a solution because it could not be factored. The written curriculum asked the actual students to discuss if Kira's response was correct and why, thus providing an opportunity for students to listen to one another, to discuss, and to familiarize themselves with the process used to evaluate Kira's reply. As analysis of the problem continued, another fictitious team member, Katelyn, suggested using a graphing calculator to see if there were solutions. Next, the written curriculum asked the actual students to discuss what Katelyn meant by using the graphing calculator to see if there were solutions. This scenario presented another opportunity for the actual students to observe other's thinking and approach to solving the problem. Then, another fictitious group member, Janelle, suggested completing the square to help them solve the quadratic equation. Several steps for this process were shown in the text, and again, the written curriculum asked the actual students to discuss what the next "move" should be. This event served, once again, to initiate the discussion among group members.

Finally, Lesson 14 provided opportunities to help students choose a strategy (Zero Product Property, Quadratic Formula, or Completing the Square) for solving a set of quadratic equations. Seven instances for students to enact the *Approaches of Others* Focal PSC were included in this lesson. For example, in one problem, six quadratic equations were listed, and groups were required to decide which solution strategy was most appropriate for each example. This problem created a plethora of opportunities for students to listen to one another and assess the approaches taken by others for solving the quadratic equation, including approaches that may have been different from their own. In groups, students were asked to try different strategies, and examine the different quadratic equations for clues to determine which strategy was best to use in which situation. After investigating the strategies used, the groups were brought together for a class discussion of the six quadratic equations. This class discussion, as detailed in the written curriculum, provided yet further opportunities for students to learn and understand how different groups approached the analysis and solution of the same problem. This series of events subsequently led to a class discussion about the best strategies to try first when attempting to solve a quadratic equation, and to identify situations when other methods were more preferred. Hence, I posed the question, "When is factoring easiest?" James responded, "When there's a GCF." I replied "Ok, so if there's a GCF. What else? Maybe like part C, it was already factored. Maybe if it just looks factorable because it's nice numbers." I then asked students to consider another scenario. “Completing the square. When would that be easier to use?” Tom chimed in “Perfect Squares.” Fiona added “When it’s not factorable.” I responded, “Or like in this case since the x squared term was just a one, that made it easier because you didn’t have to divide anything out and the middle term was an even number, so it cuts in half nicely.” Posing yet another scenario to students, I asked “What about Quadratic Formula?” James commented “Everything else.” Excited, I said “Good! Everything else.”

While the discussion was productive, students put their work up on the board and we talked about the different approaches available for solving the problem. Looking back, I still felt that I led and perhaps dominated the discussion too much as the teaching authority. It would have been great if a student would have said, “So how do we know when to use each method?” While this did not happen, my reluctant participation made me feel that I was still attempting to lead a student-centered classroom because in my traditional classroom I would have just said, “You use factoring when it factors nicely or see a GCF, you use completing the square when you see perfect square trinomials or notice the first term is x-squared with the middle term an even number, and you use Quadratic Formula for everything else.” However, in this student-centered classroom, I encouraged a clear and unmistakable experience in which students were, for the most part, the ones telling me when to use which method.

A possible explanation for the lower number of opportunities for this Focal PSC across the 14 lessons in the written curriculum is that this Focal PSC is best facilitated when students participate in group work, as in those situations where students present ideas and compare solution strategies and mathematical ideas with their peers. Including this *Approaches of Others* Focal PSC in a lesson via written curriculum alone (Phase I of this study) is difficult for a curriculum writer to do, because the process relies more upon the students and their engagement with the material and with each other than it does with the curriculum as written. The level of student engagement depends upon many factors. Two of these are the teacher and the learning environment created in a given classroom, factors which led into Phase II of this study. More opportunities for the *Approaches of Others* Focal PSC may arise by placing students into groups (which is a significant component of the *CPM* curriculum), providing an environment where students feel comfortable to express their ideas and question others, and through the use of carefully designed questions by the teacher, acting in the capacity of a facilitator of the classroom while the students grapple with the lesson's content.

## Summary

To recap the findings for Research Question 1 (i.e., Investigation of the written curriculum), the three Focal PSCs analyzed in the written curriculum were *Analysis, Representations,* and *Approaches of Others.* These were selected from the set of 10 PSCs based on the fact that they were found to be the most prevalent in Phase I. While there were multiple opportunities in each lesson for the enactment of the *Analysis* Focal PSC, it most often occurred in the later lessons after students were no longer learning to factor quadratic expressions. Student effort consisted mostly of analysis by concentrating on multiple representations of quadratic functions (e.g., graphing compared to equations) and the relationships and connections between these representations. Although the *Representations* Focal PSC was absent in Lesson 1 because students were learning the procedure of factoring with algebra tiles and not explicitly comparing representations, it was adequately represented in the remaining lessons. Specifically, an abundance of opportunities was found in later lessons because students were able to compare multiple representations, since they learned different representations of quadratic functions throughout the lessons. Finally, the *Approaches of Others* Focal PSC, which was promoted daily in *CPM*, was found through specific instances in each lesson. While the frequency of these instances was less than the other Focal PSCs, this occurrence could be explained because the *Approaches of Others* Focal PSC is not only difficult to write into the curriculum, but is also a concept that needs to be enacted in the classroom by the teacher acting as a facilitator of a process rather than a teacher of factual content among students as they learn to become better problem solvers.

## The Enactment of the Focal Problem-Solving Components in the Written Curriculum

The enactment of the Focal PSCs in the written curriculum was addressed through the following research sub-question: In what ways does the teachers' implementation of the quadratics unit diminish or enhance the opportunities to enact the components of the Problem- Solving Mathematical Standard provided by the written curriculum? This sub-question was answered during Phases II and III of the study (i.e., during my enactment of the written curriculum and change across time). Although the authors of the curriculum built in numerous opportunities for students to engage with the Focal PSCs, the teacher facilitating the curriculum ultimately decides, whether consciously or not, if that written opportunity will be enacted in the classroom. As both the teacher and the researcher of this study, I had a unique opportunity to not only facilitate the curriculum as the authors intended, but to enhance the lessons as well, by providing additional opportunities for students to engage with the Focal PSCs in each lesson. Unfortunately, having taught in a traditional style throughout my teaching career, the challenge of facilitating opportunities for students to enact the Focal PSCs turned out to be more demanding, but no less rewarding, than I anticipated.

When I taught in the traditional style, I somehow sensed that I was addressing the prescribed Focal PSCs. But, having taught the quadratics unit using a *Standards*-based curriculum, I came to realize that I rarely actually addressed the Focal PSCs of *Analysis*, *Representations,* and *Approaches of Others.* For instance, I would solve an example problem involving factoring equations with the students, then give them one just like it and have them factor it on their own. In my mind, I thought that this methodology was *Analysis*. But it really was not because students never considered the givens, constraints, relationships or goals of the problem in my traditional style of teaching. Instead, they were just going through the same motions that I had demonstrated for them in the previous problem, never considering that the problem could be expressed differently, because it never was. As for the *Representations* Focal PSC, I taught students multiple representations, but rarely did they use the concept of multiple representations to explain their thinking to compare the x-intercept values they just found using the "complete the square" method with values obtained using the graphing calculator. Also, we never used technology to graph or find x-intercepts in my traditional style of teaching. Instead, I had students manually determine the x-intercepts, and then graph the parabola using the x-intercepts they had found. There was no comparison or critical thought given to the solutions obtained using the manual and the graphical methods. Finally, in my traditional classroom, I thought that my teaching methodology satisfied the requirements of the *Approaches of Others* Focal PSC because my classroom was filled with students answering questions and students sitting in groups. But those questions were almost always posed by me, the sole authority of the classroom. I always responded to students by saying things such as “good” or “not quite” when they answered a question I posed. Never did I say, “What do others think about that response?” or “Zach, can you answer Sarah’s question?” I rarely provided students a chance to become the authority in the classroom, helping each other to understand the material, even though I did not consciously see myself as the sole authority in the classroom. It was not until I analyzed my teaching methodology that I realized how much of an authority role I maintained in the classroom, and how much I interjected when students would ask a question or start to answer incorrectly; without a doubt, I wanted the students to feel welcomed and safe in my classroom. Using this familiar and traditional approach, I came to realize that by depriving students of an authority role in the problem-solving process I was over-protecting them, that I instilled in them a fear of making mistakes or answering incorrectly, because I only displayed the correct answers or the work of students who solved the problem exactly as I had previously shown. While I believed my classroom was one where students felt safe to ask questions, in reality, this was a false sense of security because I continually praised students for being correct and immediately corrected students whose answers were incorrect. As a class, we never took the time to critically analyze the manual vs. graphical methods of working out a problem, to look over student work to see if the solution procedures were mathematically sound, or how to correct them. Instead, we only focused on the work of students who had solved the problem correctly and efficiently. This approach cannot be called analysis because nothing was analyzed, and because it reduced numerous opportunities for my students to not only serve in an authoritative capacity, but also to develop a “fearless” mindset regarding the development of skills needed to become critical and proficient math problem solvers. While I thought we were tackling problem solving together as a class, in reality, I was leading them every step of the way, diminishing their opportunities to analyze problems, to compare multiple representations, and to understand the approaches of others in the classroom.

In the next sections I will present examples from each Focal PSC in which I performed the following: (1) I diminished an opportunity for students to engage with a Focal PSC, (2) I followed the written curriculum, thus providing opportunities for students to engage with the Focal PSC, and (3) I enhanced the lesson by providing students additional opportunities to practice each Focal PSC beyond those identified in the written curriculum.

Because I was new to teaching from a *Standards*-based curriculum and teaching is complex in nature, I knew it would be challenging to enhance every lesson by providing additional opportunities not in the written curriculum for the students to practice each Focal PSC. It would also be difficult to enact every opportunity as suggested by the written curriculum because curriculum writers are unaware of particular contextual circumstances in any particular school. They are also not familiar with my students or the norms established in the school or in my classroom. Finally, as a teacher, I make instantaneous decisions in my classroom based on many factors; many of these would not be foreseeable by the curriculum authors.

Because this study, in part, represents an account of my journey involving a marked change in teaching styles, I feel that it is important to share the failures, as well as the successes of my experience in learning to teach using the new approach. While I believed my identity as a teacher was one where students felt safe to ask questions and make mistakes and learn from those mistakes, through this journey I learned that my true identity in the classroom was that of the mathematical authority figure, the knowledge provider, and the only person able to determine if something was correct. This false identity became very apparent when examining daily videotaped lessons and reflecting on the events captured with my critical friend during our weekly meetings. Even when watching the recorded video sessions on my own, I still thought that I was providing the prescribed Focal PSCs' opportunities for my students. It was not until my coding partner would point out, in Phase III, why certain situations were not really *Analysis*, or *Representations*, or *Approaches of Others*, that I became fully aware of my false sense of identity in the classroom.

### Enacting the *Analysis* Focal PSC

In Lesson 5, the written curriculum suggested assigning each group several quadratic expressions to factor. The written curriculum required students to put each problem on poster paper and tape them to the board. Students then grouped similar problems using guidelines provided by the written curriculum. However, in my enactment, I diminished an opportunity for students to analyze how the problems in each group were similar. The videotapes show this happened when I unwittingly instructed students that one group of problems was of an unusual type, commonly known as "perfect square trinomials." Students grouped the quadratic expressions together simply because the trinomials factored into products that could be written as the quantity squared (i.e., $(x + 5)^2$), but never looked at the terms in the trinomials. I could have let them analyze the middle terms to determine that the pattern of the middle terms was twice the product of the square roots of the first and last terms. However, I intervened and showed the students this pattern instead, thus diminishing their opportunity to analyze the quadratic expressions and discover the unusual pattern on their own, which would have had been more meaningful. The dialog that captured my oversight included the following:

> Yesterday right when the bell rang, you guys had just come up with putting these into groups, I said. These are called perfect square trinomials because all of these can be written as something times itself, or you write it as the quantity squared. Now the reason they are called perfect square trinomials is actually because isn't four a perfect square? And so is x squared? And 25 is a perfect square as well, right? Students responded with a simple "yeah." I went on to say "If you look in the middle, we have 2x and five. If we do 2x times five and then we multiply by two, that actually gives us 20x. Ok same thing down here. If you do 3x times two and multiply it by two, that gives you 12x. If you do x times negative five and multiply it by a two that gives you negative 10x. Down here, if you do x times negative three and multiply it by two you get negative 6x. So perfect square trinomials all kind of have this pattern that when you multiply them together and multiply it by two you get the middle number. That's just kind of something interesting that could be a quick way you could factor it.

Notice that during this explanation, I was doing all of the talking. The only input students had was "Yeah." It could have been helpful if I would have asked students, "Do you notice anything about the middle terms of these grouped expressions in comparison with the first and third terms? Do you notice a pattern that they all share with respect to the middle term?" Perhaps this would have helped students notice that the middle term was double the product of the square roots of the first and third terms.

During Lesson 3, Problem 8-26, students were asked to factor $4x^2 - 10x - 6$, but they were unaware of the concept of completely factored problems (problems where all GCFs are factored as well as having the trinomial factored). Students analyzed this problem in groups, trying to build this expression using algebra tiles. Several groups found ways to factor this problem. Fiona's group came up with the expression (4x + 2)(x - 3) using algebra tiles, and James' group solution using algebra tiles was (2x - 6)(2x + 1). Fiona's group then tried solving the problem using the generic rectangle method, setting it up as sown in Figure 7. The group determined that this factored form was incorrect because a check of their work by multiplication of the greatest common factors resulted in an $8x^2$ term instead of the desired $4x^2$ term. These events brought the class to a discussion about completely factored and factoring out a greatest common factor before creating a generic rectangle.

| | | |
|---|---|---|
| -6 | -12x | -6 |
| 2x | $4x^2$ | 2x |
| | 4x | 2 |

Figure 7. Fiona's generic rectangle

This was an example in which I enacted the lesson as presented in the written curriculum. I did not help Fiona's group create its algebra tiles, nor did I encourage the members of the group to check their work by using a generic rectangle. They performed this analysis on their own. While they struggled to come up with the idea of factoring completely, they recognized that their factored form from the algebra tiles did not work in the generic rectangle when they checked their solution by multiplying (2x-6) and (4x+2). This lesson could have been enhanced by having students determine that they needed to factor out a greatest common factor before constructing a generic rectangle for factoring this problem. I could have asked the students, "Is there anything you notice about all of the terms? Could we do something first before completing a generic rectangle, to help?" Instead, once groups presented their work, I instructed the students to factor completely; I told them that they needed to factor out a greatest common factor first before completing the generic rectangle.

Sometimes I enhanced the lesson by providing students an additional opportunity to engage with the *Analysis* Focal PSC. One such occasion involved Lesson 12, Problem 9-15. I had students work in their groups to solve four quadratic equations using the Quadratic Formula. I enhanced this lesson by assigning four of the five groups the task of putting a problem on the board for discussions, while the fifth group would serve as the "checkers." "You guys are going to check everyone's work when they put their problem on the board," I said. "Compare your work with the work students put on the board, and you guys determine if there are any errors." Using this approach, I placed the solution evaluation authority into the students' hands, and I became a facilitator. In this *Analysis* Focal PSC example, the students in the "checkers" group took their role very seriously. They solved the problems together as a group, and then when another group brought their solved problems to the checker group, the checker group analyzed each step of the submitted work to help determine if the problem was solved correctly. "Does this look good", Mary asked the checker group. "'A' looks good" Ansel told the class. "So does 'B'", Jackson said. "'C' and 'D' are ok too" Ansel replied after checking over their work. Rather than the teacher being the authority and the one to determine correctness, a group of students became the teacher and authority for validating the solution of the problem. The concept of the "checker" group was something that I thought of in the moment, as a way to get all five groups involved because there were only four problems.

The technique proved to be successful and provided an additional opportunity for analysis of students' work.

**Enacting the *Representations* Focal PSC**

In the enactment of Lesson 2, students were exploring shortcuts to factoring without algebra tiles, using generic rectangles instead to factor the assigned problems. Fiona determined that the product of the two missing terms along the diagonal was equivalent to the product of the other two terms present in the generic rectangle representation of a quadratic trinomial expression. In her explanation of the identified pattern, she said, "We knew that 8 times 3 is 24, and then the only 2 numbers that equal 10 and are multiplied to be 24 are 6 and 4." Instead of asking her to generalize her findings to any quadratic trinomial placed into a generic rectangle, I interrupted her explanation and said, "Oh, ok, so you're saying if you multiply the diagonals." Although unintentional, my comment diminished the chance for Fiona to explain the pattern she perceived in the representation to her classmates.

After reflecting and analyzing the student responses, I realized that adjustments to the lesson were needed. Instead of acting as the authority, standing in front of the classroom by the example on the board, I should have had Fiona stand at the board to point and show her classmates where she saw the pattern as she explained the rationale behind her solution technique. I would then have asked her to generalize her findings or have asked her to put into written words how to find the pattern, hopefully getting her to say something like "the product of the missing terms' diagonal is the same as the product of the other terms' diagonal."

In Lesson 8, students investigated methods, other than factoring, to find the x-intercepts of a quadratic equation. The example used in this lesson provided me with an occasion to enact the lesson as presented in the written curriculum, thus providing students with the opportunity to represent the x-intercepts using multiple representations, through factoring and the use of the Zero Product Property when the quadratic function factors, and through approximation on a graphing calculator whenever the quadratic function was not factorable. For example, in Problem 8-77 students tried to use the Zero Product Property to find the roots of the quadratic function $y = x^2 - 3x - 7$. "It doesn't factor", Oscar exclaimed. Once students determined this, the problem asked students to graph the quadratic function with a graphing calculator to see the roots and estimate them. "Oh, they're decimals", Fiona said to her group. Finally, students needed to use the table feature on the graphing calculator to estimate the roots. This is an exceptional exercise for problem solving because students found roots through multiple representations. It provided opportunities for students to observe that when the quadratic equation did not factor, the x-intercepts must be approximated. Upon reflecting on this lesson, I realized that this problem could have been enhanced by encouraging students to graph two quadratic functions (one that factored and one that did not) and then use the Zero feature of the calculator to compare the computed values of their x-intercepts. Also, students could have brainstormed in their groups to come up with ideas to determine how they thought these two examples could generalize for various quadratic functions. This methodology would have helped the students understand that when the quadratic function does not factor, the x-intercept values are expressed as decimal approximations (Note: students had not learned about the Quadratic Formula at this point in the unit).

Unfortunately, I did not create or enhance an opportunity for students to enact the *Representations* Focal PSC. One explanation for this can be attributed to the fact that in my traditional classroom, I did not facilitate this type of problem solving, and I was uncomfortable in my own teaching of the *Representations* Focal PSC. Also, my perception of the *Representations* Focal PSC content was that the authors of the *CPM* curriculum did a commendable job of facilitating this Focal PSC themselves, that it would take an experienced teacher of this curriculum to find ways to enhance the curriculum even further.

### Enacting the *Approaches of Others* Focal PSC

Lesson 2 provided an example in which I enacted the lesson as described in the written curriculum. In this lesson, students were asked to develop a method for factoring trinomials without algebra tiles. Because students were placed in groups daily, the nature of working together afforded many opportunities for enacting the *Approaches of Others* Focal PSC. In one group, Erin asked Jase for help. Jase was not using the tiles but was attempting to analyze the problem by drawing his own tiles on paper. Jase stopped his work and checked Erin's work by multiplying out the product she created with the tiles to see if it worked. Erin was able to create the rectangle with the tiles but did not understand how to write the sum as a product, given her newly created rectangle. "But you have to go down then over because it's length times width," he explained to Erin. Jase was interrupted by Erin exclaiming "Oh!" Erin could represent the situation with algebra tiles but did not understand how to write the sum as a product. Jase was able to explain to her that she had to write what she saw on the length and again on the width of her algebra tiles.

The observations serve to demonstrate that the *CPM* curriculum enactment created a situation that had the students working together to determine how to build the quadratic expression with tiles and then write the product as a sum. While the written curriculum did not necessarily anticipate Erin's struggle, the nature of working together in groups as suggested by the curriculum afforded an opportunity for Erin to understand what Jase was saying to her. Prior to this unit, if Erin would have gotten "stuck", she would have asked me, the authority, for help to quite possibly, solve the problem rather than understand the underlying principles that require a properly defined methodology to solve the problem. Instead, because students were working in groups, Erin and Jase worked together to help her understand how to write the sum as a product and why they needed to do this. This interaction illustrates the nature of students working in groups that led to many opportunities for students to understand the approaches of their peers. I could have enhanced this opportunity by asking Erin to explain to Jase how she created the rectangle, since he did not create it yet. This would have given Jase an opportunity to understand Erin's thinking, regarding the creation of the rectangle, as well. Although I missed the occasion to enhance this opportunity, my experience was a valuable one, and one that would likely not have evolved in a traditional classroom setting.

In Lesson 7, students explored the connections between graphs and quadratic equations written in standard form. This provides another example of a situation where I diminished an opportunity for students to learn from each other. Problem 8-64 asked: "Can you sketch a parabola if you only know where its y-intercept is? Consider the following example; if the y-intercept of a parabola is at (0, -15), can you sketch its graph? Why or why not?" (Dietiker et al., 2013, p. 391) Students wrestled with these questions in their respective groups.  For instance, Fiona said to her group, “No, if you only know its y-intercept you can’t,” while Jackson said, “But you could do it one way.”  He proceeded to get up and point to the Cartesian coordinate plane white board where someone had graphed a parabola from last night’s homework.  Fiona argued, “So you’re telling me that you can take that y-intercept right there and know exactly where the vertex and exactly where the x-intercepts are?”  Jackson responded, “Yeah, if you graph it.”  Jackson tried to argue his point by drawing a parabola that opened up, going through the y-intercept, and another that opened down, going through the y-intercept.  Fiona asked, “So how do you know if you can graph it?”  At this point, I heard a back-and-forth exchange from the group and approached them.  Instead of listening and letting them work it out on their own, I interrupted and said to Jackson, “When you were explaining it, you said it could go this way or this way, right? So how do you know which way?”  After I said this, Jackson agreed that one exact parabola could not be determined, but multiple possibilities existed, so the answer was no, a y-intercept was not enough information.  In this instance, if I would not have gone over to the group, Fiona may have been able to explain to Jackson that a single parabola could not be determined; thus, the y-intercept alone is not enough information to construct the parabola.  This was a perfect example of where Tyminski’s (2009) notion of teacher lust came into play and took over, hence eliminating the chance for Fiona’s group to solve the problem together through an *Approaches of Others* moment where Fiona potentially could have gotten Jackson to understand that one single parabola cannot be formed from the y-intercept alone.  Instead, if I would have stayed away from the group, they may have figured it out on their own; or

I could have asked Barry, another member in the group, for his thoughts on the situation since most of the debate was between Fiona and Jackson.

Lesson 14 is an example when I enhanced the opportunity. During this lesson, students were trying to determine which method to use to solve a quadratic equation. More specifically, the group that was assigned the quadratic equation $x^2 + 12x + 27 = 0$ chose the Quadratic Formula as a method to solve the problem. When discussing the problems as a class and looking at the work on the board for this problem, I asked a simple question, “What do you guys think?” which led several other students to suggest that factoring would have been easier. “It factors, so that would have been a lot faster than Quadratic Formula,” said Oscar. Asking this question was a “spur of the moment” decision, because I did not plan, ahead of time, to divide up the examples among the groups nor did I select the group who chose the Quadratic Formula option, when factoring would have been easier. It was a planned question though, in the sense that once I saw that the group had used the Quadratic Formula on the problem, I wanted to ask the opinions of others to see if any of them would have thought that another method would have been more appropriate, and they did. If I had not asked that question, students may have thought that the Quadratic Formula was the best method for that problem since that was what the group wrote on the board.

## Summary

To review, just because a Focal PSC was provided by the written curriculum, there is no guarantee that it will be enacted by the teacher, who can diminish the opportunity for enactment by interrupting, helping too much, not allowing students sufficient wait time, or answering questions of students, instead of responding with an open-ended question or redirecting the student to consult his/her peers. Because of my own teacher lust, there were times during the study in which I diminished the opportunity for students to engage in a Focal PSC by interjecting my help too quickly. While I did not find evidence of a clear pattern of repeated interjections, my first and natural response was to help the students. However, there were also times in which I enacted the lesson as prescribed by the written curriculum, thus providing students with the opportunity to engage with the Focal PSCs. Finally, I enhanced lessons whenever possible by providing additional opportunities to engage with the Focal PSCs. My experiences with enactment taught me that curriculum writers can provide opportunities in their lessons to engage with the Focal PSCs, ultimately, it is the teacher who facilitates the lesson and has the capacity to recognize/determine whether meaningful opportunities for engagement exist. My experience also taught me to be observant and receptive to the possibility of providing additional opportunities.

## The Change Across Time of the Teacher's Enactment of Problem-Solving Opportunities

Numerous researchers have written about changes in teaching style that a teacher can experience throughout his or her career (e.g., Adams, 2008; Bonner, 2006; Edwards & Hensien, 1999; Jaworski, 1998; Keazer, 2012; Khiat, Chia, Tan-Yeoh & Kok-Mak, 2011; Raymond & Leinenbach, 2000, Timmerman, 2003; Tinto et al., 1994). While teacher change is a desired outcome of action research, it is not easy. Recall that one of the goals of this study was to improve my own classroom implementation of a *Standards*-based curriculum and to refine my teaching methods of Algebra I in such a way that it was consistent with *CCSSM*, especially with the Problem-Solving SMP. The critical question that action researchers pose to themselves is, "How do I improve what I am doing?" (Whitehead, 1993). This is the question that I was striving to answer about my own classroom practice. More specifically, how could I develop strategies to facilitate students' problem-solving activities? In this section I will outline the changes that occurred over the course of this study, thus addressing Q3. These changes included the enactment of problem-solving opportunities, collaborative problem solving, mathematical discourse, questioning, and facilitation.

### Change in Enactment of Problem-Solving Opportunities

In the beginning of the study, as the teacher, I often did not know how to create opportunities for students to engage with the Focal PSCs. For example, in Lesson 4, Jase was at the board explaining his work to his classmates. When he finished, I simply said "Questions for him? Did you come up with anything different?" Jase was a student who often volunteered and typically had a correct answer. As a result, my perceived observations of Jase led me to believe that students may have felt intimidated by his strong knowledge of math, were reluctant to ask, or simply avoided asking questions, or that, perhaps, I had chosen Jase to put his work on the board because it was generally correct; all perceptions typical of me as a traditional teacher. Thus, even if students did have something different from Jase, they might have dismissed it as incorrect and assumed their way to be wrong and his way to be right. Looking back, I could have enhanced this lesson and thus provided additional opportunities for the students to engage in problem solving by asking another student to put his or her work on the board also, and then have the students discuss the similarities and differences. This would have provided an opportunity for students to engage with the *Approaches of Others* Focal PSC. Of course, not all of my questions were student specific. Some of the questions I typically asked the class early on included the following: “Questions?” “Does that make sense?” These questions seemed open-ended and provided me with a sense of encouragement before this study. Looking back, these questions are hardly adequate for initiating or maintaining critical dialog among students and the teacher in a mathematics classroom. In the context of the study, these are insufficient questions because they have simple yes or no responses instead of inviting students to think critically about their work. Some better questions would have been “Thoughts? Do you agree or disagree and why?”

By the middle of the study, I found myself attempting to provide students with opportunities to engage with the Focal PSCs, as in this conversation with Cole and James during Lesson 8. “When you have to graph, like, the points are, well, I don’t get how to find the two points,” Cole asked. I responded “Ok. Usually your y-intercept is not your vertex. So how do you find the vertex? James, can you help him out?”

By asking James if he could help Cole, I was attempting to engage the students in an *Approaches of Others* Focal PSC. Rather than implying that Cole needed help, I could have said “James, what are your thoughts?” This could have begun a discussion between James and Cole, rather than my original question “James, can you help him out?” which implied that Cole was wrong in his thinking.

Near the end of the study, while I was still developing my skills, I did enact problem solving opportunities suggested in the written curriculum. For example, in Lesson 13, the authors suggested to do one problem as a “carousel.” Six example problems of “solving quadratic equations” were put on the board. Groups were assigned one problem each, with specific instruction to “write the next step in the solution of the problem." The groups were then rotated, completing yet another step on a different problem. This suggestion by the *CPM* curriculum was very creative and by doing these problems in a carousel fashion, students were engaging in the *Approaches of Others* Focal PSC. I maintain this claim of engagement because students had to understand the work done by the previous student in order to be able to complete the next step of the problem. A carousel approach for student participation in problem solving is something that I had never done before. While I did not enhance the lesson by providing this opportunity myself, I chose to try the carousel method, as suggested by *CPM*. This method was not only fun for the students but challenging and engaging as well. The carousel was challenging because students had to analyze how others were thinking about the particular problem in front of them, understand the approach to the problem that the previous group took, and then pick up where the previous group left off by writing the next step of the exercise. It was engaging because students were not only jointly working examples in groups but were also analyzing the work of other groups at the same time without actually speaking to the other group. This was a great example of group problem solving. My decision to use the suggested carousel led to the enactment of the problem-solving opportunities in this instance.

Coming into this study, I thought I asked open-ended and encouraging questions that provided an environment in which students felt comfortable sharing their ideas. However, I quickly realized that I asked basic comprehension questions that created simple yes or no responses. Changing my practice to focus on multiple solution methods (not always the correct methods) or changing my questions to get students to discuss solution strategies with each other were simple concepts, yet difficult to enact. The changes did not always create an environment in which students felt comfortable with being uncomfortable with new knowledge, and that unfamiliarity would incite curiosity and provoke deeper learning. As for implementing the changes, it did not take much extra effort to walk around while students were working, to choose different methods to solve the same problem, and to have them put their work on the board. It only took a few extra minutes to compare the examples and talk about all methods so that students could learn from each other's mistakes. Taking the time to do this analysis could have answered questions that other students had, or it could have stopped other students from making similar mistakes.

While these changes did not take much extra effort, they did not come naturally to me. I was still concerned about embarrassing students by having them put their incorrect work on the board. This internal struggle was never resolved and made it difficult to change. In my teacher-centered classroom, I never would have tried a carousel. I would not have felt that students would have taken it seriously or engaged with the problems the way that they did. As a traditional teacher, I spent a considerable amount of time focused on the correct work and answer, and often underestimated the students' potential to analyze another person's work without me guiding them. My teacher lust, that desire to help them succeed, interfered with the real learning that could have taken place regardless of the fact that students had or did not have correct answers. Even if they would have completed the problems incorrectly, that opportunity could have been used to discuss the mistakes and the proper approach needed to solve problems correctly. These outcomes were observed during the session when the carousel method was used. Here, the students created meaningful learning experiences that were evident by their interactions with each other during the activity, by their responses to the work others had done, and by focusing on the next added step to the problem. Consider the following dialogue segment captured during a carousel session: Mary, talking to Erin in her group, said “Is this right? Can you do that?” Erin responded “Yes, the last group just put in 0x since that was missing.” Mary exclaimed “Oh! I get it.” “Rotate.” I yelled, “Go to the next one.” Jase, looking at the same problem Mary and Erin had just been working on said, “Don, you write the next step.” Don replied “Ok. (writes next step) is that right?” Jase commented “Oh yeah, that was important to square the negative. Good.” Students appeared to be finished with their next step, so I asked “Ok, are we good? Rotate!” Oscar, looking at the same problem now, seemed puzzled. “Why did they do that? There was an easier way to do this. But let’s do the next step now.” Rachael, confused by Oscar’s comment, asked “Oscar, how could it be easier?” “Factoring,” Oscar replied. “Oh yeah!” Rachael exclaimed.

The dialogue stream shows that students were able to solve problems together, to follow and interpret what the preceding groups were doing, to pick up and continue the solution logic made available to them and to write the next step in the solution of the problem. This *Approaches of Others* opportunity had students actively working together to solve problems and strive to understand each other’s thinking. From this activity, students learned firsthand that they can solve quadratic equations in multiple ways, and that sometimes, one method is easier to use than another, as was pointed out by Oscar to his group and by others later during a class discussion. In addition to the change in the enactment of the problem-solving opportunities, I also noted other changes including changes to response and participation behavior of different members in the groups, questioning, and facilitation. These important changes resulting from adoption of a new teaching technology based on a problem solving, *Standards*-based approach were documented as part of this study as they influenced and shaped the on-going transition of my teaching practice.

**Changes in Collaborative Problem Solving**

Facilitating collaborative problem solving in my classroom turned out to be a challenge worthy of note in this thesis. Other teacher-researchers have also written about their challenges with teaching in a student-centered classroom (e.g., Cady, 2006; Umbeck, 2011). I, too, struggled with the transition from a teacher-centered to a student-centered classroom. Just like Cady (2006), I found sharing ideas and talking with each other to be a major hurdle for my students. Although research indicates that students working in small groups learn more of what is taught and retain it longer than when taught through a teacher-centered, lecture format (e.g., Beckman, 1990; Lumpkin et. al., 2015; Slavin, 1987), the task of creating collaborative groups took more than physically putting student desks together. When students were placed in groups, they did not immediately interact and engage in discussions with one another. Instead, I encountered four individuals seated near each other, doing their own work, rarely talking or collaborating with one another. In another instance, when I was called over to a group, the request was typically made by one student and that one student had often failed to ask the group for help prior to calling me over for assistance. Additionally, I found struggles similar to those cited by Umbeck (2011) involving changes in classroom norms during this study. Initially, groupwork was unproductive, and students struggled with the new routine and wanted me to give them the reassurance they were used to from previous math classes (either with myself or other mathematics teachers). After several days, if not weeks, of this new methodology, students did finally accept the new norms and began to ask each other for help before raising their hand for my assistance. Part of this change could have occurred because I was not familiar with the concept of facilitating mathematical discussions in groups it simply took me longer to talk with each group and help them adjust to the changes introduced, thus students waited longer to have their questions answered. My assessment of the changing situation was that this delay could have led students to seek more immediate answers from their groupmates before asking for my assistance, or perhaps, students were accepting the new norms and were willing to give them a chance. Another explanation is that teaching in this new way is often more about what we do not do as teachers, than what we do; standing back and letting the students take control is at the center of *Standards*-based curricula. For the most part, by the middle of the study, collaboration among the groups was on the rise as evidenced by the fact that students were spending more time working on the assigned problems together rather than individually, checking with one another after getting stuck, and offering reassurance to each other. “I need help on this problem,” Erin said as she raised her hand, looking for my assistance. “Here, what do you need, ask me,” Jase replied. This exchange seems so simple, yet it is so powerful in terms of students accepting the new classroom norms and routines and finally helping each other first, before seeking assistance from the teacher.

**Changes in Mathematical Discourse**

Another closely related challenge encountered in the study was that of promoting mathematical discourse among students in the classroom. In my teacher-centered classroom, I often had students put their work on the board, but I was always the person going over the work, discussing it with the students, pointing out what was done correctly and fixing mistakes. In the new student-centered environment, I found that students would volunteer to put their work on the board but did not want to discuss their work with their classmates as I indeed, hoped they would. I genuinely struggled with this unanticipated event. Being mindful of students' insecurities in front of their classmates and wanting them to feel safe and secure in my classroom, I often encouraged students to explain their work on the board. However, if they were unwilling to do so, I did not push the issue or force them to discuss their work. Subsequently, I noticed that once other students realized that I would not force them to discuss their own work, more and more of them declined to explain the work they had volunteered to put on the board. This pattern of behavior is one that continued throughout the study. I never found a balance or a solution to promote student discussions of their work. However, I did find something to slightly help with this struggle. For example, I did notice that students were decidedly more comfortable talking about their work when seated in their respective groups, compared to standing at the board. Additionally, after the study had ended, my Phase III coding partner, a teacher experienced in facilitating rich mathematical discourse, suggested the following: have one student write up the work and have the entire group stand at the board ready to explain the work, or simply have another student in the group explain the work that was written on the board. My hope had been that students would have felt comfortable in my classroom, so that they would have readily followed my instructions to discuss their work. Unfortunately, the fear of standing in front of their peers, the fear of being judged, and/or their own insecurities, were larger than that of a high school student's desire to comply with an instructor's directions. Perhaps if I had started with this student-centered classroom environment with students discussing their mathematical ideas in front of the class at the beginning of the year (instead of the beginning of the $2^{nd}$ semester), then the reluctance to explain their work would not have been as challenging. While I anticipated mathematical discourse to be a struggle, I did not anticipate being so conflicted about how to handle the resistance from students when it came time to discuss their work. Initially I thought I would insist that they explain their work. However, I struggled imposing such a demand; after all, I wanted my classroom to be one in which students felt comfortable in all aspects of the learning experience, including the discussion of their work. These fears and uncertainties led me to giving in and not forcing students to openly discuss their work with their peers, even though I knew their mathematical understanding would improve by doing so.

**Changes in Questioning**

Cady (2006) used an adapted list of Driscoll's (1996) questions to help promote students' mathematical thinking during her study. Thus, I also adopted this practice, choosing Cady's adapted list of questions to help promote mathematical thinking in my classroom. See Table 5 (repeated from Chapter 3) for a list of these questions.

Table 5. Questions to Promote Mathematical Thinking (Cady, 2006, p. 461, adapted from Driscoll, 1996)

| **Organize information and thinking** | **Clarify responses** | **Keeping students focused** | **Promoting reflection** | **Exposing students' understanding** |
|---|---|---|---|---|
| What strategy could/did you use? | Why is this a reasonable answer? | What is the problem asking? | What other ways might work? | What would happen if …? |
| How could/did you organize your information? Your thinking? | How did you reach that conclusion? | How did you begin to think about this problem? | How does this relate to …? | What is the pattern or rule? |
| Could you organize your information another way? | Make a drawing to show that. | Tell me more about what you did. | What are some possibilities? | How did you think about the problem? |
| What information is needed? Not needed? | Explain how you did this part. | What else could you try? | How is this different from (same as) ______'s? | What predictions can you make? |
| What else do you know from the information given? | Does anyone have the same answer but a different way to explain it? | How can you check your answer? | Does that always work? | How could you prove that? |
| | | | What do you think about what ______ said? | What strategy was most helpful? Why? |

Unfortunately, I quickly realized that trying to look at the list in its entirety was overwhelming, and as a result, I found that I was not utilizing the list as I originally expected. I then tried to anticipate questions or struggles that students would have during the lesson that day and write down a few key questions from Cady's list for use during the lesson. I found this approach to be much more manageable during the lesson, but I still struggled with naturally asking students a question that would promote mathematical thinking or discourse in their group. For example, in my teacher-centered classroom, when students would pose a question, I immediately responded with a reply that featured an answer, a reassurance, a correction, or occasionally a question to challenge their thinking. In this new student-centered classroom, when students posed a question, I hesitated in giving a reply, pondered their question or looked for some "appropriate" question to ask them. Whether true or not, I felt that my use of hesitation made the student lose confidence in me as if I did not know the answer, rather than simply not knowing how to respond in a way that would elicit mathematical thinking, especially when trying to elicit a Focal PSC response. This insecurity was an internal struggle, and one that I did not realize or feel before in my years as an authority figure in the teacher-centered classroom. Yet, knowing how to respond to a student in the form of a question was something I did not anticipate being such an insecurity and difficulty during this study.

**Changes in Facilitation**

Like Umbeck (2011), I experienced an internal struggle releasing some control of the classroom. While I made the change at the beginning of the semester, to make the transition easier, some students had a pre-determined notion of expectations for a teacher-centered classroom, which consisted of familiarity, routine, and structure. I found myself missing this comfortable routine. However, I knew that this student-centered approach yielded enduring understandings of mathematics through mathematical discussions and problem solving with peers. I was determined to stick to the plan of implementing this norm for the entire quadratics unit.

Also, Boole's notion of teacher lust, which Tyminski (2009) warned about, was something I struggled with immensely during the study. Just as Tyminski pointed out, I diminished many opportunities for students to engage in mathematics, especially the Focal PSCs. While not intentional, I often vacillated between quickly answering a question for a student, providing the student affirmation or critique, or, restating the question for another student. Observations of my teaching and student responses led me to believe that this teaching style was not sustainable and one that warranted change; I discussed it frequently with my critical friend as I attempted to change. I even went as far as to cover my mouth with my hand while a student was talking, to remind myself not to interject and offer my opinion or guidance. I was genuinely determined to change, although in the moment of teaching, change was harder than I expected.

## Summary

While I knew how I wanted to change my classroom teaching style, implementing the change was more challenging than anticipated. From the onset, I never thought of myself as an absolute classroom authority, because I always maintained a relaxed classroom atmosphere and tried to make my students comfortable to approach me. However, this study showed me that students did view my presence as the authority and looked to me for correct answers, for approval, and/or for reassurance that they were on the right track. Teaching for many years in this manner made it difficult to change. Being empathic of students' fears and insecurities conflicted with my ideal mindset of empowering them to do something that would lead to mature growth in mathematics. While I wanted students to feel secure in my classroom, I wanted them to develop a level of confidence that would lead to discussions of mathematics with their peers. My own insecurities of not knowing how to respond to students in a way to get them to interact more with their group and stop looking to me for answers, while not being comfortable forcing students to talk in front of their peers, complicated this study. While it was easy to anticipate how I would handle situations in the classroom, it was hard to change in the moment.

# CHAPTER 5: IMPLICATIONS AND CONCLUSIONS

When I started this research, my intention was to improve the problem-solving skills of the students in my classroom through my facilitation of problem-solving activities. I employed action research methods of data collection and analysis in order to capture the nature of changes that were made in my classroom with my teaching practice. This chapter will review the findings of my research study, discuss the implications of this research, and conclude with suggestions for future research.

## A Summary of the Findings

My research revealed the following salient finding: while a written curriculum can provide opportunities for students to engage with the mathematics, the teacher's role is pivotal in determining what opportunities are enacted in the classroom. Through the teacher's enactment of the curriculum, opportunities for students to engage with the Focal PSCs can be diminished, enacted as written, or enhanced, all by the teacher. Thus, the teacher's role in the classroom is central to students learning to become better problem solvers. Additionally, change is gradual and challenging due to the complexities of teaching in the mathematics classroom and the school context. Continuous reflection and adaptation are essential when it comes to changing one's practice.

From the Pre-Implementation stage of my study (i.e., a review of the written curriculum), I discovered that the *CPM* curriculum contained numerous opportunities for potential student engagement with the Focal PSCs (i.e., *Analysis*, *Approaches of Others, Representations*). In fact, *Representations* was the only Focal PSC absent from any of the lessons; it was absent in the first lesson, which concentrated on students learning the process of factoring with algebra tiles instead of analyzing quadratic equations or the characteristics of quadratic functions. *Analysis* and *Approaches of Others* were both present in all lessons, with *Analysis* occurring the most often (63 instances throughout the 14 lessons). *Approaches of Others* was found the least number of times (20 instance, while 40 instances of *Representations* were found in the written curriculum. Reflecting upon the written curriculum, I found it notable that the *CPM* curriculum implemented all three Focal PSCs to such a great extent. Sixty-three opportunities for *Analysis* in one quadratics unit is a lot of analyzing for students. However, what is not as surprising, given the nature of this *Standards*-based curriculum, is that students are analyzing the mathematics on their own, in groups, and as a whole class.

During the Post-Implementation phase of the study (i.e., Data Analysis and Reflection), I learned that the teacher's role is crucial in the enactment of the curriculum to preserve the opportunities for students to engage with the Focal PSCs as found in the written curriculum. The teacher can diminish, enact as written, or enhance each opportunity and thus the teacher is vital in ensuring that students have as many opportunities as possible to engage with the Focal PSCs and mathematical problem solving. In fact, there were 14 instances where engagement with the Focal PSCs could not be coded because I diminished the opportunity for students to engage with the particular Focal PSC by leading the discussion too much, instead of letting students lead the discussion and learn from each other. As can be seen from this particular study, the classroom teacher can attempt to ensure that the opportunities to engage with the Focal PSCs, as written, are preserved and implemented through daily reflection upon the lessons and adapting their teaching to mirror these reflections, which was a key factor to promoting change in my own practice. Additionally, this reflection and adaptation facilitated my improvement and professional growth in teaching.

Over the course of the quadratics unit, my enactment of the problem-solving opportunities changed. First and foremost, I came to understand and acknowledge that my “teacher lust” was impeding student engagement with the Focal PSCs. Once I became aware of this, learning to overcome it and not diminish students’ opportunities became the next step. While I was never fully able to stop interrupting students to help them completely, this internal struggle was one that I was unaware of prior to this study; now that I have been made aware of it, I can continue to work on improving my practice and heeding Tyminski’s (2009) warning. If I want my students to become better problem solvers, it is important that I step back from the role of “mathematical knowledge giver” and become a facilitator in the classroom. Becoming this facilitator requires me to allow students to grapple with the mathematics more and to let them work together to discover the mathematics instead of the relying on the teacher being the holder of the knowledge. For example, in Lesson 7, Mary was asking for assistance and Erin started to explain the problem to her. Instead of letting this interaction play out, when Mary still expressed confusion, I interrupted Erin and began explaining the problem to Mary. Also, a shift in my thinking was required. I know that learning in this manner will result in students learning to persevere in problem solving. Part of this perseverance is going to include students struggling and grappling with the mathematics. My goal of wanting to help my students feel successful must shift to one in which I strive to help them to be successful by allowing them the freedom to understand the mathematics and discover it together.

While changing my mindset may be difficult, it is necessary for my own personal growth as a teacher and for my students' growth as problem solvers of mathematics.

## Post-Study Reflections

Since this study concluded, I have had a plethora of feelings. First, I felt successful for sticking with the study for an entire nine weeks of the school year. Next, I felt overwhelmed with the enormous amounts of data that I had collected and the daunting task of sifting through it. Additionally, I felt concerned about finishing the semester and still covering the remainder of the standards that my colleagues were teaching. Never having taught in this manner before, I was slow to learn the balance, pacing, timing, and other nuances of teaching with a *Standards*-based curriculum. Thus, this quadratics unit took much longer for me to teach than if I had taught it traditionally. Students were also not used to learning in this manner, so they were somewhat resistant and were slow to get comfortable learning this way. Once students accepted that this learning style was the new norm in our classroom and they got familiar with their groupmates, class began to have a new normalcy and flow to it, which helped to accelerate the daily routines in the classroom. Reflecting on the study, there were many days I felt like a failure or that students accomplished nothing in class, although this was not the case. Students were learning on a new, deeper level than ever before, and this learning looked different than in a traditional classroom. There were other days in which I felt like the learning was very impactful and that students understood concepts more deeply. For example, for the first time in my teaching career, students not only understood the process of completing the square for quadratics, but they enjoyed it and preferred it. Reflecting on this moment gave me satisfaction that what I was trying to accomplish was worth pushing toward and continuing my journey of growth and change.

### What Were the Contextual Challenges of Conducting the Study?

I faced many contextual challenges over the course of the study. First, I was required to teach all of the same topics as my colleagues who were also teaching Algebra I. Even though I was covering each topic in greater depth in the quadratics unit than others, I still needed to cover all of the same topics. Second, the weather during the study was very cold and snowy, thus impacting numerous teaching days, often shortening our class periods from an already short 50 minutes to an even shorter 25 minutes. Teaching with a *Standards*-based approach is not easy in a 50-minute class period and is even more challenging during a 25-minute class, thus lengthening the duration of the study. Finally, I felt pressure from colleagues to abandon the study and teach traditionally again due to the length of the study.

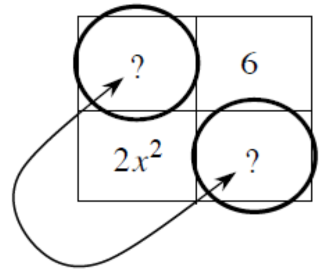
Once the study was over, I still needed to cover the rest of the semester's material into the remaining nine weeks, which was difficult especially when that timeframe included missed classroom time for standardized testing. Learning to teach with a *Standards*-based approach takes more planning time than I have during a teaching day. One 50-minute period a day to prepare for three different courses, grade, create lessons, and attend various meetings is not substantial enough time to plan when learning to implement rigorous tasks for students, nor does it allow for much time to reflect on a day's lesson, especially in the manner that is critical in order to improve my practice.

**What Were the Impacts of the Study on my Practice?**

In spite of the fact that I felt contextual burdens during and after the study, my teaching practice was positively impacted in many ways because of this study. It taught me a new appreciation for *Completing the Square*, as I now understood why the process was named as such. Before this study, I simply thought that the title *Completing the Square* just meant students were trying to create a perfect square trinomial. But during this study, I realized that students were physically creating a square with the algebra tiles during the process of *Completing the Square*.

It also showed me a new way to teach factoring to students, using the diamond-rectangle method described in the text. While I had seen multiple ways to teach factoring, I had never seen this diamond-rectangle method before the study. This diamond-rectangle method was especially useful for my struggling students, who normally did not do well with the traditional "guess and check" approach to factoring. It was useful for struggling students because the diamond became an organizational tool for students and multiplying the diagonals of the rectangle for checking purposes also reassured students that they had found the correct combination of factors. *Figure 8* below shows a portion of problem 8.14 from Lesson 2 which demonstrates an example of the diamond-rectangle method for factoring.

c. Kelly wants to find a shortcut to factor $2x^2 + 7x + 6$. She knows that $2x^2$ and $6$ go into the rectangle in the locations shown at right. She also remembers Casey's pattern for diagonals. Without actually factoring yet, what do you know about the missing two parts of the generic rectangle? **[ Their sum is $7x$, and their product is $12x^2$. ]**




d. To complete Kelly's generic rectangle, you need two $x$-terms that have a sum of $7x$ and a product of $12x^2$. Create and solve a Diamond Problem that represents this situation. **[ The product $12x^2$ should be placed at the top of the diamond problem, $7x$ at the bottom, and terms $3x$ and $4x$ should be in the middle. ]**

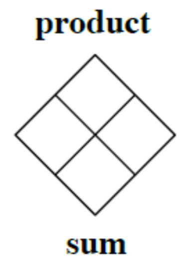



Figure 8. The diamond-rectangle method for factoring.

This study taught me to incorporate new formative assessments into my practice. I had never considered a "carousel" before, nor had I thought about teaching factoring and then having students discover how to group the factoring into categories such as perfect square trinomials or difference of squares. This lesson, Lesson 5, was probably the most powerful to me in the unit because it was exciting to watch students discover ways to group the quadratic expressions into categories and it was something I had not ever considered doing before. Students were eager to group the factored problems and they came up with a lot of ideas for why the problems in the groups were similar.

Most importantly, this study opened my eyes to the reality of my classroom and teaching. I was not previously asking open-ended questions, nor was I engaging students in higher level thinking. Instead, my questions typically elicited a yes or no response and it was not until I watched the classroom recordings that I realized that often, students were answering yes without putting much thought behind their response. Those responses led to me having a false belief that students were understanding the material. I also did not realize how often I diminished opportunities for students to help each other, instead allowing my desire to help overtake the conversation as I remained as the sole authority in the classroom. In my traditional style classroom, I was the “giver of knowledge” and how I saw things was how my students were expected to see things. In this new student-centered classroom, my students were creating the mathematics that I once just told them. They were understanding the math more, and remembering the small details, reminding each other by saying “remember that game we played with the Zero Product Property” or “remember the diagonal’s trick to check your work”. Before, it was always me saying those things to them and they would not remember. But now, in this new student-centered classroom, students would respond “oh yeah!” and I could tell that they actually did remember, rather than just saying “yes” to appease me.

Challenges after this study are still present; I still grapple with the balance between having students feel comfortable in my classroom and having students put their work on the board, even if it does not lead to a correct answer. During the study, students would happily volunteer to put their work on the board but were resistant to explain their work to their peers, fearful of being incorrect or judged by their classmates. While I was happy that they volunteered to put their work on the board, I did not push them enough out of their comfort zone to explain their work to their peers. Instead, I was torn by wanting my students to feel comfortable in my classroom, and they were still anxious of being judged by their peers, especially once they realized that sometimes work with mistakes was being put on the board for learning purposes. I never found a balance to this struggle. As reported in Chapter 4, while I believed that my classroom was one where students felt safe to ask questions, because I typically praised correct answers, and immediately corrected students who were incorrect, this was a false sense of security. I still find that my teacher lust can interrupt and diminish opportunities for students to explain their understanding to others. After teaching for so many years in a traditional manner, it is difficult to change what comes so naturally to me now. Changing my mindset to one where I allow students to grapple with the mathematics and learn to persevere is not something that comes naturally to me because of my teacher lust; instead I want to immediately help them and steer them in a correct direction whenever I see them struggling.

Also, it is easy to plan to allow students to persevere with problem solving, however, in the moment, during the enactment of the lessons, it is impossible to script what students will do, thus making my responses even that much more difficult to control. I want to be quiet and allow students to work out the problems together, without my help; I want to question students in ways that guide them without helping too much. But in the moment, it is difficult for me to think of ways to do this naturally and quickly.

I still feel pressure from my colleagues, students, parents, and school administrators to teach in the same manner that my colleagues are teaching: the traditional approach. At PHS, the tradition is for everyone to be doing the same thing, at relatively the same time, giving the same assessments, in an attempt to ensure continuity in depth and topics among teachers. Because of this tradition, I feel the pressure to conform, even though my research and experience with this study has shown me different approaches and ways of teaching that I previously was unaware of in my traditional classroom. Going against the tradition is difficult in such a large school, especially when parents talk and have a large influence on the school climate.

Teaching in this manner is still a challenge for me, especially with the breadth of topics I must cover in a semester and the numerous interruptions to my class time. In a public K-12 setting, in a large school like PHS, there are numerous interruptions to my class time for testing, convocations, scheduling, the office calling for students, shortened classes for weather, etc. All of these interruptions take away from an already short 50-minute class period. Additionally, teaching from a *Standards*-based curriculum takes additional time – students need time to persevere in problem solving, especially when they are not used to learning in this way. The *CPM* curriculum had students make great connections among topics, more so than in a traditional classroom, but making these connections takes time. However, because I am not the only algebra teacher at PHS, I am required to teach the same topics that my colleagues are teaching. They are teaching in a traditional style, so they do not cover topics in as much depth, or if they do, they do not allow time for students to create the mathematics, thus they are able to cover topics more quickly than I can teaching from a *Standards*-based curriculum.

One aspect I find significant from this study is how I see students in terms of their perseverance. Before this study began, I never would have thought my students would persevere in learning in a student-centered, problem-based curriculum. I anticipated students giving up and going to the principal and that I would be forced to stop my study to return to teaching traditionally. Fortunately, this did not happen. Instead, I was pleasantly surprised with my students' perseverance. They struggled together, learned together, and found success together. They learned to help each other learn, more than any other semester of my entire teaching career.

## Implications

Based on my findings, I conclude with implications of this study for my practice, for teachers' practices and for researchers. In terms of my practice, I will discuss how this study serves me as professional development, how videotaping effected the classroom, how reflection affected me, and how coding impacted me and others. I will also discuss how I learned from my critical friend, and through the process of action research. I conclude with changes to my teaching, how this study can serve other teachers and other researchers too.

### How Can This Study Serve My Practice?

**Professional development.** For starters, simply engaging in the process of action research has helped me to develop professionally on multiple levels. Choosing to change the textbook in my classroom to a *Standards*-based curriculum was a large step out of my comfort zone. Teaching in this style for the first time in my career was a huge risk in my mind, but a necessary step towards obtaining a classroom style in which my students would become better problem solvers. For the first time since my first year of teaching, I no longer felt comfortable in my skin as a teacher, and I was learning new things every day alongside my students in terms of developing new classroom norms and ways of thinking about learning mathematics. My own "teacher lust" was brought to light from this change in my classroom. I continue to work on refraining from interrupting students' mathematical thinking, as that had been diminishing their opportunities to explore and discover the mathematics firsthand.

**The effects of videotaping.** Additionally, videotaping my classroom daily for an entire nine-week period was rewarding and challenging all at the same time. While both the students and I forgot about the cameras and recorders during the lessons as they became part of our routine, their impact on our classroom was not forgotten. For instance, as the teacher during the enactment of the curriculum, I could only hear so much from student groups as they worked together. I could not sit at each group, listening to their whole conversation and watching them work. Instead, I circulated around the classroom and checked in with each group, hearing bits and pieces of their conversations while I was near or present in their group. The videotapes that I watched each day afforded me the chance to see more of the interactions within each group. It brought attention to those quieter students who do not often speak up when the teacher is near. It shed light on the moments where students helped each other to grasp the material instead of relying on the teacher to assist. The videotapes also provided insight into students' thinking and misconceptions more so than could be discovered by simply walking around the classroom. Spending this time to get to know my students on a more intimate level was exciting and eye-opening. I learned that often the quieter students do know what is going on with the mathematics, they just take more time to engage with their group members and feel comfortable sharing their ideas with their classmates. However, after working in groups on a routine basis, these students sought the assistance of their groupmates, offered suggestions and solutions to them, and shared their insights and struggles with problems in the same ways as the more outgoing students. Often, I wished there was a camera on each group, instead of just two of them situated in the classroom to capture as much as possible, so that I could sit and get to know each student better as a mathematics learner in my classroom. Additionally, it would have been nice to have someone operating the cameras instead of them just sitting on tripods because I often forgot about the cameras and did not stop to change their position to capture student work on the boards from a better angle.

**Daily reflection.** Another aspect of my professional growth and development came from reflecting daily in my TRF. Teachers do not often take the time to formally sit and reflect about their classroom lessons each day, let alone journal about the mathematics that students were learning for the day. Taking time daily to reflect on the lesson, how it went, what went well, what did not go well, what went according to plan, and what was surprising was all very intriguing to me. Thinking carefully about each lesson, the adjustments I needed to make for the next day, and whether or not students had the opportunity to engage with each Focal PSC were all things I often overlooked and did not spend time thinking about simply because I was concentrating on other aspects of teaching. Having dedicated time to do this reflection helped me to get to know my students as learners much more quickly than in other semesters. It also helped me to acknowledge the challenges I faced that day implementing the lesson as well as provided me with structured time to reflect on how I would overcome those challenges the next day. Taking this time to reflect on the lesson daily provided me more insight into whether or not my students were comprehending the material. It allowed me the opportunity to think about how to help them the next day and how I could rephrase a question or respond to a student question from that day. Additionally, creating the TRF was a challenging task that helped me to grow professionally. Originally, when constructing my TRF, I asked questions that were too vague or complex to answer. For example, “How did the lesson go?” was too vague and did not scaffold my thinking enough to challenge me to grow and develop professionally. Instead, asking if the lesson went according to plan helped me to think more purposefully about my lesson planning as well as the adjustments I needed to make. Thinking about my own challenges was something I had not done previously; instead I often just focused on where my students struggled. Unexpectedly, looking at myself as a teacher brought feelings of vulnerability about my teaching. This new feeling was one I was uncomfortable with because I had not challenged myself as a teacher in this way prior to this study. This vulnerability was something I did not anticipate; however, it was a welcomed feeling as it helped me to grow professionally as a teacher, confronting my insecurities head on instead of hiding behind the authoritative role of a traditional teacher in which “I know all mathematics” is often the projected tone. Instead, this new feeling of vulnerability allowed me to learn about myself as a teacher and person and to grow from it, gaining more confidence in my new classroom role.

**The process of coding.** The process of coding was also a rewarding professional experience for my coding partners and me. While I had helped others code their data before, I had never been the one in charge of the coding process prior to this study. Due to the complexities of teaching along with the interconnected nature of the Focal PSCs, coding was not an easy process for my coding team. Often, deciphering which Focal PSC to code took discussion and agreement between the coders, and sometimes multiple Focal PSCs were coded from the same interaction. During the coding process, I learned from my coding partners and they learned from me. We helped each other through coding and grew together as researchers, learning more about the SMP and Focal PSCs as we delved into the data. With my coding partner in the Pre-Implementation phase, I learned how to code and work to consensus when our coding differed. She taught me how to code written data and I taught her about the Focal PSCs. Together we learned what it would look like in writing for students to engage with a Focal PSC. This was not an easy process because we had to create our own codebook, sorting our ideas and decisions in ways that would help us to increase our level of agreement in subsequent lessons. The process of creating our codebook also helped us to develop as researchers together. During the Post-Implementation phase of this study, my coding partner and I were again learning from each other and developing a codebook for the implementation of the lessons. During this phase, my new coding partner was experienced with teaching in a student-centered classroom, but was new to the SMPs and Focal PSCs, so once again, a lot of learning and growth took place between us. She had never coded before in research, so I also took the role of teaching her how to code research. As we coded together, I taught her about the SMPs and Focal PSCs, and in turn, she helped me better understand when students were actually engaging in *Analysis, Representations,* and *Approaches of Others* compared to when my teacher lust impeded the students' engagement with those Focal PSCs.

**Learning through my critical friend.** In addition, significant learning and professional growth took place while working with my critical friend during this study. While my critical friend was experienced with research and analysis, having a Ph.D. in Mathematics Education herself, she was not familiar with action research and the role of a critical friend. Through reading articles and having discussions, I taught her about the role of a critical friend, the process of meeting with a critical friend, and its importance to my study. She taught me how to analyze the clips I chose and how to see beyond the surface of the video and hone-in on the small interactions that I was not initially seeing when viewing the videotapes. She helped me to think critically about my classroom teaching and encouraged me when I felt discouraged in the times where I diminished opportunities for students to engage with the Focal PSCs. Her guidance and support helped me solve problems and look for ways to improve for the next week as we journaled critically and positively about the growth and learning that I was experiencing as a teacher. We engaged in professional growth by viewing the classroom video clips and analyzing the clips in a critical manner.

**The process of action research.** Finally, action research is more about the process than the product in many ways. While some researchers focus on the outcomes of the study, action researchers focus on the learning from the process of engaging in action research itself. Simply by conducting this research in my classroom, I learned, my students learned, and my colleagues learned. I learned more about myself as a teacher, I learned more about my students as learners, I learned more about problem solving, and I learned more about teaching my students to become better problem solvers. From the critical friend meetings, the daily reflections, the nightly watching of the videotapes, and the Post-Implementation phases of analyzing the videotapes, I learned and experienced growth as a teacher. Never before had I spent such constructive time analyzing and reflecting on my teaching. This purposeful reflection and analysis challenged me while helping me to grow professionally for the first time in my career in a way that was meaningful in my classroom. At the same time, my students learned to be better problem solvers; they learned another way to learn mathematics: from collaborative problem-solving in groups and learning through discovery. They learned to analyze problems, to characterize problems using multiple representations, and they learned to understand the approaches of their peers. They learned that their teacher cared about them as learners enough to go out of her comfort zone and learn to teach in a new manner. Most importantly, my students and I learned to construct the mathematics together as a class instead of the teacher as the authority of the classroom. Finally, my colleagues learned that good teachers reflect on their practice, and there is a way to bridge the gap between research and practice. They learned about the importance of problem solving in the mathematics classroom and about *Standards*-based curriculum as an alternative approach to the traditional lecture style classroom. They learned that teachers can be researchers too and exposing the teacher's own insecurities and vulnerabilities in the classroom is invigorating and rewarding because it helps you to grow professionally as a teacher.

**Changing my practice.** Change in the classroom is gradual and oftentimes difficult. It is not easy to critically reflect on your teaching, expose your insecurities, bring to light your vulnerabilities, and then attempt to change your practice. Therefore, I must continue to teach in this fashion if I plan to continue to change my practice and teach in alignment with the *CCSSM* standards. If I want my students to learn to persevere in problem solving, I must continue to work to minimize my own teacher lust, letting go of authority in the classroom and helping students to understand that learning through incorrect answers and mistakes is much more important than simply getting a correct answer. I need to work on my own struggles in terms of the internal conflict I feel when it comes to wanting students to feel safe in my classroom yet face their insecurities by discussing the mathematics with their peers rather than looking to me for guidance and step-by-step explanations. None of this can be solved overnight. Change takes time, and only through disciplined dedication will change actually take place in my classroom. This study has served as a catalyst to changing my own practice. Continued effort in this regard is the necessity in my own efforts to change my practice permanently.

**How Can This Study Serve Other Teachers?**

Obviously, I benefited a lot as both a teacher and researcher from this study. As explained previously, I experienced tremendous professional growth from conducting this action research study. I learned a great deal about myself as a teacher. I confronted my insecurities as a teacher and began the process of changing my teaching to reform my classroom to one that is student-centered, so students become problem solvers. Others helping with my study benefited as well. They learned about the SMPs, Focal PSCs, and action research and grew professionally themselves.

While this study was an action research study focused on the process of learning to facilitate problem solving in my classroom by implementing the Problem Solving SMP, it can be said that I also implemented this SMP on my own personal teaching. Personally, I learned to make sense of problems in my classroom and I persevered in solving them by completing the study for the entire nine weeks duration and continuously working to improve using action research methodologies, including improving my questioning, improving upon interrupting students and assisting them too much, and improving my facilitation of the *Standards*-based curriculum. Of course, conducting the study was not enough as an action researcher. As is important with any action research study, I must disseminate my research to make Altrichter, Posch and Somekh's (1993) model of action research complete. This dissertation serves as one form of dissemination for my study. Additionally, presenting at ICTM in the fall of 2019 will serve as another form of dissemination. From there, I hope to formally present my research to my colleagues during a professional development session at PHS, where this study was conducted and where I still teach. The goal of the professional development at PHS will be to help other teachers learn to step out of their comfort zones of traditional teaching and incorporate a student-centered approach into their own classrooms. Additionally, I hope to help them to see the importance of teaching students how to problem solve in the mathematics classroom and how a *Standards*-based curriculum helps to meet this need through rigorous problems that challenge students to think about the mathematics and better understand together without the authority of the teacher to tell them the mathematics.

Other teachers would benefit from the outcomes of this study as well. An improved understanding of the complexity of the Problem-Solving Standard from *CCSSM* would be beneficial. The Problem-Solving Standard contains 10 components of mathematically proficient students; I only examined three of these components. Another researcher could look at other components or all 10 at the same time. Also, examining another teacher's attempt at changing her practice by implementing the Problem-Solving Standard in her classroom could encourage others to change their own practice in similar ways. I am just one teacher; my experience is unique to me. It would be beneficial to have another teacher complete a similar study to compare their experience with my own. Investigating how I enacted the Problem-Solving SMP into my classroom serves as an example for others to follow suit. Others may not know how to enact the Problem-Solving SMP into their classrooms, but after this study, they now have an example to look at and use for their own implementation. It affords them the opportunity to see how others are delving into the *CCSSM* Standards of Mathematical Practices and implementing them in the classroom. While teachers are told to implement the *CCSSM* SMPs, they are not given guidance on how to implement these SMPs into their classroom, thus this study serves as one example of implementing one of the SMPs.

**How Can This Study Serve Researchers?**

This study has implications for mathematics education research as well. Acting as both the researcher and the teacher at the same time provided an opportunity for me to bridge the gap between research and practice. Being an insider in my own classroom presented a unique opportunity that most researchers do not have. Other researchers could learn, or be reminded of, the challenges that classroom teachers face when it comes to implementing research, and teachers can learn how to unpack research and implement it in their classrooms. Researchers could discover the struggles that teachers have while trying to put research into practice due to the many complexities of the classroom and school context. They could also witness that, even when curriculum is written with particular intentions, the enactment of the teacher has direct impact on whether that intention occurs in the classroom. Often, teachers do not realize they are diminishing opportunities for students to take the lead in the classroom because it is not our aim to squander these opportunities, but teacher lust, insecurities, and numerous other factors get in the way of the intended curriculum or research. If someone else were to do a similar study, I would recommend choosing different Focal PSCs to concentrate on or choosing a different unit or even a different SMP altogether, thus adding to our understanding of the intersection of the SMPs, mathematics content, and classroom practice.

Additionally, Vaiyavutjamai & Clements (2006) pointed out that little attention has been paid to the teaching and learning of quadratic equations in mathematics education literature, as evidenced by a lack of research on the subject. This study focused on teaching a quadratics unit, thus adding to the literature about teaching quadratic functions and equations, since little attention has been paid to quadratic equations in the literature. Commonly, students struggle with solving quadratic equations, especially when they are expected to solve them in different ways (Kotsopoulos, 2007; Tall, de Lima, & Healy, 2014). Ellis (2011) found that when students work with an open-ended problem with multiple entry points, thus having the opportunity to work collaboratively in groups, their generalization processes were enhanced and reinforced. So, this study, focusing on teaching a quadratics unit in a manner suggested by Ellis (2011), adds to the literature.

Cady (2006) wrote about the challenges she experienced when she attempted to teach in a student-centered classroom with a *Standards*-based curriculum. She brought to light the complexity of teaching with the paperwork, meetings and planning being overwhelming. I too shared this struggle. Videotaping my classroom and finding time daily to both reflect on that day's lesson in my TRF as well as watch the classroom videotapes for reflection was challenging. Continuing this process for an entire nine weeks was exhausting. I originally planned to reflect in my TRF during my planning period, but as is the life of a teacher, that planning period was often taken up by meetings, paperwork, and numerous other tasks, which forced me to change my reflection to daily after school.  I used the same list of questions that Cady used, adapted from Driscoll (1996).  My thoughts on using the same list of questions was that since the list was so vast and comprehensive, it would be easy to have a premade question for a variety of situations.  However, when I discovered that carrying the entire list around was overwhelming, I tried to anticipate student questions and thus only carried a few chosen questions with me as I circulated around the room.  Unfortunately, when students did not respond as I had planned, I found it challenging to promote their mathematical thinking by coming up with my own question in the moment.

Umbeck (2011) also wrote about her struggles when she tried to change classroom norms.  Her struggles were mine as well.  It took several weeks for students to begin to work productively within their groups.  It took even longer to establish the norm that I would not tell them a correct answer, and that the discovery of the mathematics within their groups was the key to success.  Similar to her students, my students wanted those recognizable routines of the lecture-style classroom back, and looked to me to provide those routines, although I did my best not to deviate from the *Standards*-based curriculum and student-centered classroom.  Also, while I did not think I would struggle with letting go of control of the classroom, that is something that I did end up finding to be an internal struggle.  I tried to minimize this by setting the new norms in the classroom at the start of a semester, but since multiple students were in my classroom the previous semester, they expected the old routines and norms.  Furthermore, since they were not used to learning in a student-centered classroom, they did not understand the norms and struggled with the teacher not being the authority in the classroom as well

Probably the most salient, yet unexpected thing I learned from this study was about my “teacher lust.” Boole’s (1931) notion of teacher lust was something I was not aware of in my own teaching prior to this study. Tyminski (2009) warned that teacher lust results in removing an opportunity for students to engage with the mathematics for themselves. While I knew that I had a natural desire to help my students, I did not think of myself as imposing my own understandings upon my students; I did not think I would diminish opportunities for students to grapple with the mathematics. However, after this study, I learned that my biggest issue was just that: removing opportunities for students to engage with the Focal PSCs because of my desire to help students. Essentially, due to my desire to see my students succeed, I removed their chance to understand the mathematics on their own. Each day I recommitted myself to heed his warning, but it continued to be a struggle throughout the study.

## Limitations

Every study of course has its own limitations; the complexities of teaching, accounted for many of the limitation of this study. The first limitation was the short time span. This study only focused on one unit’s worth of problem solving, which lasted nine weeks of the school year. While this is one-fourth of the school year, or half of a semester, one unit and nine weeks is not a long time to focus on problem solving in a high school algebra class. Because PHS changes courses at the semester break, the study could have potentially been a semester long. However, I chose to focus on one unit so that I could investigate the Focal PSCs more closely over a short time period.

A second limitation of this study was the small sample size of approximately 30 students. Unfortunately, since I only taught one section of algebra, my sample size was limited to this one classroom of approximately 30 students. Finally, because I served as both teacher and researcher of this study, potential bias could be presented into the results. This limitation was addressed by having coding partners and a critical friend while analyzing my data throughout this process. The limitation of being both the teacher and researcher limits the reader to only one viewpoint—my lens and interpretations of this study—but it is an important voice for other teachers to hear.

## Suggestions for Future Research

In moving forward with this research, it is important for me to consider a longitudinal study of teaching practice change, as well as a study examining the impact of student learning in this problem-solving approach. A longitudinal study may provide insight into whether I continue to teach in a problem-solving approach or if the complexities, insecurities, extra duties, and pressures of teaching force me back into a traditional style of teaching. A study examining the impact of student learning in this *Standards*-based approach may provide insight into whether students understood and retained the mathematics more because they learned the mathematics in a more meaningful way. Additionally, it would be interesting to look at whether learning mathematics in this manner impacted student scores on standardized tests. Finally, if other teachers are implementing the Problem-Solving SMP in their classroom, it would be important to see what it looks like in their classrooms. They would likely have some similarities to my own experience, but could be radically different, and undoubtedly will be different in some fashion. Future research could look at the differences in the experiences. I would be interested in looking at the differences and further developing professionally by seeing how I could adapt my own classroom teaching to incorporate the differences, or even a collaboration with other teachers.

It is important to remember that to teach is to learn. Reflection is critical for teacher change, but teacher change is needed for more impactful learning of mathematics for students.  This change is imperative if students are to remain competitive internationally in the mathematics community, to develop as learners who are expected to be more college and career ready, and of course, to become problem solvers invested in lifelong learning.

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# APPENDIX A.

## TEACHER REFLECTION FORM (TRF)

Lesson:

Page Numbers:

Did the lesson go according to plan? If not, what kind of adjustments did I have to make?

*PART I:*

**ANALYSIS PSC**

1. During the lesson, were students provided with opportunities to engage with this Focal PSC? If not, why not? If so, how did the students engage with the PSC?

2. What challenges did I face with facilitation of this Focal PSC?

3. What challenges did the students face with this Focal PSC? How do I know (what is my evidence)?

**REPRESENTATIONS PSC**

4. During the lesson, were students provided with opportunities to engage with this Focal PSC? If not, why not? If so, how did the students engage with the PSC?

5. What challenges did I face with facilitation of this Focal PSC?

6. What challenges did the students face with this Focal PSC? How do I know (what is my evidence)?

**APPROACHES OF OTHERS PSC**

7. During the lesson, were students provided with opportunities to engage with this Focal PSC? If not, why not? If so, how did the students engage with the PSC?

8. What challenges did I face with facilitation of this Focal PSC?

9. What challenges did the students face with this Focal PSC? How do I know (what is my evidence)?

*PART II:*

**FACILITATION & QUESTIONING**

10. Which pedagogical strategies did I use to facilitate collaboration and promote problem solving? Did they seem effective? What is the evidence? (in both informal and formal assessments)

11. Which questioning strategies did I use to promote problem solving?

    a. Which of Cady's (2006) questions did I ask?

    b. How am I using Cady's questions?

    c. What other questions did I ask?

    d. How am I using other questions?

    e. Which questions did I ask that elicited a PSC response?

    f. What is the nature of the questioning that elicits a Focal PSC response?

    g. How did I use (and not use) facial expressions and tone?

*Part III:*

12. How do I see my questioning? Does my questioning that results in a Focal PSC response increase over time?

13. What might I try to do differently in the next lesson?

14. Other thoughts/comments?

# APPENDIX B.

## THE CRITICAL FRIEND CONVERSATION FORM (CFCF)

Date:

Lesson covered in chosen snippet:

**My Desired Outcomes from this meeting: (These outcomes will be determined at the end of each week to allow this conversation to benefit me in the areas where I determine that change is important/desired…**

**Description of Practice**:

I will "describe a practice and request(s) feedback" (Costa & Kallick, 1993, 50) using video snippets from the week's lessons. These snippets will be chosen based on when the Focal PSC of *Analysis, Representation or Approaches of Others* is visible.

**Critical Friend Clarification Questions: (in order to further understand what happened)**
**(i.e., How long did this lesson take? Did it take longer than you originally planned? Were all students engaged? Why or why not?).**

**Critical Friend Feedback:** At this point, my critical friend will provide feedback to me about "what seems significant about the practice in order to provide a lens that helps to elevate the work" (Costa & Kallick, 1993, 50).

**<u>Critical Friend Questions and Critiques (sample questions below)</u>**

1. After watching the lesson, what do you think went well in terms of the Focal PSCs?

2. After watching the lesson, what do you think you would do differently next time in terms of implementing the Focal PSCs?

3. After watching the lesson, do you notice a change in your teaching? Describe this change.

4. What challenges are you experiencing? How will you overcome these challenges?

**<u>Written Reflection by Teacher and Critical Friend</u>** (additional questions)

This is where the teacher and critical friend write notes about the meeting such that the teacher "does not have to respond or make any decisions on the basis of the feedback. Instead, the learner [teacher] reflects on the feedback without needing to defend the work to the critic" (Costa & Kallick, 1993, 51).

Costa and Kallick (1993). Through the Lens of a Critical Friend. Educational Leadership, October 1993, pp. 49 – 5