LEARNING TRANSFER IN THE DIFFERENTIATION USING THE CHAIN RULE AND ITS RELATIONSHIP TO MOTIVATION AND PERFORMANCE

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Damji Heo

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THE PURDUE UNIVERSITY GRADUATE SCHOOL
STATEMENT OF DISSERTATION APPROVAL

Dr. Tim Newby, Co-Chair

Department of Curriculum and Instruction, College of Education

Dr. Muhsin Menekse, Co-Chair

School of Engineering Education and Department of Curriculum and Instruction, College of Engineering and College of Education

Dr. Darryl Schneider

Department of Psychological Sciences, College of Health and Human Sciences

Dr. James McClure

Department of Mathematics, College of Science

Approved by:

Dr. Janet Alsup

Head of Department of Curriculum and Instruction, College of Education

I dedicate this dissertation to my grandparents, Gi-wan Yuh (유기완) and Oak-soon Lee (이옥순). Please rest in peace.

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SYMBOLS

a	discrimination index in an IRT model
b	difficulty index in an IRT model
θ	latent trait in an IRT model

ABBREVIATIONS

AGQ-R	Achievement Goal Questionnaire-Revised
ICC	Inventing with Contrasting case for Calculus learning
IRT	Item Response Theory
TP	Tell and Practice
MCMC	Monte Carlo Markov Chain method
MSLQ	Motivated Strategies for Learning Questionnaire
PFL	Preparation for Future Learning
SRL	Self- Regulated Learning

GLOSSARY

GR model	Graded Response model which deals with ordered Polytomous categories
MCMC	Monte Carlo Markov Chain method which are the algorithms for sampling from a probability distribution
2PL model	2 Parameter Logistic model

ABSTRACT

Heo, Damji Ph.D., Purdue University, December 2019. Learning Transfer in the Differentiation Using the Chain Rule and Its Relationship to Motivation and Performance. Major Professors: Tim Newby, Muhsin Menekse.

Previous studies indicated that calculus courses are considered 'weed-out' courses as a lot of students in STEM majors struggle to pass. Instructors and researchers explored various instructional methods to facilitate calculus learning, however, more tailored instructional strategies are still needed. Inventing Contrasting Case is a strategy that has been proven effective in transfer, yet, its effect when combined with the motivational factor and across various content areas should be investigated further. Therefore, this study investigated the relationship between participants' motivation, instruction condition, and the performance on the direct application and transfer problems using Calculus 1 content. The data was collected from undergraduate students in STEM majors at a Midwestern university who were required to complete a Calculus 1 course to attain their degree. Eighty-one students participated for the study. Participants were assigned to either the iCalCulus (iCC) group or the Tell and Practice (TP) group. The study consisted of two separate sessions. In Session 1, participants were provided with a motivation survey, calculus course experience survey, pre-requisite knowledge check test, ICC task or TP task, and post-test. Seven days later, participants took a delayed post-test (Sesson 2). Google Forms was used to create study materials. The results from Bayesian independent sample t-test analyses indicated that the iCC group did not outperform the TP group in direct application problems. In addition, the iCC group did not outperform the TP group in PFL problems in either test. However, the ICC group outperformed the TP group in the further PFL problems from the delayed post-test ($BF_{01} = .096$, $p = .003$). The results from Bayesian one-way ANCOVA analyses indicated that there was the moderate evidence that supports the effect of group condition on direct application, Preparation for Future Learning (PFL) performance from the post-test, while controlling for the average pre-requisite knowledge check test score and motivational level. The results also indicated that there was from moderate to strong evidence to support that group condition had an effect on PFL performance from the delayed post-test (Session 2), and the further PFL performance from both post-test and delayed post-test while controlling for the average pre-requisite knowledge check test score. In addition, motivational level was shown to not be an effective moderator between instructional condition and performance in PFL problems. The results from GLM repeated measure analyses showed the ICC strategy had a more significant effect on the participants regarding PFL performance and further PFL performance over time as there was a significant cross-over interaction effect between the time and the instruction condition (p = .012, η_p^2 = .08 for PFL performance and p = .003, η_p^2 = .11 for further PFL performance). The direction for potential future studies is addressed in the conclusion section including the importance of developing curriculum to train students' transfer ability; and a new type of assessment to measure transfer is offered for consideration.

1. INTRODUCTION

Calculus is commonly considered a “weed-out” course, and student performance in it is a significant predictor for attainment of many STEM degrees. High rates of failure in calculus lead many students to change disciplines (Suresh, 2006; Tyson, 2011). Previous research studies also pointed out that Community College (CC) transfer students in four-year institutions for STEM degrees experienced GPA shock and showed that their GPAs in calculus courses strongly affected CC transfer students' retention rate in four-year institutions (Laugerman, Rover, Shelley, & Mickelson, 2015; Laugerman, Shelley, Rover, & Mickelson, 2015).

Although the importance of learning calculus has been emphasized by many researchers, there are still insufficient research studies which suggest tangible instructional methods that produce strong and reliable effects for learning and teaching calculus (Ramussen et al., 2014). Specifically, research studies which provide a clear understanding of how students can learn key concepts for calculus are needed. Moreover, it is important for students to transfer their calculus knowledge to real settings when they work in a STEM field. Therefore, learning environments where students can understand the key concepts of calculus and learn how to transfer the knowledge to other settings should be provided. For instance, only a few studies provide the clear understanding of how students differentiate composite functions and apply the chain rule to find the derivative of each function (Clark et al., 1997; Kabael, 2010). Recognizing the structure of composite functions and differentiating them correctly are important to find integrals as well. Accordingly, the chain rule is one of the important concepts in learning and understanding calculus.

Inventing with Contrasting Cases (ICC) is one of the pedagogies that has been introduced as effective for learning transfer by previous research studies (Schwartz, Chase, Oppezzo, & Chin, 2011; Schwartz & Martin, 2004). ICC is a combination of ‘contrasting cases’ activities and an ‘inventing’ activity (Schwartz et al., 2011). ‘Contrasting cases’ refers to instructional materials which are designed to facilitate learners in discerning information which might be overlooked otherwise. In other words, learners can find the difference between two cases and think about how the difference are related to finding solutions. The contrasting cases is a carefully designed instructional strategy designed to help students notice key features of a problem and to avoid overly simplistic solutions. Inventing activities is an instructional strategy requiring students to invent aids by themselves to help them understand given tasks more efficiently or effectively.

While research on ICC has gained prominence recently, many questions about how and when it works remain. For example, Schwartz and Martin (2004) pointed out that ICC might seem inefficient when measuring learners' performance with standardized assessments which focus on measuring how much learners understand the task at the moment. The authors argued that assessments should include how students can prepare themselves to learn in a new context to assess the effect of the ICC strategy. They introduced an instructional design called ‘Preparation for Future Learning’ (PFL) as a new type of assessment to measure learning transfer more accurately. Another example is that ICC from previous research studies has not incorporated a motivational factor to explain the mechanism of learning transfer even though learning transfer requires persistence and motivation throughout the process (Perkins & Salomon, 2012). Belenky and Nokes-Malach (2012) found that ICC in statistics promoted mastery goal orientations in college students, and mastery goals predicted transfer. This result addresses the call to consider the intersection of transfer and motivation (Goldstone & Day, 2012).

ICC also has been shown to have a limited effect on learning transfer when performed individually (Sears & Pai, 2012). While research studies usually combined ICC with collaborative learning condition to maximize its effectiveness (Schwartz, Bransford, & Sears, 2005; Schwartz et al., 2011), there are research studies with individual learners that used strategies such as providing metacognitive feedback to facilitate the effectiveness of ICC (Roll, Aleven, McLaren, & Koedinger, 2011). Roll et al. (2011) developed an intelligent tutoring system (ITS) that could substitute for the function of peer feedback during an ICC condition. They designed metacognitive feedback to help learners gain better help-seeking skills. In their study, they found the effect of metacognitive feedback in Geometry Cognitive Tutor software program on help-seeking skills of students when learning Geometry.

Implications from research studies on ICC and its effect and applications offer some directions to researchers. For instance, the ICC strategy incorporated with motivational factors should be explored further. In addition, the effect of an ICC strategy should be tested with different types of content.

The purpose of this research study is to investigate the effect of an ICC strategy on the performance of near transfer using chain rule content. In this research study, I investigated the effect of ICC strategy and testing surveys called ICalCulus. Few if any published studies have addressed the use of ICC with calculus content or with attention to learning and motivation; ICalCulus offers an important test case which could provide a basis for future studies. Therefore, in this study, the following research questions are addressed:

1. Do students in the inventing contrasting cases for iCalCulus (iCC) group outperform Tell and Practice (TP) group in direct application?

2. Do students in the inventing contrasting cases for ICalCulus (ICC) group outperform Tell and Practice (TP) group in Preparation for Future Learning (PFL) target problems?

3. Is the students' motivational level a significant moderator between the instructional condition and their Preparation for Future Learning (PFL) target problem performance?

Ultimately, this research study will provide valuable information to researchers, instructors, and administrators in STEM fields as well as an applicable learning tool to facilitate the achievement of students on calculus.

2. LITERATURE REVIEW

In this chapter, the previous literature on calculus concepts and instructional methods used for calculus subjects, self-efficacy, achievement goal theory, learning transfer, and the methods used to measure learning transfer are discussed. The first section discusses crucial calculus related concepts and the chain rule, and the instructional methods that have been used for calculus subjects. Next, the motivational constructs focusing on self-efficacy and achievement goals are discussed and how they have been investigated in relation to the other learning-related constructs. Learning transfer and its relationship between the motivational constructs are discussed as well. Lastly, how the researchers have explored and developed the methods to measure the construct is discussed is discussed.

2.1 Literature in Calculus Learning

Calculus courses are one of the required courses to complete STEM degrees such as engineering degrees (Felder, Forrest, Baker-Ward, Dietz, & Mohr, 1993; Suresh, 2006; Tyson, 2011; Young et al., 2011). Previous research studies on engineering students and attrition rate found that performance in calculus courses strongly predicts the attrition rate and degree completion (Felder et al., 1993; Suresh, 2006; Tyson, 2011). Although the importance of learning calculus has been emphasized by many researchers, there are still insufficient research studies which suggest concrete instructional methods with the strong empirical evidence for learning and teaching calculus (Rasmussen et al., 2014). For instance, there have not been many studies that provided the clear understanding of how students understand decomposition of functions and apply the chain rule to find the derivative of each function (Clark et al., 1997; Kabael, 2010). One possible assumption for the reason that there is not much literature on teaching/learning calculus, in general, is that the population who need to learn calculus is quite limited compared to math branches taught in K-12 level. Thus, it is possible that researchers in math education might focus more on the other math branches that are taught in k-12 level as there is the much wider target audience. In addition, as Rasmussen and his colleagues (2014) suggested, the calculus curriculum reform movement in the 1990s was led by mathematicians who do not have an extensive educational research background. Mathematicians who conducted research studies on learning calculus might not have been much introduced to researchers in education due to different disciplines. Therefore, it is important for researchers to provide strong logic with the audience to understand why more research studies for calculus learning should be conducted even though it is already expected that the targeted audience would be limited. Then, finding crucial concepts for better understanding calculus should be followed, based on the support while communicating with mathematicians who have expertise in calculus.

In this literature review, research studies on calculus learning focusing on trigonometry, limits, and the chain rule from differential calculus are discussed. Trigonometry and limits can be categorized as pre-calculus as they are covered in pre-calculus course in college and pre-requisite knowledge that students should have before they take calculus courses (Weber, 2005). The chain rule is one of the calculus concepts.

2.1.1 Concepts in Pre-Calculus

Trigonometry

Trigonometry takes a large portion of pre-calculus textbooks (Cohen, Lee, & Sklar, 2006; Sullivan, 2014). Moreover, trigonometric knowledge is used for the entire calculus contents (Stewart, 2012; Weber, 2005). Furthermore, trigonometric knowledge is used widely in engineering and physics disciplines (Weber, 2005). In other words, trigonometry is the crucial concept in order to understand calculus more smoothly. According to calculus standards from Indiana Academic Standards, Department of

Education for AP calculus, finding the derivative of a trigonometric function is listed in the table that students who learn calculus should know and be able to do (see Figure 2.1 C. D. 3.. Full Table is attached as an appendix A).

DIFFERENTIATION	C.D.1: Understand the concept of derivative geometrically, numerically, and analytically, and interpret the derivative as a rate of change.
	C.D.2: State, understand, and apply the definition of derivative.
	C.D.3: Find the derivatives of functions, including algebraic, trigonometric, logarithmic, and exponential functions.
	C.D.4: Find the derivatives of sums, products, and quotients.
	C.D.5: Find the derivatives of composite functions, using the chain rule.
	C.D.6: Find the derivatives of implicitly-defined functions.
	C.D.7: Find the derivatives of inverse functions.
	C.D.8: Find second derivatives and derivatives of higher order.
	C.D.9: Find derivatives using logarithmic differentiation.
	C.D.10: Understand and apply the relationship between differentiability and continuity.
	C.D.11: Understand and apply the Mean Value Theorem.

Figure 2.1.: Calculus standards from Indiana Academic Standards, Department of Education. Retrieved from `https://www.doe.in.gov/sites/default/files/standards/mathematics/calculus-standards.pdf`

Despite the importance of understanding trigonometric functions, students often go through difficulty with understanding trigonometric functions as they are different from algebraic functions that involve arithmetical procedures (Breidenbach, Dubinsky, Hawks, & Nichols, 1992; Weber, 2005). Also, they would need to mentally draw a unit circle to calculate the value of a function (Weber, 2005; Weber, Knott, & Evitts, 2008). Students are not generally familiar to this kind of process and reasoning (Weber, 2005; Weber et al., 2008). The current instructional methods that are used for trigonometry class encourage memorization of formulas and angles which eventually makes it more difficult for students to think and mentally draw the images to solve trigonometric functions (Weber, 2005). In the study of Weber (2005), the author used procept to measure college students' understanding of trigonometric function. Procept refers to the combination of “concept and process represented by the same symbol” (Gray & Tall, 1994). Weber (2005) collected the data from college students who enrolled in a college-level trigonometry course but in two different sections; one with a traditional method vs. the other section with an experimental method. The traditional method was a lecture with contents straight from the textbook. In the experimental section, the instructor taught trigonometry emphasizing procedure, process, and precept. That is, students learned trigonometry through the step-by-step algorithm, meaningful method, and see the mathematical symbols as procepts. Students in both sections had pre and post-test. Students in the experimental method section performed better. However, the author mentioned that the study was not an experimental study, and thus, argued that a comparison of the results between two sections might not accurately represent the true difference. Nevertheless, students in the experimental section showed better performance in their post-test. Understanding trigonometric concepts as procepts can be beneficial to understand calculus later as students would learn trigonometry in calculus for hyperbolic functions which could be confusing and even more complicated than before (Perrin, 2007). Therefore, more research studies for trigonometry regarding effective instructional methods with empirical evidence are needed.

Limits

The limits one of the crucial concepts to understand calculus (Williams, 1991). Yet, students struggle to understand them accurately (Liang, 2016; Tall, 1980; Williams, 1991). Researchers have been tracking down the cognitive processes that students go through when learning limits and the obstacles that they experience and the instructional methods that facilitate deeper understanding and application (J. F. Cottrill, 1999; Liang, 2016; Maharaj, 2010; Sofronas et al., 2011; Williams, 1991). One of the reasons that students have difficulty understanding the limits is due to their misconception regarding the limits (Liang, 2016). That is, students think that a sequence never reaches its limit (Liang, 2016; Merenluoto & Lehtinen, 2004). Moreover, when students learn limits with an example such as '$x \to 3$ (p. 37)', they tend to think x reaches very close to 3 from one side only. Liang (2016) pointed out that the term limit used in everyday experience and even in some textbooks strengthen this misconception. Other researchers have also pointed out that students experience difficulty understanding the concept of limits because of the definitions of limit (Liang, 2016;

Tall, 1992). That is, “any”, and “there exist” (Liang, 2016, p. 39) or “approaches” (Tall, 1992, p.2) are the examples of the words in the definitions of the limit that could confuse students to understand the definitions. Liang (2016) argued that the deficiencies of the definition of the concept of the limit interrupt clear understanding. Tall (1992) argued that the words used in the formal definitions of the limit have colloquial meanings that could conflict with the meaning of the definitions, and thus, students could be misled.

Although many students have difficulty understanding the limits as researchers suggested, it is crucial to understand the concept fully since limits are related to the other important calculus concepts (Liang, 2016; Williams, 1991). Continuity, uniform continuity, convergence, and derivative are the examples of the calculus concepts that are closely related to the limits (Liang, 2016; Stewart, 2012). For instance, the rate of change is the fundamental concept to understand the derivative (Stewart, 2012) which uses the limits. Therefore, the concept of limits is crucial to understand the differential calculus.

Researchers explored various instructional methods to facilitate a better understanding of the limits (Liang, 2016; McGuffey, 2017; Sylvestre, 2016; Craig Swinyard & Sean Larsen, 2012). McGuffey (2017) investigated the effect of guided reinvention on students’ understanding of the concept of the limits. The author conducted a 1-hour session for five weeks. In session 1, participants were asked to describe the change of the quantities in five given realistic situations. In session 2, the students were asked to look for similarities and differences among the situations given in session 1. Students indicated that they would like to use graphs and formulas to solve the problems. In session 3, the author gave either the formulas or graphs to solve each problem. In session 4 and 5, students were asked to predict the value of the quantity that was not displayed on the graph. The researcher also introduced two new situations which involve exponential growth and circular motion to contrast their performance on prediction in the previous examples. The author found that the participants showed their understanding of the formal concept of infinity.

Researchers also have investigated the theoretical frameworks to understand students' cognitive processes and mental model when learning limits (J. Cottrill et al., 1996; Maharaj, 2010; Williams, 1991). Maharaj (2010) used Action-Process-Object-Schema (APOS) theoretical framework to investigate undergraduate students' understanding of the concept of a limit of a function. In the action stage, learners react to the external stimuli, such as when learners first receive the limit of a function $\lim_{x \to a} f(x)$, the learners merely input the values to the x for the function that are close to a. In the process stage, learners repeat and reflect on an action and internalize the knowledge into a mental process. Therefore, learners would understand the transformation of the function without entering any specific values into x for the function. In the object stage, learners would recognize the process as a totality and be able to encapsulate the limit of the function concept and make an object (e.g., a new function) out of it. Last, in the schema stage, learners understand the concept processed through action – process- object process and link each step into a coherent framework. Therefore, learners begin to understand the limit of a function fully, the input values are close from left and right to a, and the output values go through the transformation following the function.

Researchers have tried to explore the instructional methods to enhance students' understanding of the concept of limit as it is crucial to understand other calculus concepts as mentioned above. Nevertheless, there is still room to be improved in terms of instructional methods for the limit. For instance, as previous research pointed out (Liang, 2016; Craig Swinyard & Sean Larsen, 2012), how the definitions of the concept of the limits could be revised and how the instruction to teach the definitions of the concept could be improved should be studied more. Furthermore, researchers should explore more effective instructional methods that could facilitate not only students' understanding of the concept but also applying to the calculus lessons for learning.

2.1.2 Concepts in Calculus

Chain Rule in Differentiation

Differentiation is finding a derivative of a function (Orton, 1983; Stewart, 2012). If a function is a simple function of x, then students can differentiate the function with respect to x. For instance, if there is a function:

$$y = 5x^2$$

you can differentiate the function $5x^2$ with respect to x, and the answer is $y\prime = 10x$. However, if a given function is a composite function, you cannot use the same method that you use when you differentiate simple functions. Composite function is a function inside of another function. That is, when there are two functions $f : X \to Y$ and $g : Y \to Z$ and they can be denoted in one function $h(x) = f(g(x))$, $h(x)$ is a composite function (Clark et al., 1997; Kabael, 2010; Stewart, 2012). Students cannot solve the composite function with respect to x as there are two functions combined in one function and one function would remain undifferentiated. In this case, the chain rule should be used to find a derivative of $h(x)$ (Stewart, 2012). By using the chain rule, $h(x)$ is differentiable at x and thus $h\prime(x) = f(g(x))g'(x)$. One more technique that we can use while differentiating the composite function with the chain rule is Leibniz's notation. Leibniz's notation is invented by the 17th century German mathematician Gittfried Wilhelm Leibniz (Stewart, 2012; Tall, 1985) which uses the dx and dy as infinitely small increments of x and y (infinitesimal). Therefore, we can use $y = f(u)$ and $u = g(x)$ in order to make the composite function to be the form that can be denoted as Leibniz's notation. Thus, the notation would be:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

For instance, if a composite function is:

$$y = 2x^{5x},$$

and if we solve the function as the simple function, then $5x$ could be remained undifferentiated. Instead, by using Leibniz's notation and the exponent rule, $y = 2e^{5xln(x)}$, $\frac{dy}{dx} = 2\frac{d}{dx}e^{5xln(x)}$. By using the chain rule, $y = f(u) = e^u$, and $u = 5xln(x)$, $\frac{df(u)}{dx} = \frac{df}{du} \times \frac{du}{dx} = 2\frac{d}{du}(e^u)\frac{d}{dx}(5xln(x))$, and we need to first differentiate e^u and $5xln(x)$ separately, and $(e^u)' = e^u$ and $(5xln(x))' = 5ln(x) + 5$. Therefore, $2\frac{d}{du}(e^u)\frac{d}{dx}(5xln(x)) = 2e^u(5ln(x) + 5) = 2e^{5xln(x)}(5ln(x) + 5)$, and the answer $y' = 2x^{5x}(5ln(x) + 5)$.

Despite the usefulness and the importance of understanding the chain rule, differentiating composite functions using the chain rule can be challenging especially for those students who just learned composite function for differentiation since finding the inner function and outer function from the composite functions can be intricate. Indeed, researchers have suggested the differentiating composite functions using the chain rule as one of the difficult parts in Calculus to understand (Clark et al., 1997; Dubinsky, 2002). Thus, some researchers investigated effective instructional methods to teach the chain rule (Clark et al., 1997; Kabael, 2010). Clark et al. (1997) investigated students' understanding of the chain rule with the Action-Process-Object-Schema (APOS) framework. The framework was based on the triad mechanism of Piaget and Garcia (1989) which were intra-, inter- and trans- stage for the analysis. Students were in the intra- stage would have a collection of rules to find derivatives including special cases that would require the chain rule but would not recognize the relationships between the general formula and the special cases. In inter- stage, students would begin to recognize the special cases that would require the chain rule are related to each other. The chain rule schema starts to be formed. In the last stage, trans- stage, students would understand that composite functions would require the chain rule to differentiate and the chain rule would be formed as a single rule. Students can form their schema development through the triad mechanism. In their discourse analyses, the authors were able to find that students formed the schema of the chain rule following this stage.

Tall (2008) and Kabael (2010) used "three worlds of mathematics" framework that Tall (2008) developed to analyze students' understanding of differentiation.

Tall (2008) suggested three different models which are "conceptual-embodied world", "proceptual-symbolic world", and "axiomatic-formal world" (p. 7). Conceptual embodied world refers to the world formed based on the perception of properties of objects and reflection on them. The examples would be imagining a triangle and perceiving it as a prototype of the concept of "triangles". Proceptual-symbolic world is formed based on the action of using the symbols. Differentiating functions in calculus would be the example. Axiomatic-formal world is formed based on the formal definitions and proofs. The chain rule would be one of the examples. Figure 2.2 illustrates these three worlds.

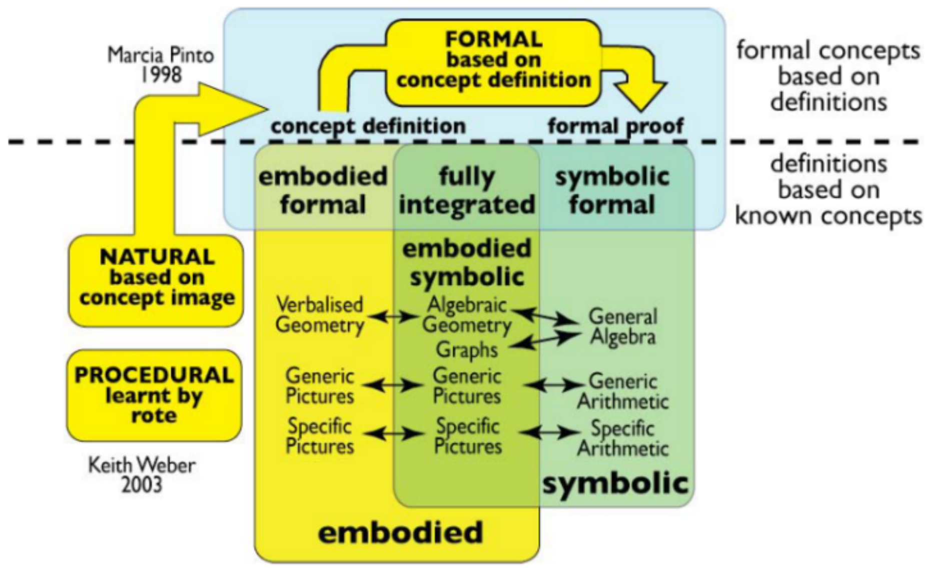


Figure 2.2.: Three worlds of Mathematics. Adapted from "The Transition to Formal Thinking in Mathematics," by D. Tall, 2008, *Mathematics Education Research Journal, 20*(2), p. 8. Copyright 2008 by Mathematics Education Research Group of Australasia Inc. Reprinted with Permission

Kabael (2010) used the case of chain rule lesson combining Tall's (2007, 2008) three worlds of mathematics and APOS framework. He reported how students apply the chain rule, take the second order derivative, embodied route, symbolic route and the combinations of these routes in the cognitive development of the chain rule in his study with those frameworks. The author found that students developed their understanding of the chain rule based on the three worlds of mathematics. The data was collected from twenty-seven students who were taking calculus 1 level course. After the test, students were divided into five groups based on the level of difficulty that they indicated regarding composite functions. Students were supposed to find the solution of the test items with four steps based on the chain rule. Then, one student from each group was selected for the interview. The results indicated that students initially were able to differentiate composite functions when they were given separated single equations. However, they did not realize that they were solving the functions following the chain rule. They were in the conceptual-embodied world. Students' difficulties were mostly related to symbolic or structural difficulties stemming from applying the chain rule. Students began to understand that the chain rule is used for the composite functions. However, they still have not figured out fully the underlying schema that goes through the differentiation process of composite functions. As they realized that the functions that would require the chain rule were related to each other, they entered into the inter- stage. Finally, they were able to symbolize the notion of the chain rule, indicating that they were in the symbolic world and the trans- stage. Figure 2.3 below represents the model of cognitive development for the chain rule schema that students developed combining triad mechanism and the mathematical world. The author particularly focused on the embodied world and symbolic world for the study.

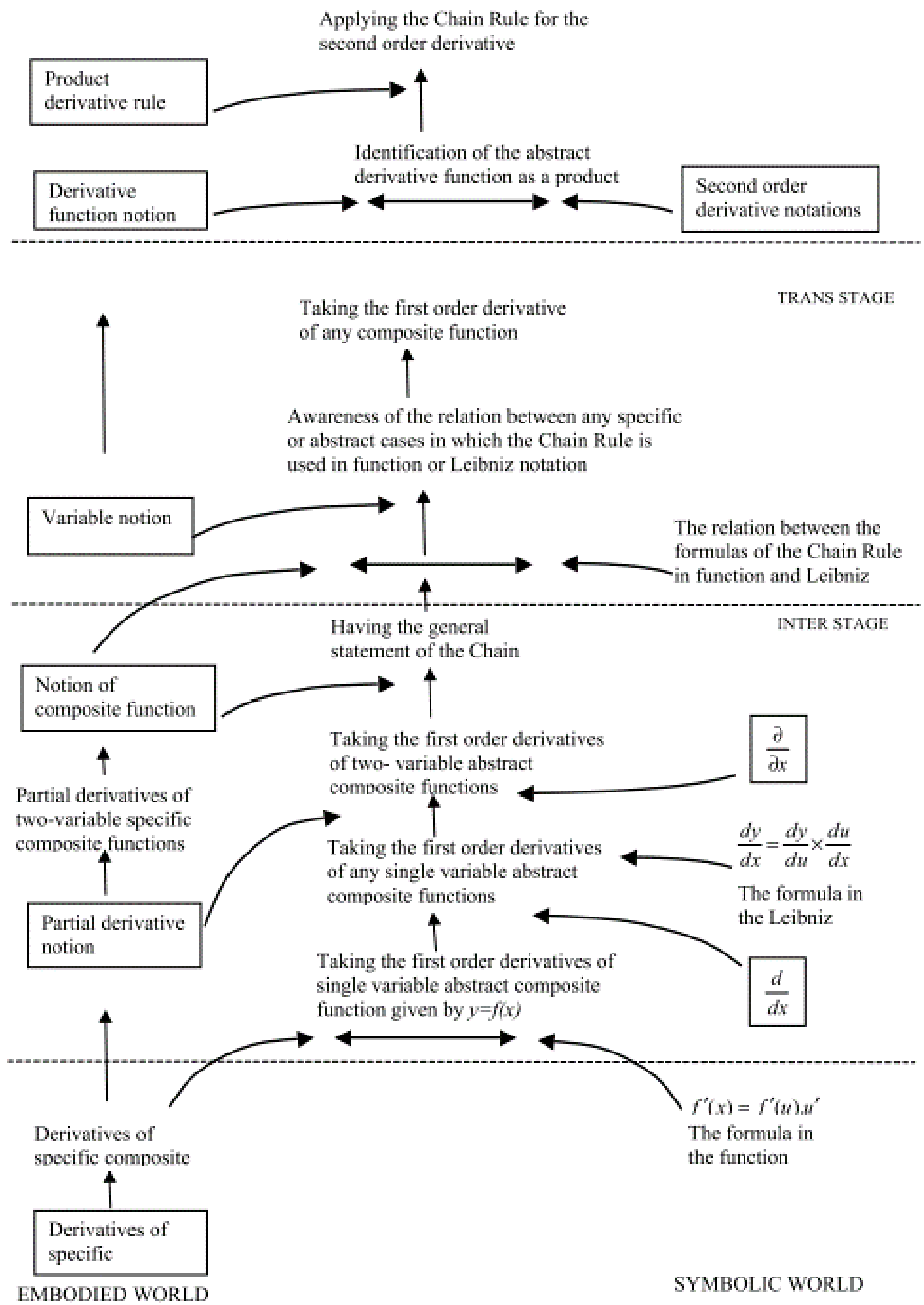


Figure 2.3.: Cognitive development of the chain rule. Adapted from "Cognitive Development of Applying the Chain Rule through Three Worlds of Mathematics," by T. U. Kabael, 2010, *Austrialian Senior Mathematics Journal, 24*(2) p. 25. Copyright 2010 by Australian Association of Mathematics Teachers.

The chain rule is important as various phenomena in natural science that could be represented as mathematical models that consist of composite functions, and we can get the rate of change of models that do not have constant derivatives (Lutzer, 2003). Also, the chain rule is used in many other areas of calculus such as implicit differentiation and integration by parts (Stewart, 2012). More research studies tracking students' learning of the chain rule and instructional methods with empirical evidence should be followed. Specifically, types of composite functions that students have trouble applying the chain rule to, or how the students stumble when they apply the rule to implicit differentiation should be explored.

2.1.3 Instructional Methods in Calculus

Researchers have explored the instructional methods to facilitate students' understanding of calculus contents (Rasmussen, Marrongelle, & Borba, 2014). Especially during the calculus reform movement in the United States back in the 1990s, researchers have explored various instructional methods and promoted new methods, particularly in the U. S. (Hurley, Koehn, & Ganter, 1999; Rasmussen et al., 2014; Sevimli, 2016). Flipping classrooms is one of the instructional methods that have been the most popular alternative method for traditional lecture method (Jungi, Kaur, Mulholland, & Xin, 2015; Sahin, Cavlazoglu, & Zeytuncu, 2015; Wasserman, Quint, Norris, & Carr, 2017). Flipped classrooms are a type of blended learning that flips the traditional learning environment by adopting online learning environment and deliver the course materials to the outside of the classroom (Abeysekera & Dawson, 2015). Pioneers of the reform movement, such as the Harvard Calculus Consortium, emphasized the necessity of conceptual understanding (Hurley et al., 1999; Sevimli, 2016). As a result, they suggested that the instructional context should be represented in multiple formats and be supported by technology (Sevimli, 2016). Naccarato and Karokok (2015) suggested that the flipped classroom model is based on “distance learning, Just-In-Time Teaching, Problem-Based Learning, and Inquiry-Based Learning” (p.

969). Therefore, students watch online lectures, participate in classroom activities, and/ or participate in the online discussion with peers and ask questions to the instructors via online in typical flipped classrooms (Jungi et al., 2015; Sahin et al., 2015; Wasserman et al., 2017). Some researchers reported a positive effect of flipped classrooms on students' academic achievements in calculus (Anderson & Brennan, 2015; Kadry & Hami, 2014; McGivney-Burelle & Xue, 2013). On the other hand, other researchers reported there was little difference between students in the traditional lecture-based classroom and flipped classroom in their performance (Bagley, 2014; Ziegelmeier & Topaz, 2015). Kadry and Hami (2014) also pointed out that there is little evidence from experimental research studies that could strongly support that flipped classroom enhance students' academic performance and achievement. Thus, more research studies that could support the effect of flipped classrooms, as well as the other types of instructional methods for calculus on students' academic performance, are needed. The incongruent results of flipped classroom on students' performance on calculus raises another issue regarding the instructional strategies adapted in non-traditional lecture-based classrooms such as flipped classrooms. Specifically, there are few research studies on instructional strategies for calculus and how these are incorporated in the reformed classrooms such as flipped classrooms in detail. Online lectures are still the crucial part for students' learning in flipped classrooms as students learn the course contents from the lectures for the first time. Moreover, calculus classrooms include traditional lecture format no matter if it is traditional lecture-based classrooms or reformed classrooms such as flipped classrooms (Sahin et al., 2015). Therefore, online lectures in flipped classrooms and face-to-face lectures in traditional lecture-based classrooms are similar other than they are delivered in different formats. Naccarato and Karakok (2015) insisted that it is necessary to speculate the flipped classrooms with explicit learning objectives to evaluate pedagogical practices. Indeed, there are insufficient research studies that described reformed classrooms for calculus with learning objectives for course contents concretely and reported empirical evidence to prove the effects on students' performance. Last but not least, instructional methods for calculus learning is that the traditional lecture method is still the prevailing instructional method in calculus classrooms, although alternative instructional methods have been explored and introduced. Thus, the calculus reform movement is still on-going.

Therefore, it is necessary to explore *tailored* instructional strategies with empirical evidence for each core concept of calculus courses to facilitate students' understanding, guiding their cognitive process precisely, regardless of the instructional methods.

Suggestions for Potential Future Research on Calculus

As briefly mentioned at the beginning of the chapter, research on learning calculus has a lot of room for improvement. For instance, most studies referred in calculus learning literature are case studies with qualitative research methods. Accordingly, the findings from their research studies cannot be generalized to other settings. Thus, there should be more research studies on instructional methods for calculus learning with reliable empirical evidence.

Another suggestion is conducting research studies to develop instructional strategies for each of the core calculus concepts with empirical evidence.

In this section, previous literature on calculus learning and students' understanding of calculus were discussed, focusing on trigonometry, limits, and the chain rule, followed by the suggestions. As mentioned above, research studies on calculus learning have a lot of room to work on. Researchers in math, math education, and learning sciences, and LDT should conduct collaborative research studies to provide calculus instructors with effective instructional methods with empirical evidence.

2.2 Self-Efficacy and Achievement Goals as Motivational Constructs

Self-Efficacy

Self-efficacy has been vigorously explored as a crucial motivational construct in the educational psychology field (Pajares, 1996; Reeve, 2009). Bandura (1977, 1986, 1993, 1997; Reeve, 2009; Zimmerman, 2000) is one of the key scholars who investigated the construct in learning settings. Self-efficacy is defined as an individual's judgment of ability to achieve certain goals while dealing with a given situation (Bandura, 1986; Reeve, 2009). Self-efficacy consists of the level, generality, and strength of the efficacy belief (Bandura, 1977, 1997; B. J. Zimmerman, 2000). The level indicates the self-efficacy beliefs based on the perceived difficulty of a given task. Generality refers to the transferability of the beliefs across various activities. Strength indicates how strongly a learner believes that he/she can perform a given task. In addition, self-efficacy focuses on performance capabilities rather than personal characteristics. Last, self-efficacy relies on a mastery criterion (Bandura, 1977, 1997; B. J. Zimmerman, 2000). That is, self-efficacy depends on the criteria that evaluate whether a learner fully masters a certain task or learning. Therefore, self-efficacy is multidimensional and can be flexible to given situations.

Self-efficacy has been explored regarding students' learning and academic achievement in Educational Psychology (Chemers, Hu, & Garcia, 2001; Honicke & Broadbent, 2016; Schunk, 1996). Researchers showed that self-efficacy had a strong correlation with other learning-related constructs during the learning process and as a result, affected student's academic performance. For instance, Bouchard, Parent, and Larivèe (1991) found the strong effect of self-efficacy on self-regulation. The authors argued that self-efficacy had a significant correlation with self-regulation including monitoring working time, persistence during task performance, and performance regardless of cognitive ability and school grade. Zimmerman (2000) suggested that self-efficacy and Self-Regulated Learning (SRL) closely interact with each other throughout the entire learning process. Also, self-efficacy was positively correlated with academic achievement (Multon, Brown, & Lent, 1991; B. J. Zimmerman, Bandura, & Martinez-Pons, 1992). Pintrich developed a survey Motivated Strategies for Learning Questionnaire (Pintrich & De Groot, 1990; Pintrich, Smith, García, & McKeachie, 1991) which measures the components that affect academic performance such as intrinsic goal orientation, self-efficacy, critical thinking, and metacognitive self-regulation. The model that is embedded in the MSLQ survey considers how students involve in SRL by using different cognitive strategies (Pintrich & De Groot, 1990). Pintrich and De Groot (1990) reported that self-efficacy, along with intrinsic value, was positively related to performance.

Achievement Goal Theory

Another extensively studied motivational constructs achievement goals. Achievement goal theory considers that an individual's motivation is driven by a specific purpose (Ames, 1992; Covington, 2000), and encompasses various types of beliefs that are related to goals (Pintrich, 2000). Achievement goal theory incorporates both affective and cognitive factors of goal-directed behavior (Ames, 1992; Nicholls, 1984). Accordingly, this theory can be considered as an integrating theory that provides a profound framework to explain students' goal-directed behavior to enhance learning outcomes. Mastery and performance goals were first developed in this theory and had been extensively examined in the prior literature (Ames, 1992; Covington, 2000; Nicholls, 1984; Pintrich, 2000). The mastery goal focuses on learning and understanding materials, whereas the performance goal focuses on performing well compared to others, as it involves an individual's ego (Covington, 2000). These different goals influence students' achievement in different ways based on their self-regulation strategies and learning processes (Covington, 2000; Pintrich, 2000; Zimmerman, Barry, 1990).

Researchers adapted the ‘approach versus avoidance‘ distinction later on into achievement goal theory to better explain performance goal-related results (Harackiewicz, Barron, Carter, Lehto, & Elliot, 1997; Rawsthorne & Elliot, 1999). For instance, Wolters (2003) suggested that performance-avoidance goals may be associated with negative academic outcomes, whereas performance-approach goals are considered beneficial in some cases to enhance learning outcomes (Harackiewicz et al., 1997). Despite the attempts to distinguish achievement goals in more detail, however, findings from different research studies on these constructs are rarely congruent to corroborate the distinction. The researchers included different variables as a measure of response, or for the evaluation criteria that they used to categorize performance goal or mastery goal with approach and avoidance distinction (Elliot & Moller, 2003; Wolters, 2003; Harackiewicz et al., 1997; B. Zimmerman, 2002; B. J. Zimmerman & Schunk, 1989). Elliot and McGregor found that the performance approach was a significant predictor of students' learning outcomes (Elliot & McGregor, 2001). Other studies found that mastery approach affected intrinsic motivation (Elliot, A.J. & Harackiewicz, 1994; Elliot & Murayama, 2008). Also, some of the previous studies found a positive correlation between these elaborated achievement goal orientations and SRL behaviors (Elliot, 1999; Elliot & Moller, 2003). On the contrary, Elliot and Moller (2003) analyzed previous research studies regarding the relationship between SRL and performance approach and found that there was no significant relationship between SRL and performance approach. However, they also suggested the results might be affected by a difference in perspective about the performance approach. That is, the performance approach has been considered either positive or negative for students' learning due to a difference in focus by researchers when evaluating goal orientation in their studies. In the meta-analyses study of Rawsthorne and Elliot (1999), the authors found that confirming and non-confirming feedback were related differently to performance and mastery goals. That is, learners' performance goals affected learners' persistence negatively when confirming feedback was given compared to mastery goals. In contrast, performance goals and mastery goals affected learners' persistence similarly when non-confirming feedback or when feedback was absent. The existence of mastery-avoidance has been investigated to clarify more the existence and role on learners' academic performance (Baranik, Stanley, Bynum, & Lance, 2010; Bartels

& Magun-Jackson, 2009; Yperen, Elliot, & Anseel, 2009). Baranik et al. (2010) performed a meta-analysis to measure the construct validity of mastery-avoidance. The authors found that mastery-avoidance was distinct from the other three achievement goals. Particularly, the authors found that mastery-avoidance goals were strongly related to the increase in cognitive and somatic anxiety, such as test anxiety. In short, "mastery approach focuses on learning and understanding materials, performance approach focuses on performing well compared to others, mastery avoidance focuses on avoiding failure of learning or understanding, and performance avoidance focuses on avoiding performing worse than others" (Heo, Anwar, & Menekse, 2018, p. 1635).

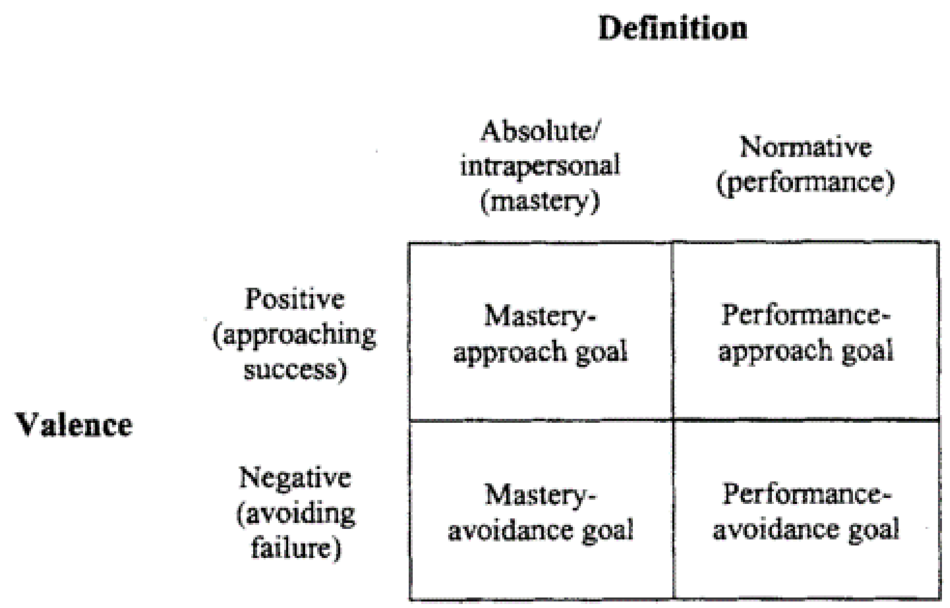


Figure 2.4.: The achievement goal framework. Adapted from "A 2 x 2 Achievement Goal Framework," by A. Elliot and H. A. McGregor, 2001, *Journal of Personality and Developmental Psychology, 80*(3), p. 502. Copyright 2001 by the Americal Psychological Association. Reprinted with permission.

Self-efficacy and achievement goals have been discussed by researchers for more than a few decades. In this section, the previous literature on these constructs is discussed as one of the key motivational constructs and how they are related to other crucial learning related constructs, such as self-regulated learning, reflection, and academic achievement. In addition, research studies in educational psychology that explored the relationship between self-efficacy, achievement goals and other crucial learning related constructs and their relationship with students' academic performance are discussed. In the next section, the relationships between learning transfer, self-efficacy, and achievement goals are discussed. Also, the importance of including learning transfer and motivational constructs into one conceptual framework to better understand students' direct performance as well as their performance on learning transfer is addressed.

2.3 Learning Transfer

Learning transfer has been investigated by many researchers in the Learning Sciences field; prior studies have developed strategies to enhance students' transfer skills and found significant effects on learning transfer of certain subjects such as statistics and physics concepts (Schwartz et al., 2011; Schwartz & Martin, 2004). However, research studies on learning transfer are incongruent and used various terms to refer to similar cognitive concepts (Schwartz et al., 2011; Schwartz & Martin, 2004). Also, the definition of learning transfer should be discussed differentiating from other transfer concepts such as analogy transfer or knowledge transfer (Gick & Holyoak, 1983; Mowery, Oxley, & Silverman, 1996). Research studies on learning transfer and analogy transfer are overlapped largely as the learning transfer is rooted from the studies of analogy transfer in cognitive psychology (Schwartz & Martin, 2004). Therefore, the definitions of analogy transfer and learning transfer share similarities. Analogy transfer refers to transferring information from a prior source that individuals learned to a new target (Holyoak & Thagard, 1997; Novick, 1988). Analogy transfer was the term used in cognitive psychology often regarding artificial intelligence (Forbus, Gentner, Markman, & Ferguson, 1998; Holyoak & Thagard, 1997, 1997). On the other hand, knowledge transfer refers to transferring knowledge or skills from an experienced partner, or a unit, to a novice (Argote, Ingram, Levine, & Moreland, 2000; Wijk, Jansen, & Lyles, 2008). The term is commonly used in organizational studies (Wijk et al., 2008). Researchers who investigate knowledge transfer are often interested in how to effectively form the network structure for knowledge transfer or effective methods of transfer (Argote & Fahrenkopf, 2016; Mowery et al., 1996; Spraggon & Bodolica, 2012; Szulanski, Ringov, & Jensen, 2016). Learning transfer refers to learning when the prior learning in one context either enhances or undermines a related performance in another context (Perkins & Salomon, 1992). Learning transfer captures wider phenomena than analogy transfer since learning transfer includes locating resources to learn a new learning context in addition to the transferring process (Schwartz & Martin, 2004). In other words, researchers who study learning transfer are ultimately interested in how learners recognize what to learn first to understand a new learning context. An example could be realizing that they would need to learn the chain rule (or recognizing something similar even though they cannot exactly pinpoint the chain rule) to solve implicit differentiation problems. Furthermore, learning transfer focuses on educational settings compared to research studies on knowledge transfer (see Table 2.1).

Table 2.1.: Summary of learning transfer and related concepts

	Analogy Transfer	Learning Transfer	Knowledge Transfer
Field	-Cognitive Psychology -Engineering design studies	-Educational Psychology	-Organizational management
Similarity	Focus on the transferring process		
Difference	-Focus on transferring skills and elements of knowledge -The data were usually collected in lab settings	-Focus on transferring skills and knowledge and locating resources to learn a new context, often in educational environment	-Focus on transferring skills and knowledge in working environments
Examples of key studies	-Arthur Markman (1997) -Dedre Genter (1997) -Keith J. Holyoak (1995) -Kevin Dunbar (2001)	-Bransford (2004) -Dan Schwartz (2004) -David N. Perkins (2004) -Gavriel Salomon (1989)	-Argote (1993)

There are several sub-concepts to understand learning transfer. One is near transfer vs. far transfer (Perkins & Salomon, 1992). Near transfer refers to transferring learning to closely related contexts or the same domain. Far transfer, on the contrary, refers to transferring learning to different contexts or the different domains. The near vs. far transfer is similar to within domain vs. between domain analogies (Holyoak & Koh, 1987; Dixon & Johnson, 2011). Within domain analogies refer to transferring analogies from the same domain. That is, the source and the target subjects share surface features for the within domain analogies. On the contrary, between domain analogies refer to transferring analogies drawn from different domains. The source and the target subjects rarely share the surface features but share the structural similarities for the between domain analogies. Researchers in the transfer domain are ultimately interested in facilitating the far transfer or between domain analogies for creative thinking (Barnett & Ceci, 2002).

Other crucial sub-concepts for learning transfer are positive transfer vs. negative transfer (Novick, 1988; Perkins & Salomon, 1992; Schwartz, Chase, & Bransford, 2012). Positive transfer occurs when learning was successfully transferred to a new context and enhanced the following performance. Negative transfer, on the other hand, occurs when learning transferred to the new context impaired the following performance. Negative transfer, thus, occurs when learners overgeneralize the prior learning and apply it to a new context inappropriately (Schwartz et al., 2012). Especially, the overzealous transfer is an example of a negative transfer that Schwartz and his colleagues (2012) suggested that interrupts students from learning something new. That is, students are occupied with applying prior knowledge, solutions or skills that worked in their previous experience to a new learning context and miss the opportunity to learn the new information. Researchers in transfer have investigated the mechanism of positive vs. negative transfer to understand how to facilitate the positive transfer and prevent or reduce negative transfer (Novick, 1988; Schwartz et al., 2012). The research studies are especially beneficial to train novice as novices are more prone to produce negative transfer when they observe the source and the target subjects that share surface features but do not share any structural similarities (Novick, 1988). Researchers were interested in comparing experts versus novices regarding usage of analogies, such as in engineering design domain (Ball, Ormerod, & Morley, 2004; Christensen & Schunn, 2007; Dixon & Johnson, 2011). Other prominent and productive authors in learning transfer, Perkins, and Salomon (2012) suggested the detect-elect-connect model that represents the process of learning transfer. Detect refers to recognizing the possibility of whether the prior learning could be connected to a given new context. Elect refers to determining to pursue the possible connection. Connecting is the most challenging step among these three steps since it is still difficult to find how the prior knowledge and the new learning context can be connected to each other.

Indeed, researchers on learning transfer were interested in how to facilitate learning transfer effectively, especially for novices (Kapur, 2008). Kapur(2008) experimented with ill vs. well-structured problems in a computer-supported collaborative learning setting. The 11th-grade science students were divided into two groups to solve either ill-structured or well-structured problems. Then, they were instructed to solve well-structured problems individually. The author reported that students who were in the ill-structured problem group produced a poor quality of solutions compared to well-structured problem group. However, when they solved well-structured problems individually, they outperformed students who were in the well-structured problem group regarding the near and far transfer. The author suggested that the initial failure that the students experienced helped students transferring their problem-solving skills in the new well-structured problem setting. Similarly, Schwartz, Bransford, and Sears (2005) have also investigated how to facilitate learning transfer while balancing innovation and efficiency in their performance. The authors suggested the figure below to locate the optimal area of learning in instruction and assessment to facilitate both innovation and efficiency. If an instruction only facilitates high efficiency, the transfer can be restricted to near transfer. On the other hand, if the instruction only facilitates innovation or creative thinking, the structure of transfer would be too loose and would not be efficient to solve various problems in many real-world situations either. Therefore, the authors suggested the area in the efficiency x innovation graph where instructions should be designed and assessments should evaluate the optimal balance between these two constructs.

Preparation for Future Learning (PFL) includes a unique approach to learning transfer (Bransford & Schwartz, 1999; Schwartz & Martin, 2004). Bransford and Schwartz (1999) pointed out that previous studies on transfer considered transfer as the direct application of prior knowledge to a new task, which the authors named it as "sequestered problem solving" (p. 68). The authors suggested broadening this view by including learners' PFL. PFL facilitates "knowing with" (p. 69). "Knowing with" refers to the solution process generated by a person as he/she thinks and judges the new learning context with the prior knowledge or skills even though he/she does not recall specific prior context that he/she was in. PFL, therefore, is associative and interpretive compared to other conventional transfer concepts. Schwartz and Martin (2004) suggested a new direction to assess transfer by introducing PFL. In other words, the authors suggested the new direction to measure how learners prepare themselves for a new learning context for transfer instead of using standardized assessment methods that had been used for academic performance.

In general, research studies on learning transfer still have a lot of room to improve. As mentioned above, defining the construct more concretely should be continued. Also, the instructions that facilitate efficiency yet create transfer, and methods to measure learning transfer accurately with high validity and reliability should be further investigated.

2.4 The Relationship between Learning Transfer, Self-Efficacy, and Achievement Goals

Although there is substantial literature on self-efficacy, achievement goals, and how they are related to the other crucial learning related constructs, there is insufficient literature that attempted to understand learning transfer regarding either of these constructs (Belenky & Nokes-Malach, 2012). In fact, learning transfer itself has not been investigated as much in educational psychology compared to self-efficacy and achievement goal theory. One conjecture of insufficient attention to learning transfer by researchers, even though the previous literature infers that there could be a large overlap between each construct, is that learning transfer is not a direct learning outcome. As Schwartz and Martin (2004) pointed out in their study, learning transfer cannot be measured with the same approach as assessing academic achievement. In this sense, it is not surprising that there are even some views that deny the existence of transfer (Van Oers, 1998). Therefore, researchers might doubt the justification of investigating learning transfer in the first place. Accordingly, much less attention has been drawn to the relationship between achievement goals and transfer, or between achievement goals, transfer and academic achievement (Belenky & Nokes-Malach, 2012; Ford, Smith, Weissbein, Gully, & Salas, 1998). Another possible assumption for the lack of literature on the relationship between learning transfer, self-efficacy, and achievement goals is that learning transfer and achievement goals have been perceived that they belong to two distinct categories. In other words, learning transfer has been conventionally perceived as a cognitive factor whereas self-efficacy and achievement goals are motivational factors. Also, both learning transfer and motivational constructs such as self-efficacy and achievement goals have various aspects by themselves that researchers should consider to design their research. Thus, it is possible that researchers limit their scope for their research to either learning transfer or motivation.

Ford and his colleagues (1998) used mastery orientation and metacognitive activity and their relation to knowledge acquisition, skilled performance and self-efficacy. The authors found that mastery orientation was significantly related to metacognitive activity and metacognitive activity to those three training outcomes. Lastly, they also found the positive relationship between these three training outcomes with transfer task (see Figure 2.5). Figure 2.5 indicates that mastery orientation is positively related with metacognition, activity level, and these two were positively related with knowledge, final training performance, and self-efficacy, and these three were positively related with transfer performance.

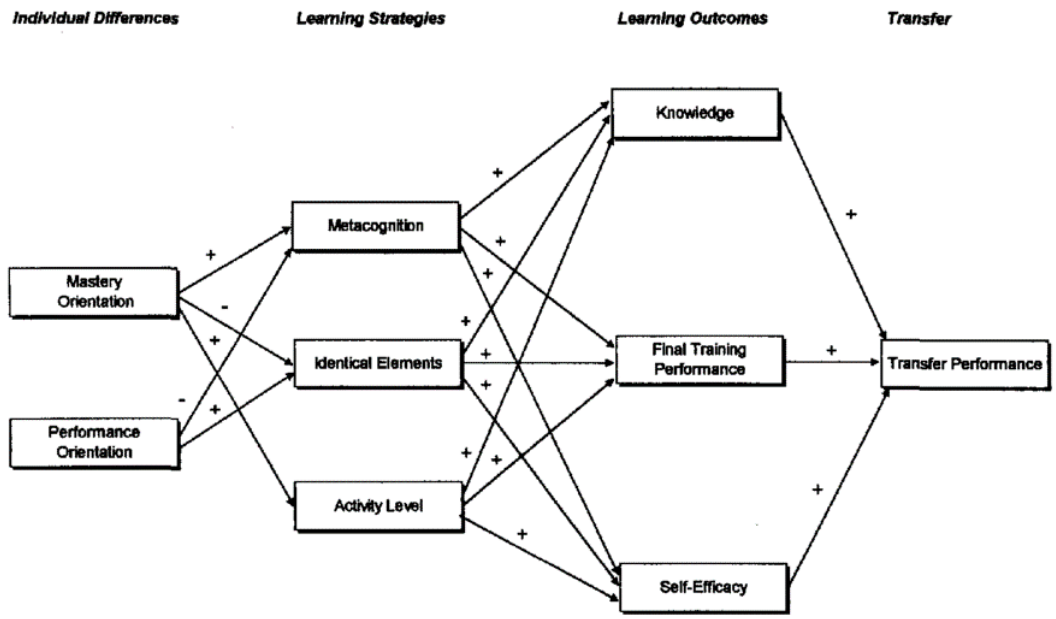


Figure 2.5.: The impact of goal orientation factors and learning strategies on learning and transfer outcomes. Adapted from "Relationships of Goal Orientation, Metacognitive Activity, and Practice Strategies With Learning Outcomes and Transfer," by J. K. Ford, E. M. Smith, D. A. Weissbein, & S. M. Gully, 1998, *Journal of Applied Psychology, 83*(2), p. 219. Copyright 1998 by the American Psychological Association. Reprinted with permission.

The research study of Belenky and Nokes-Malach (2012) also helped to inform other researchers to understand the relationship between mastery approach and learning transfer. Belenky and Nokes-Malach (2012) have investigated how mastery approach was related to training strategies on the learning transfer and learning outcome. The authors found the significant relationship between mastery approach and transfer. In addition, previous research studies that investigated learning transfer and achievement goal (Belenky & Nokes-Malach, 2012; Ford et al., 1998) and metacognition and achievement goal (Pintrich & De Groot, 1990; Vrugt & Oort, 2008) infer that there could be significant overlaps among these three constructs. Therefore, more research studies should be conducted to enlighten the relationship between these constructs.

2.5 Methods Used in Previous Studies on Learning Transfer

Learning transfer occurs when learners apply what they learned in the past to a different and potentially new context, whether it is very similar from the previous context or not (Martin & Schwartz, 2013; Schwartz & Bransford, 1998). Inventing with Contrasting Cases (ICC) is one of the pedagogies that have been proven effective for learning transfer from previous research studies (Schwartz et al., 2011; Schwartz & Martin, 2004). ICC is a combination of 'contrasting cases' activities and 'inventing' activity (Schwartz et al., 2011). First, 'contrasting cases' indicates instructional materials which are designed to facilitate learners to discern the information which could be overlooked otherwise. In other words, learners can find the difference between those two cases and think about how the difference is related to find solutions. Second, 'inventing activities' indicate that students invent aids by themselves to understand given tasks more efficiently or effectively. In the study of Schwartz and his colleagues (2011), students were asked to invent a quantitative index to understand density. Students in ICC group were not given a lecture about density. However, they were asked to come up with their index to represent crowdedness. The authors found that the ICC group showed better transfer compared to a Tell and Practice (TP) group in which participants received conventional instructions.

Although research studies demonstrated the effect of ICC for learning transfer, there have been controversies about whether learners actually perform better by learning with ICC strategy (Bransford & Schwartz, 1999; Schwartz et al., 2005; Schwartz & Martin, 2004). Schwartz and Martin (2004) pointed out that ICC might seem inefficient when measuring learners' performance with standardized assessments which focus on measuring how much learners understand the task at the moment. The authors argued that assessments should include how students can prepare themselves to learn in a new context to assess the effect of ICC strategy. The authors introduced PFL as a new type of assessment to measure learning transfer more accurately. As mentioned in the previous section, PFL is an instructional design which measures how learners prepare to learn from a given direct instruction/resources. In other words, PFL measures learners' ability to think about ‘what do I need to learn to solve this task?’. The authors used a special type of assessment which was called “dynamic assessments” to measure the specific ability. By this assessment, teachers can evaluate learners whether they can improve their ability to detect what they need to learn and prepare for the future task.

ICC has proven that it has prominent features for effective learning transfer. However, there are limitations and has aspects that need to be more explored as well. For instance, ICC from previous research studies hasn't been incorporated into a motivational factor to explain the mechanism of learning transfer even though learning transfer requires persistence and motivation throughout the process. Belenky and Nokes-Malach (2012) pointed out the importance of motivation in their experimental study. The authors argued that mastery goals could be a mediator of ICC and learning transfer. In their study, the researchers adapted the ‘double transfer paradigm’ by Schwartz and Martin (2004) to compare students in four conditions (ICC vs. TP, resources vs. no resources) to find the effectiveness of ICC and measure PFL more accurately. Belenky and Nokes-Malach also adapted mastery approach items from Achievement Goal Questionnaire (Elliot and McGregor, 2001) to measure the level of motivation regarding the content that participants received during the experiment. They compared the mastery approach score from the initial phase and the final phase. The researchers found that ICC facilitated participants' motivation. Interestingly, they also found that students who had high mastery approach transferred what they learned regardless of condition.

2.6 The Study of Belenky and Nokes-Malach (2012)

The study of Belenky and Nokes-Malach (2012) on transfer and mastery approach is mainly based on the study of Ford et al. (1998) and Schwartz and Martin (2004). Belenky and Nokes-Malach (2012) pointed out a lack of literature on the relationship between motivation and learning transfer. Ford and his colleagues (1998) first adapted goal orientation regarding students' performance and learning transfer. Schwartz and Martin (2004) introduced "preparation for future learning approach" to assess learning transfer accurately. Based on these two studies, Belenky and Nokes-Malach (2012) included mastery approach and investigated its relationship with learning transfer and ICC activities. The authors measured the ability to use the standard deviation formula, the ability to represent the data visualization, and reasoning ability with the raw dataset. The purpose of measuring these concepts was measuring students' preparation for future learning ultimately, the same as Schwartz and Martin's (2004) study. Also, mastery approach orientation was measured using the AGQ survey after modification (Elliot and McGregor, 2001).

They mainly used Schwartz and Martin (2004)'s work with some modification. That is, 2(learning activity: invention vs. TP) x 2 (learning resource: present vs. not), between-subjects, pre/post- test design. Students initially received an AGQ survey, took the pre-test, the variability activity, the activity questionnaire, watched a video, the standardization activity, a post-test, an AGQ survey, and a demographics survey. Figure 10 represents the flow of the experiment.

2.6.1 Analytical Procedures

Two raters coded 40% of the data from the problems, and they had 100% agreement. Thus, the remaining data was coded by the one rater. They tested three hypotheses. For H1, "existing mastery-approach orientation will lead to better transfer" (p. 406), the authors used binary logistic regression to examine the relationship between students' initial mastery-approach orientation and their likelihood of solving the transfer problem successfully. For H2, "invention activities will lead to more mastery-related goal adoption than tell-and-practice activities as well as more attention to important conceptual features of the learning problems" (p. 607), t-tests were used to compare invention activities vs. TP on learning transfer (learning performance) and mastery-related goal adoption (activity questionnaire). For H3, “there will be a moderation effect of invention activities on the beneficial effect of mastery-approach orientation for transfer, such that the effect of mastery approach orientation will be a stronger predictor of the likelihood of transfer for the tell-and-practice activities than for invention” (p. 407), the authors used a binary logistic regression as well. The authors found that students’ existing mastery approach orientation led to better transfer and ICC activities led to more mastery related goal adoption. Also, there was a moderation effect of invention activities on the mastery approach and transfer.

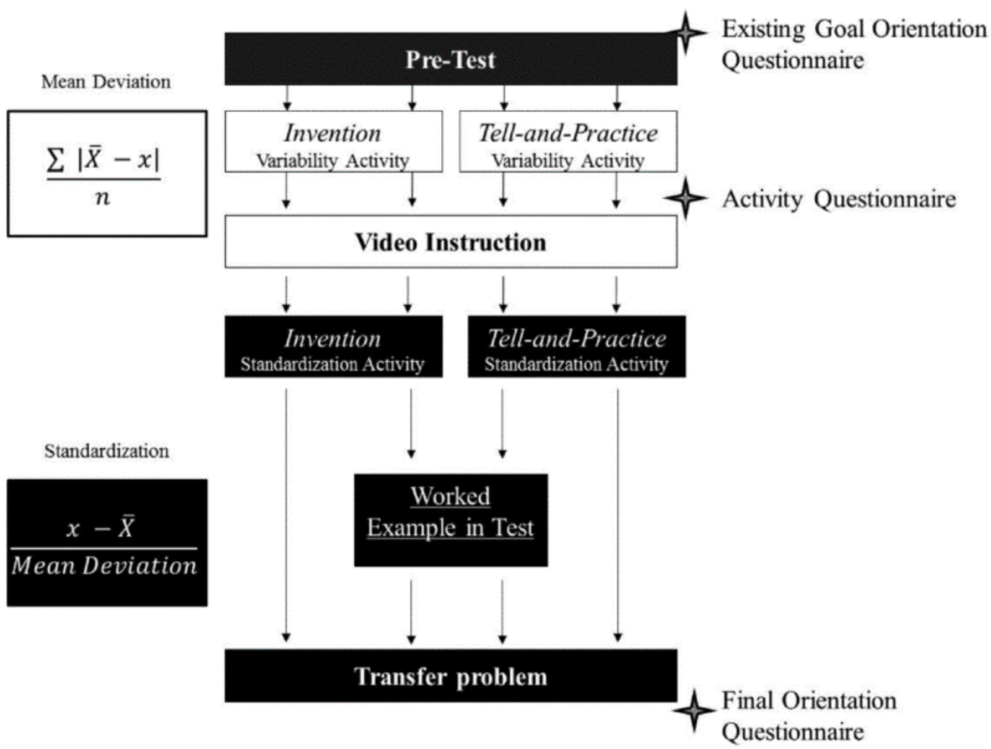


Figure 2.6.: Chart representing the student activities during the experiment. Adapted from ”Motivation and Transfer: The Role of Mastery-Approach Goals in Preparation for Future Learning,” by D. M. Belenky and T. J. Nokes-Malach, 2012, *The Journal of Learning Sciences, 21*, p. 409. Copyright 2012 by Taylor & Francis. Reprinted with permission

2.6.2 Suggestion based on Belenky and Nokes-Malach's (2012) Study

One of the limitations of this study is that they used the same testing material as the Schwartz and Martin (2004) so that the generalizability of the results from this study can be limited. Therefore, different contents area can be tested with the same experimental format to find if the effect is also observed in other contents area.

2.7 Summary

In this chapter, an extensive literature review on pre-calculus concepts, chain rule, and the instructional methods, mainly flipped classroom, for calculus learning, learning transfer, self-efficacy, achievement goal theory, and calculus learning was included. The constructs were included to cover two themes: (1) more tailored instructional strategy is needed with empirical evidence to facilitate the chain rule which is one of the hurdles to understand calculus 1, and (2) ICC strategy with motivation and various contents should be tested.

Based on the literature review and the implications from the previous research studies, the following research study was suggested. In the next chapter, the proposed research study and the findings are discussed. The study is to inform other researchers and stakeholders about a new instructional method for calculus learning with the empirical evidence, more research based-information about the relationship between learning transfer and the motivation factor, and testing ICC strategy with the different contents.

3. METHODOLOGY

In the previous chapters, the literature on learning transfer, achievement goal theory, self-efficacy, calculus concepts, and instructional methods were reviewed. Based on the literature review and the lack of research studies on the relationship between learning transfer, motivation and instructional methods with empirical evidence for calculus learning, the current study was conducted. In this study, Preparation for Future Learning (PFL) was adopted to measure students' performance on near transfer using calculus content, particularly the chain rule. In addition, students' motivation scores were measured to investigate the relationship of motivation to their performance on PFL target problems. iCalCulus (iCC) is the testing survey system that facilitates participants recognizing the difference between simple differentiation versus composite differentiation. The purpose of iCC is to help students figure out what knowledge that they should know in order to solve the composite differentiation problems. iCalCulus (iCC) is an adapted ICC strategy which provides participants with the contrasting cases and an inventing case with a hint. Therefore, the iCC group refers to the group that received the ICC strategy in this study. In addition, the post-test included PFL problems which measures how students locate the resource to solve the transfer problem that they cannot solve with the current knowledge level to measure the learning transfer. This study also includes a theoretical framework that integrates motivational constructs and learning transfer to understand the transfer process more profoundly. The implication of this study might be useful in the design of instructional methods for other calculus contents as well.

3.1 Research Hypotheses

This study is focused on iCC and TP conditions. Therefore, there were two groups: individuals in iCC group and TP group. The response variables were the direct performance and PFL target problem performance. The motivation was the moderator variable. The hypotheses which were tested in the study are:

1. Do students in the inventing contrasting cases for iCalCulus (iCC) group outperform Tell and Practice (TP) group in direct application?

2. Do students in the inventing contrasting cases for ICalCulus (iCC) group outperform Tell and Practice (TP) group in Preparation for Future Learning (PFL) target problems?

3. Is the students' motivational level a significant moderator between the instructional condition and their Preparation for Future Learning (PFL) target problem performance?

3.2 Methods

3.2.1 Participants

The data was collected from 81 undergraduate students at a Midwestern university (iCC group: 40, TP group: 41) who are in STEM majors requiring calculus courses for degree attainment. To define the sampling frame for the data collection, the criteria of STEM majors were examined. Previous literature included either natural science or engineering majors as STEM majors (Chen, 2009). The characteristics of STEM majors can be further defined based on the characteristics of the jobs that students would find after they graduate. In other words, students learn skills and gain knowledge from the higher education institutions to work in STEM job settings. Thus, in order to define STEM majors more clearly, the list of STEM majors was obtained from the website of the U.S. Immigration and Customs Enforcement (ICE) office (Department of Homeland Security, 2016). ICE office assigned the Classification Instructional Programs (CIP) codes to STEM designated degrees for foreigners who apply for H-1B visa for employment in the United States. In addition, STEM majors that require calculus courses to earn the degrees were then extracted from the comprehensive list of majors in the university. Therefore, in this study, STEM majors refer to the majors that are listed in the CIP code list by ICE and require calculus courses. In particular, the Calculus 1 course was chosen as a the determinant degree requirement because the study focuses on content found in a Calculus 1 course. The full list of STEM majors included in this study can be found in appendix B. Thus, the sampling frame is the list of the STEM majors at the Midwest university that require taking Calculus 1s for Engineering and Science majors, and Applied Calculus 1, which is more application focused. Most first-year students in STEM majors were already exposed to calculus material when they were in high school, including the chain rule and implicit differentiation. For this reason, I also included undergraduate students who either passed or waived a Calculus 1 course for this study. The data was collected during one academic year.

3.2.2 Instruments

Google Forms was chosen to form to create study materials including a post-test and a delayed post-test since Google Forms is one of the Google online applications commonly used in educational settings. It allows teachers/ instructors to administer a questionnaire online and assess students' performance promptly (Ballew, 2017; Lin & Jou, 2013). Each student was provided with pieces of blank paper and a pen so that they could use it for calculation.

Motivation Survey

The motivation survey includes two constructs: self-efficacy and mastery approach. To measure participants' self-efficacy, I included self-efficacy for learning and performance items from MSLQ (Pintrich et al., 1991). Among eight self-efficacy items, the item "I believe I will receive an excellent grade in this class" was excluded as participants did not receive grades from this study. The other seven items were modified based on the current study, such as from "…in this class." to "…in this study." (please see table 3.1). The seven items had high Cronbach's α, ranging from .63-.89 based on the study of Pintrich, Smith, Garcia, and McKeachie (1993). For the mastery approach, the mastery approach subcategory items of AGR-Q by Elliot and Murayama (2008) were included. Participants' motivation was measured at the beginning of the task as the items are included in the pre-requisite knowledge check survey. In order to keep the reliability and validity of the items, the items was used without any modification. There are 10 items total in the survey.

Calculus Questionnaires

The full list of the test items are attached as appendix (see appendix **??**). The questions to test the ability of direct application (post-test item 1, 2, 4) asked to differentiate composite functions that were not trigonometric. In the post-test, the learning resource was provided which showed how to differentiate trigonometric composite function using the chain rule. The questions to test the ability of PFL (post-test item 3, 5, 6) were trigonometric composite functions. The question to test the ability of further PFL (post-test item 6) asked to differentiate an expression that have implicit functions that have x and y variables which are also trigonometric and composite function. The delayed post-test included only PFL questions (one PFL and one further PFL). The chart of the study flow and the components can be seen in figure 3.1.

Table 3.1.: Motivation survey

*Please check the most appropriate answer regarding this study.

Items	1 (Strongly disagree)	2	3	4	5	6	7 (Strongly agree)
1. My goal is to learn as much as possible.							
2. I am striving to understand the content as thoroughly as possible.							
3. My aim is to completely master the material presented in this study.							
4. I am certain I can understand the most difficult material presented in the readings for this study.							
5.I'm confident I can understand the basic concepts taught in this study.							
6.I'm confident I can understand the most complex material presented by the researchers in this study.							
7.I'm confident I can do an excellent job on the assignments and tests in this study.							
8. I expect to do well in this study.							
9. I'm certain I can master the skills being taught in this study.							
10. Considering the difficulty of this study, the researchers, and my skills, I think I will do well in this study.							

<table>
<tr><th>iCC group</th><th>TP group</th></tr>
<tr><td colspan="2">a-1. Survey for motivation and calculus course experience (10min): demographics about calculus experience (5 items), motivation survey (10 items)</td></tr>
<tr><td colspan="2">a-2. Pre-requisite knowledge test (10 mins): pre-requisite knowledge check items regarding the chain rule (definition, usage, total 2 items), pre-requisite knowledge check (1 simple differentiation using the quotient rule item, 1 simple differentiation using the product rule item, 2 trigonometry items, 1 trigonometry simple differentiation using the product rule, 1 limit item)</td></tr>
<tr><td rowspan="2">b. iCC with a hint (10 mins): 2 items for contrasting case (simple differentiation vs. composite differentiation that might look similar to simple differentiation), one inventing case (inventing the way to differentiate a composite function), one hint, and asking the same inventing case again</td><td>c. calculus lesson only (5 mins): Chain rule lecture note (same as the iCC group)</td></tr>
<tr><td rowspan="2">d. a worked example (10 mins): a worked example of a procedure of differentiating a composite function using the chain rule with one isomorphic problem for practicing</td></tr>
<tr><td>c. Calculus lesson only (5 mins): chain rule lecture note</td></tr>
<tr><td colspan="2">e. immediate post-test (20 mins): two direct performance items of the chain rule with isomorphic problems as the worked example and the inventing case problem (Q. 1, 2, 4) two target PFL problems (Q. 3, 5), one further target PFL problems (Q. 6)</td></tr>
<tr><td colspan="2">f. delayed post-test (15 mins): two items that ask the definition of the chain rule and how it is used (same as in the pre-requisite knowledge check test), one target PFL problem and one further PFL problem (isomorphic problems as the post-test problems)</td></tr>
</table>

Figure 3.1.: The chart of the study flow and the components

3.2.3 Procedure

Participants were engaged in the study voluntarily. They received the monetary compensation ($16 in cash) at the end of the second session of the study. The advertisement was posted on the school website where any student has access. Students who also registered in calculus 1 courses received advertisement emails as well. In addition, physical advertisement flyers were posted on bulletin boards in engineering buildings, math buildings, science buildings, and the College of Technology building.

Participants were randomly assigned to either of the groups. The demographic survey, and motivation survey data were collected at the beginning of Session 1. The participants in each group performed the tasks individually. A post-test was provided at the end of Session 1. Seven days after Session 1, they came to the computer lab again and took the delayed post- test (Session 2). Two participants in the TP group did not come to the second session for the delayed post-test.

After the data collection, the data was coded based on the code book that I developed (see appendix D). Since there is no previous research study that contains scoring rubric to grade participants' answers to calculus questions regarding learning transfer, the scoring rubric was fully developed after completing data collection. I used hypothesis coding and provisional coding methods as the codes was revised, and expanded further based on the data from the pilot study and the data which was collected for the current study to assess the given hypotheses (Saldana, 2009). Pre-requisite knowledge check test items were coded ranging from 0 to 2 (complete miss, near miss, and correct). iCC tasks were coded in two ways: for ICC task Q1 and 2, it was coded from 0 to 1; and, for tasks Q3 and Q3-b, coding ranged from 0 to 2. The TP task was coded ranging from 0 to 3. For the post-test (Session 1), Q1, 4, 5, and 6, responses were coded ranging from 0 to 3 (complete miss, not there yet, near miss, and correct). For items 2 and 3, responses were coded ranging from 0 to 2. For the delayed post-test (Session 2), the chain rule questions (definition, how to use it) were coded ranging from 0 to 2. The other two items were coded ranging from 0 to 3. The second coder who was not involved in this research study as a key personnel coded approximately 17% of the entire data set of pre-requisite knowledge check test items, iCC task items, TP task item, post-test items, and delayed post-test items to calculate the inter-rater agreement. The inter-rater agreement for each item was significant with κ ranging from .52 to 1.00. Below is the table that denotes the kappa values for each items.

Table 3.2.: Inter-rater agreement for the tasks and the tests

Item	κ
Pretest	
Chain rule definition	.68
Chain rule usage	.56
Q1	1.00
Q2	.58
Q3	.63
Q4	.59
Q5	.61
Q6	1.00
Tasks	
iCC task Q1	cannot be computed since all iCC participants completed and coded as the same.
iCC task Q2	cannot be computed since all iCC participants completed and coded as the same.
iCC task Q3	.72
iCC task Q3-b	.78
TP task	.52
Post-test	
Q1	.90
Q2	.75
Q3	1.00
Q4	.77
Q5	.78
Q6	.80
Delayed post-test	
Chain rule definition	.64
Chain rule usage	.74
Q1	.79
Q2	1.00

3.2.4 Analyses

Five steps of statistical analyses were performed in this study. First, descriptive statistics were performed to find the demographics of participants in each group, such as majors, year, and the level of calculus completion. The distribution of motivation survey score (a.k.a. motivation level), pre-requisite knowledge check test scores, task accuracy, post-test score, and delayed-post-test score of each group were also derived. The IRT model was used to test the item discrimination value and difficulty value of the pre-requisite knowledge check test items, post-test items, and delayed post-test items prior to including the scores in any statistical analyses. Specifically, Bayes 2PL and Graded Response (GR) models using MCMC technique were chosen based on the distribution of the data per coding category. IRTPro version 4.2.1. was used for the procedure.

The mathematical function of (DeMars, 2010) 2PL is denoted below:

$$P(\theta) = \frac{e^{1.7a_i(\theta - b_i)}}{1 + e^{1.7a_i(\theta - b_i)}},$$

where $P(\theta)$ is the probability of scoring of item i. The model includes item difficulty and discrimination parameters.

The mathematical function of GR model (DeMars, 2010)is denoted below:

$$P*_{ik}(\theta) = \frac{e^{1.7a_i(\theta - b_{ik})}}{1 + e^{1.7a_i(\theta - b_{ik})}},$$

where $P*_{ik}(\theta)$ is the probability of scoring in or above category k of item i (given θ and the item parameters). a_i is the item slope, and b_{ik} is the category boundary for category k of item i.

Neither model includes guessing parameters and these two models were chosen because the test items in this study were open-ended questions which were coded into two to four categories of answers. For the pre-requisite knowledge check test, 6 items (excluding the questions about the definition and the usage of the chain rule) were included for the analyses. For the post-test, all 6 items were included for the analyses. For the delayed post-test, 2 items (excluding the questions about the definition and the usage of the chain rule) were included for the analyses. For the GR and 2PL model, the MCMC technique was used by setting prior distributions for the parameter estimates (Hsieh, Proctor, Hou, & Teo, 2010; Zhu & Stone, 2011). Based on the study of Zhu and Stone (2011), the prior was set for the discrimination parameter which is a_j˜*Lognormal* (0, 1) for GR model. For 2PL model, the prior was set for the discrimination parameter which is a_6˜*Lognormal* (0, 0.001) (Hsieh et al., 2010). For the pre-requisite knowledge check test, 2PL model was used for item 6 since most of the participants' answers were coded as either 0 or 2. The GR model was used for the remaining five items. The post-test item 3, 5, and 6 and delayed post-test item 1, and 2 were combined in the GR model as these items were developed to measure the ability on learning transfer. The post-test item 1, 2, and 4 were combined in the GR model because they were used to measure the ability on the direct application. For the MCMC technique, seed = 1,971, maximum number of cycles = 4,000, Monte Carlo size for final log-likelihood approximation = 10,000 were set. For tuning parameters, the default setting of IRT pro program which were 2,000 for burn-in and 3 for thinning was kept for the analyses.

Second, Bayesian independent sample t-tests were performed to compare the difference in motivation level, pre-requisite knowledge check test score between groups, the task accuracy of composite function problem (iCC task Q3 and Q3b after hint was given and TP group practice problem) for each group, direct application, PFL performance, and further PFL performance. Based on the power analysis using G*Power version 3.1.9.4, the sample size should be at least 102 with effect size d = .5, power .8 and assuming that the group sizes are the same for the independent mean comparison. Therefore, Bayesian approach was used for the t-test to gain more information from the model (McNeish, 2016).

Third, one way ANCOVA was performed with direct application performance variable, PFL variables, and further PFL variables as dependent variables separately and motivational level, pre-requisite knowledge check test score as covariates. In order to prevent any confounding effect of time, PFL was divided into PFL 1 and PFL 2 from post-test and delayed post-test respectively for analyses. Further PFL variable was also divided into further PFL 1 and further PFL 2 as well. Based on the power analysis, the sample size should be at least 128 with effect size $f = .25$, power = .8, numerator df = 1, and two groups, and 1 or 2 covariates. However, the sample size for this study is only 81, so the power would be .6 with the same other conditions. Thus, Bayesian approach was used for the ANCOVA to gain more information from the model (McNeish, 2016). JASP 0.11.1 was used for Bayesian ANCOVA analyses.

Fourth, mediation analyses were performed to measure the moderating effect of motivation between group condition and direct application performance and group condition and PFL performance.

Fifth, generalized linear model (GLM) of repeated measure analysis was used to find if there is any difference between groups on the *amount* of change in PFL performance and in further PFL performance from the post-test to the delayed post-test. For the analyses, PFL performance variable included the post-test item 3, 5, 6 and the delayed post-test item 1, 2 and further PFL performance variable included the post-test item 6 and the delayed post-test item 2, which asked the implicit differentiation for composite functions with trigonometric component in them. Motivational level and the pre-requisite knowledge check test score were included as covariates for PFL performance. Pre-requisite knowledge check test score was included as a covariate for further PFL performance for the GLM repeated measure analysis. Based on the power analysis, the sample size should be at least 82 in order to gain 80% power with effect size $f = .25$, with two groups and 2 dependent variables, assuming that correlation between repeated measures is .5 for frequentist GLM repeated measure. As the sample size for this study is 81, which is close to 82, Bayesian approach was not employed. SPSS 26 was used for the statistical analyses.

4. RESULTS

4.1 Results from Descriptive Statistics

4.1.1 Majors, Year, and Calculus Completion Level

Participants in iCC group were from five different colleges and nineteen majors. Participants who were from the college of engineering (n = 26) were in First-Year Engineering, Chemical Engineering, Cellular and Bio molecular Engineering, Civil Engineering, Computer engineering, and Industrial Engineering majors. Participants who were in the college of science (n = 7) were in Computer Science, Actuarial Science, Atmospheric Science, Physics, and Mathematics. One participant was in double major. Participants from the college of technology (n = 3) were in Animation, Cybersecurity, Mechanical Engineering Technology, and Visual Effects Compositing majors. One participant was in double major. Participants from the college of agriculture (n = 2) were in Biochemistry and Cellular and Bio molecular Engineering majors. Participants from the college of business (n = 2) were in Supply Chain Information Analytics and Industrial Management majors. Finally, there was one participant who were in Pre-Pharmacy major from the college of pharmacy.

Participants in TP group were also from five different colleges but with seventeen majors. Participants who were from the college of engineering (n = 27) were in First-Year Engineering, Chemical Engineering, Industrial Engineering, and Mechanical Engineering majors. Participants who were in the college of science (n = 5) were in Actuarial Science, Applied Statistics, Computer Science, Statistics, and Mathematics majors. One participant was in double major. Participants from the college of technology (n = 4) were in Aeronautical Engineering Technology, Cybersecurity, Mechanical Engineering Technology, and Robotics Engineering Technology majors. Participants from the college of agriculture (n = 2) were from Animal Science and

Biochemistry majors. The participant from the college of business (n = 1) was from Supply Chain Information Analytics major. Lastly, there were two participants who were in Pharmaceutical Sciences major from the college of pharmacy. Below are the figures (see 4.1 and 4.2) that represent participants' demographics per group.

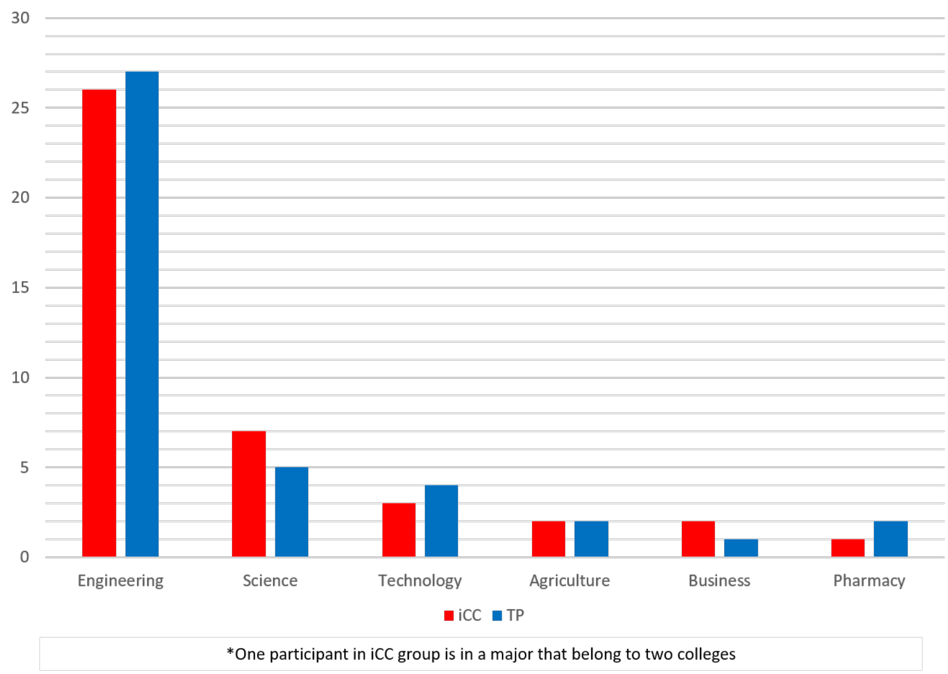


Figure 4.1.: Participants per college by group

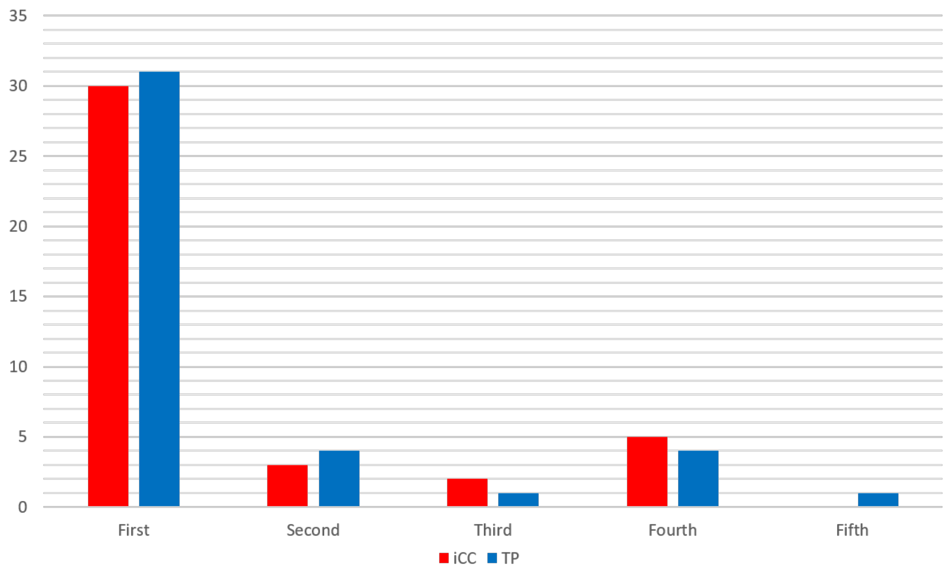


Figure 4.2.: Participants per year by group

Descriptive statistics indicated that although a lot of participants in both groups were either taking Calculus 1 or Applied Calculus 1 courses at the moment. However, none of the participants was completely new to the chain rule and the simple differentiation as they already learned pre-calculus and at least the basic level of calculus in high school.

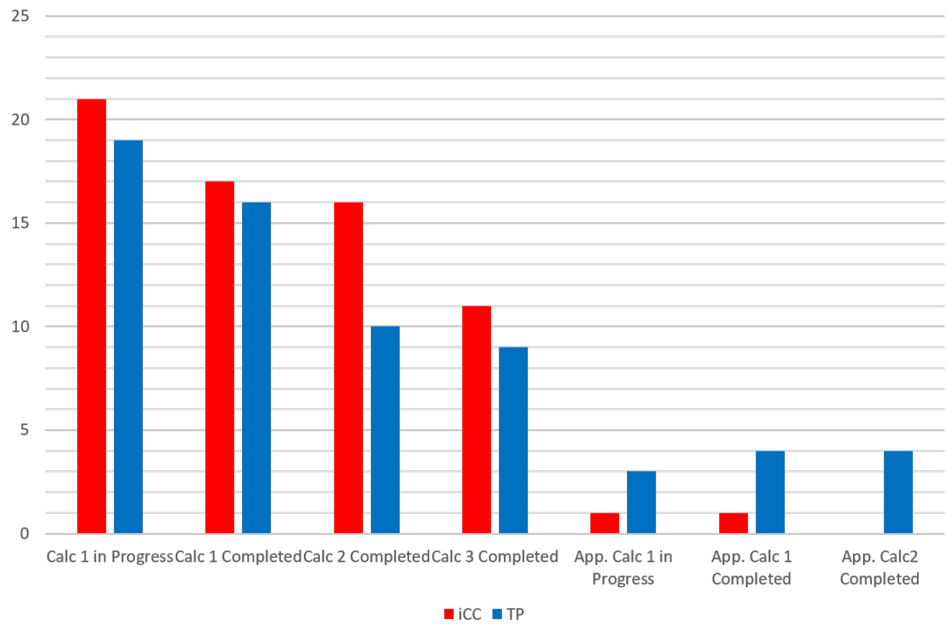


Figure 4.3.: Level of calculus completion by group

4.1.2 Motivational Level and Pre-Requisite Knowledge Check Test Score

The motivational level was negatively skewed for both groups, indicating that the distribution is concentrated on the higher score. In addition, the kurtosis were all positive for both groups, meaning that the distribution is concentrated to the mean.

Table 4.1.: Motivational level

	N	M	SD	Min	Max	Skewness	Kurtosis
iCC	40	5.46	.92	2.70	7.00	-1.00	1.49
TP	41	5.31	.95	2.30	7.00	-.60	1.29
Total	81	5.39	.93	2.30	7.00	-.77	1.16

The pre-requisite knowledge check test score was rescaled to fall between 0 to 1 so that the interpretation could be easier. The results indicated that iCC group was slightly skewed to the higher score, on the other hand, the TP group was skewed to

Table 4.2.: Pre-requisite knowledge check test score

	N	M	SD	Min	Max	Skewness	Kurtosis
iCC	40	.43	.23	.00	.88	-.08	-.53
TP	41	.35	.23	.00	.94	.52	-.41
Total	81	.39	.23	.00	.94	.21	-.69

the lower score. Both group showed negative kurtosis, meaning that the scores were slightly more dispersed than the normal curve.

4.1.3 Task Accuracy Score

Task accuracy score was also rescaled to fall between 0 to 1 for easier interpretation. Participants in TP group showed higher accuracy in their task compared to the participants in iCC group. It is possible participants in ICC group went through more cognitive load than participants in TP group since ICC group participants were provided more questions without any lecture beforehand.

Table 4.3.: Task accuracy score

	N	M	SD	Min	Max	Skewness	Kurtosis
iCC Q. 3	40	.33	.43	.00	1.00	.77	-1.23
iCC Q. 3-b	40	.41	.44	.00	1.00	.36	-1.62
TP Practice problem	41	.79	.35	.00	1.00	-1.36	.38

4.1.4 Post-Test Score and Delayed Post-Test Score

The results showed that the participants in both groups had post-test scores skewed to higher score. The scores were dispersed as similar as the normal distribution, however, iCC group participants had more dispersed distribution whereas TP group participants had more concentrated distribution than the normal distribution.

Table 4.4.: Post test score

	N	M	SD	Min	Max	Skewness	Kurtosis
iCC	40	.47	.21	.06	.89	-1.25	-.28
TP	41	.52	.22	.06	.89	-.63	.03
Total	81	.50	.21	.06	.89	-.37	-.29

The delayed post-test score of iCC group was more skewed to higher score and more dispersed than the normal distribution. TP group was slightly skewed to higher score and slightly more dispersed than the normal distribution.

Table 4.5.: Delayed post-test score

	N	M	SD	Min	Max	Skewness	Kurtosis
iCC	40	.47	.30	.00	1.00	-.11	-1.25
TP	41	.45	.25	.00	.92	-.08	-.89
Total	81	.46	.28	.00	1.00	-.08	-1.07

4.2 Results from Bayesian IRT and Bayesian Independent Sample T-Test Analyses

4.2.1 Results from Bayesian IRT Analyses

Results from Pre-Requisite Knowledge Check Test Item Analyses

The results of the analyses indicated that all pre-requisite knowledge check test items had good discrimination values, ranging from .54 to 2.70. In other words, the items discriminated test-takers who differed on the pre-calculus constructs. The difficulty values of the items were also from good overall. The characteristic curve plot for the six items overall (see figure E.3) indicated that the test had a fairly informative items.

Table 4.6.: Graded model and 2PL model item parameter estimates

Item	a	b_1	b_2
1	.54	-.64	1.02
2	.56	-1.17	.49
3	1.03	.61	4.82
4	1.19	.31	.63
5	2.70	.27	.31
6	1.00	.82	-

Results from Post-Test Item 1, 2, 4 Analyses

The results (see table 4.7) indicated that the post-test item 1, 2, and 4 had good discrimination values. For post-test item 4, all three difficulty values were negative ranging from -3.79 to -1.20, indicating it was poorly designed as it was too easy. The plots (see figure E.4) indicated that the post-test item 4 did not have much good information. The test characteristic curve plot (see figure E.5) indicates the items are overall very informative.

Table 4.7.: Graded model item parameter estimates

Item	a	b_1	b_2	b_3
post 1	.41	1.23	6.24	8.75
post 2	.92	-2.66	-.26	-
post 4	.71	-3.79	-2.05	-1.20

Results from Post-Test item 3, 5, 6 and Delayed Post-Test Item 1, 2 analyses

Post-test item 3, 5, and 6 had good discrimination values ranging from .98 to 1.36. The delayed post-test item 1 and 2 had also good discrimination values ranging from 0.99 to 1.50. The difficulty values indicated that post-test item 3, 5, and 6 had a fairly good difficulty values. Delayed post-test item 2 also had a fairly good difficulty values as well. Delayed post-test item 1 had very good difficulty values. Overall, the post-items and the delayed post-test items had the good test information. The plots (see figures E.6 and E.7) indicated that post-test item 3, 5, 6 and both of the delayed post-test items had fairly good information. The characteristic curve plot (see figure E.8) indicates that the overall items have the fairly good information.

Table 4.8.: Graded model item parameter estimates

Item	a	b_1	b_2	b_3
post 3	1.32	-1.38	.09	-
post 5	.98	.08	1.06	1.26
post 6	1.36	.01	.96	1.20
delayed 1	.99	-.58	1.27	3.11
delayed 2	1.05	-0.06	1.05	1.58

4.2.2 Results from Bayesian Independent Sample T-Test

The Bayesian independent sample t-test analyses were conducted to compare ICC group and TP group on the motivational level and pre-requisite knowledge check test score to gain the preliminary data of the participants in each group. In addition, task accuracy on composite function question (Q. 3 and Q. 3-b for iCC group and practice problem for TP group), direct application, PFL performance, and further PFL performance from the post-test and delayed post-test were also compared by t-test analyses. Diffuse prior was used which is uninformative prior (Hoff, 2009), as it gives general information about chosen variables when informative prior is not available. According to the previous literature on BF_{01} (Jarosz & Wiley, 2014), there were from anecdotal to moderate evidences for H_0 when reporting the results, thus, in this study, the terminology from Jarosz and Wiley (2014) are used for reporting. The plots of log likelihood function, prior distribution, and posterior distribution for each item are attached as appendix (see Appendix F).

Motivational Level

The result indicated that there was no difference on the motivational level between groups as $BF_{01} = 4.74$.

Table 4.9.: Results from Bayesian independent samples t-test analyses

Item	Mean Difference	Pooled SE Difference	BF_{01}	t	df	p
Motivational level	-.14	.21	4.74	.69	79	.495

* Mean Difference = iCC-TP

Pre-Requisite Knowledge Check Test Score

The items were included in an individual t-test model for the analyses. The BF_{01}s were ranging from 1.48 to 5.78 for entire test items, meaning that there is no evidence to support H_1. In other words, iCC group participants and TP group participants had similar level of pre-requisite knowledge check test score. Below is the table (table 4.10) that reported the results from the analyses in detail. The plots of log likelihood function, prior distribution, and posterior distribution for each item are attached as appendix (see Appendix F).

Table 4.10.: Results from Bayesian independent samples t-test analyses

Item	Mean Difference	Pooled SE Difference	BF_{01}	t	df	p
Pre-chain rule definition	.05	.180	5.65	.31	79	.762
Pre-chain rule usage	-.04	.21	5.78	-.21	79	.835
Pre Q. 1	-.15	.20	4.58	-7.38	79	.463
Pre Q. 2	-.30	.20	2.02	-1.53	79	.131
Pre Q. 3	-.08	.12	4.61	-.73	79	.468
Pre Q. 4	-.17	.21	4.39	-.80	79	.428
Pre Q. 5	-.24	.22	3.32	-1.11	79	.269
Pre Q. 6	-.36	.21	1.48	-1.74	79	.086

* Mean Difference = iCC-TP

Task Accuracy

Task accuracy of iCC task (Q. 3) and TP task (practice problem) and iCC task after a hint was given (Q. 3-b) and the TP task (practice problem) were compared. The results indicated that there is a substantial evidence to support H_1, indicating that the task accuracy of participants in TP group was significantly higher than participants in iCC group did.

Table 4.11.: Results from Bayesian independent samples t-test analyses

Item	Mean Difference	Pooled SE Difference	BF_{01}	t	df	p
iCC task Q. 3 vs. TP task practice problem	-.4636	.09	.000	-5.33	79	.000

* Mean Difference = iCC-TP

Table 4.12.: Results from Bayesian independent samples t-test analyses

Item	Mean Difference	Pooled SE Difference	BF_{01}	t	df	p
iCC task Q. 3-b vs. TP task practice problem	-.3761	.09	.002	-4.29	79	.000

* Mean Difference = iCC-TP

Direct Application

The result indicated that there was a weak evidence to support that a significant difference in the direct application between groups as $BF_{01} = .805$.

Table 4.13.: Results from Bayesian independent samples t-test analyses

Item	Mean Difference	Pooled SE Difference	BF_{01}	t	df	p
Direct application	-.10	.05	.805	-2.09	79	.040

* Mean Difference = iCC-TP

PFL Performance from the Post-Test

The result indicated that there was no evidence to support a significant difference in PFL performance from post-test between groups as $BF_{01} = 4.599$.

Table 4.14.: Results from Bayesian independent samples t-test analyses

Item	Mean Difference	Pooled SE Difference	BF_{01}	t	df	p
PFL performance	-.05	.06	4.599	-.73	79	.467

* Mean Difference = iCC-TP

PFL Performance from the Delayed Post-Test

The result indicated that there was no evidence to support a significant difference in PFL performance from delayed post-test between groups as $BF_{01} = 1.966$.

Table 4.15.: Results from Bayesian independent samples t-test analyses

Item	Mean Difference	Pooled SE Difference	BF_{01}	t	df	p
PFL performance	.09	.06	1.966	1.54	77	.128

* Mean Difference = iCC-TP

Further PFL Performance from Post-Test

The result indicated that there was no evidence to support a significant difference in further PFL performance from post-test between groups as $BF_{01} = 5.872$.

Table 4.16.: Results from Bayesian independent samples t-test analyses

Item	Mean Difference	Pooled SE Difference	BF_{01}	t	df	p
Further PFL performance	-.03	.27	5.872	-.09	79	.927

* Mean Difference = iCC-TP

Further PFL Performance from the Delayed Post-Test

The result indicates that there was strong evidence to support a significant difference in further PFL performance from delayed post-test between groups as $BF_{01} =$ .096.

Table 4.17.: Results from Bayesian independent samples t-test analyses

Item	Mean Difference	Pooled SE Difference	BF_{01}	t	df	p
Further PFL performance	.76	.25	.096	3.05	77	.003

* Mean Difference = iCC-TP

4.3 Results from Bayesian One-Way ANCOVA Analyses

Bayesian ANCOVA analysis chose the model with both the average pre-requisite knowledge check test score and motivational level as the covariates and the group condition as the random factor as the best model. Based on Wagenmakers et al. (2018), BF_M and BF_{01} (or BF_{10}) would be critical indexes for interpretation to report the results of the analysis. The result indicates that BF_M shows the change from prior to posterior model odds. On the other hand, BF_{01} lists the BF_{01} for null model against each model. Based on Andraszewicz and her colleagues (2015), BF_M from 3 to 10 shows that there is moderate evidence for the model, and from 10 to 30 shows that there is strong evidence, and from 20 to 100 shows that there is very strong evidence to support the model. On the other hand, BF_{01} from .33 to .10 shows that there is a substantial (or moderate) evidence for the alternative model against the null model, from .10 to .03 shows that there is strong evidence, from .03 to .01 shows that there is a very strong evidence, and from .01 and smaller shows that there is decisive evidence for the alternative model.

The BF_M is denoted as follows:

$$BF_M = P(\theta|x), \tag{4.1}$$

where x indicates a data point from the dataset, and θ indicates is a parameter. BF_{01}, can be computed as follows:

$$BF_{01} = \frac{P(x|H_0)}{P(x|H_1)}, \tag{4.2}$$

where H_1 is an alternative hypothesis and H_0 is a null hypothesis.

4.3.1 Direct Application

Bayesian ANCOVA analysis chose the model with both the average pre-requisite knowledge check test score and motivational level as the covariates and the group condition as the random factor as the best model. $BF_M = 4.972$ indicates that there is moderate evidence for the model. $BF_{01} = .085$, shows that there is strong evidence that supports the model against the null model.

Table 4.18.: Model comparison

Models	P(M)	P(M—data)	BF$_M$	BF$_{01}$	error %
Null model (incl. group)	0.250	0.053	0.166	1.000	
ave_pre_req + motiva	0.250	0.616	4.807	0.085	0.999
ave_pre_req	0.250	0.174	0.634	0.301	0.841
motiva	0.250	0.157	0.560	0.334	0.859

4.3.2 PFL Performance from the Post-Test

Bayesian ANCOVA analysis chose the model with both the average pre-requisite knowledge check test score and motivational level as the covariates and the group condition as the random factor as the best model. $BF_M = 3.228$, which indicates there is moderate evidence that supports the model. $H_0 = .014$ ($\approx 2.933e - 4$), shows that there is very strong evidence that supports the model against the null model.

Table 4.19.: Model comparison

Models	P(M)	P(M—data)	BF_M	BF_{01}	error %
Null model (incl. group)	0.250	1.525e-4	4.575e-4	1.000	
ave_pre_req + motiva	0.250	0.520	3.248	2.933e-4	1.004
ave_pre_req	0.250	0.480	2.767	3.178e-4	0.892
motiva	0.250	1.471e-4	4.414e-4	1.037	0.985

4.3.3 PFL Performance from the Delayed Post-Test

Bayesian ANCOVA analysis chose the model with the average pre-requisite knowledge check test score as the covariate and the group condition as the random factor as the best model. $BF_M = 3.403$, which indicates there is moderate evidence that supports the model. $H_{01} = .103$, shows that there is substantial (or moderate) evidence that supports the model against the null model.

Table 4.20.: Model comparison

Models	P(M)	P(M—data)	BF_M	BF_{01}	error %
Null model (incl. group)	0.250	0.055	0.173	1.000	
ave_pre_req	0.250	0.531	3.398	0.103	0.875
ave_pre_req + motiva	0.250	0.380	1.839	0.143	1.110
motiva	0.250	0.034	0.107	1.585	0.985

4.3.4 Further PFL Performance from the Post-Test

Bayesian ANCOVA analysis chose the model with the pre-requisite knowledge check test score as the covariate and the group condition as the random factor as the best model. $BF_M = 10.955$, which indicates there is strong evidence that supports the model. $H_{01} = .003$, shows that there is decisive evidence that supports the model against the null model.

Table 4.21.: Model comparison

Models	P(M)	P(M—data)	BF_M	BF_{01}	error %
Null model (incl. group)	0.250	0.003	0.008	1.000	
ave_pre_req	0.250	0.790	11.309	0.003	0.910
ave_pre_req + motiva	0.250	0.206	0.780	0.013	1.050
motiva	0.250	7.027e-4	0.002	3.819	2.538

4.3.5 Further PFL Performance from the Delayed Post-Test

Bayesian ANCOVA analysis chose the model with the average pre-requisite knowledge check test score as the covariate and the group condition as the random factor as the best model. $BF_M = 5.570$, which indicates there is moderate evidence that supports the model. $H_{01} = .199$, shows that there is substantial (or moderate) evidence that supports the model against the null model.

Table 4.22.: Model comparison

Models	P(M)	P(M—data)	BF_M	BF_{01}	error %
Null model (incl. group)	0.250	0.129	0.445	1.000	
ave_pre_req	0.250	0.648	5.522	0.199	0.807
ave_pre_req + motiva	0.250	0.192	0.712	0.673	1.108
motiva	0.250	0.031	0.096	4.157	0.995

4.4 Mediation Analyses

4.4.1 Group Condition, Motivational Level, and Direct Application Performance

The result indicates that motivational level was not a significant moderator between the group condition and the direct application performance at all, with b = -.01, t(77) = -.19, p = .846.

4.4.2 Group Condition, Motivational Level, and PFL Problem Performance from the Post-Test

The result indicates that motivational level was not a significant moderator between the group condition and the PFL problem performance from the post-test at all, with b = .05, t(77) = .72, p = .475.

4.4.3 Group Condition, Motivational Level, and PFL Problem Performance from the Delayed Post-Test

The result indicates that motivational level was not a significant moderator between the group condition and the PFL problem performance at all, with b = .01, t(75) = .08, p = .933.

4.4.4 Group Condition, Motivational Level, and Further PFL Problem Performance from the Post-Test

The result indicates that motivational level was not a significant moderator between the group condition and the further PFL problem performance at all, with b = .08, t(77) = .41, p = .686.

4.4.5 Group Condition, Motivational Level, and Further PFL Problem Performance from the Delayed Post-Test

The result indicates that motivational level was not a significant moderator between the group condition and the further PFL problem performance at all, with b = .08, t(75) = .42, p = .675.

4.5 Results from GLM Repeated Measure Analyses

For GLM repeated measure analysis, time was included as a within-subject factor (two points in time total), group condition as a between-subject factor (iCC vs. TP), and the motivational level and pre-requisite knowledge check test score as covariates for PFL problem performance. For further PFL problem performance, only the pre-requisite knowledge check test score was chosen as a covariate based on the results from ANCOVA analyses. Prior to the analyses, normality of residual, equality of covariance matrices and homogeneity of variances for each combination of the groups of two factors were checked for each GLM Repeated Measure Model. Sphericity was not checked as there were only two levels of the time point.

4.5.1 PFL Performance

Assumption of Normality, Equality, Homogeneity

The Q-Q plot (see Appendix G) with the studentized residual to check the normality of residual with PFL 1 and 2 indicates that the assumption of normality was violated, indicating that the data has more extreme values than the data with normal distribution as the points curve off in the extremeties. The Box's M test indicates that the equality assumption was not violated (p = .754). Finally, Levene's test showed that the assumption of homogeneity was not violated for either of the combinations, with F(1, 77) = .23, p = .637 for the PFL performance 1 combination and with F(1, 77) = .00, p = .985 for the PFL performance 2 combination.

GLM Repeated Measure Analysis

The results indicate that there was no significant decrease over time for participants in both groups regarding PFL problem performance, with Wilk's lambda = .10, F(1, 75) = .05, p = .830, η_p^2 = .00. In addition, there was a significant interaction effect between group condition and time, with Wilk's lambda = .92, F(1, 75) = 6.62, p = .012, η_p^2 = .08. Test of between- subject effect indicates that there was no significant group effect, with F(1, 75) = .17, p = .682, η_p^2 = .00. In short, the results indicate a cross-over interaction effect. In figure 4.4, the plot of marginal mean of PFL performance from post-test to delayed post-test after being adjusted for the covariates (motivational level, pre-requisite knowledge check test score) shows the change over time.

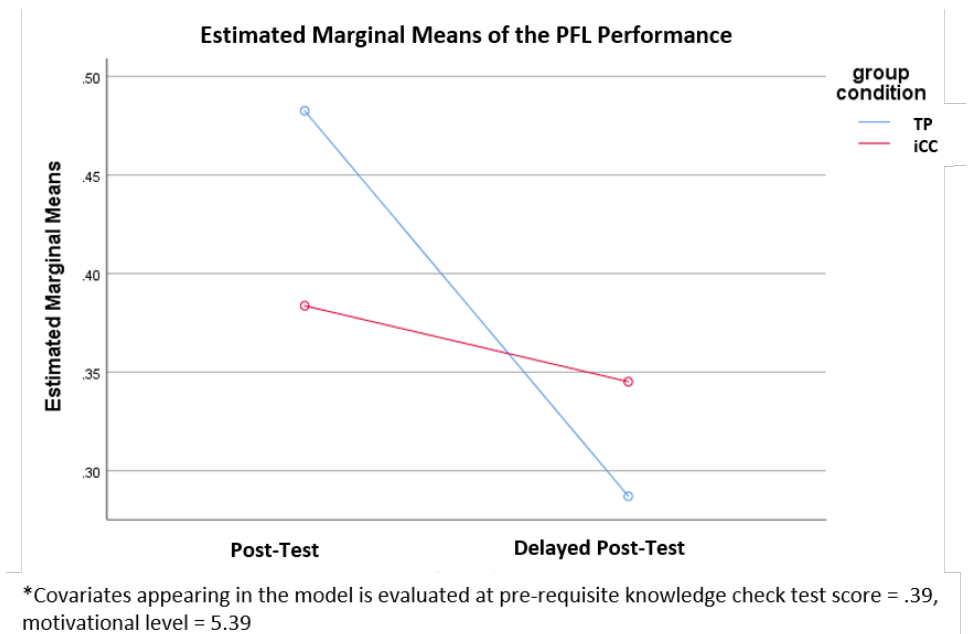


Figure 4.4.: Plot of group comparison on the marginal mean of PFL performance from the post-test

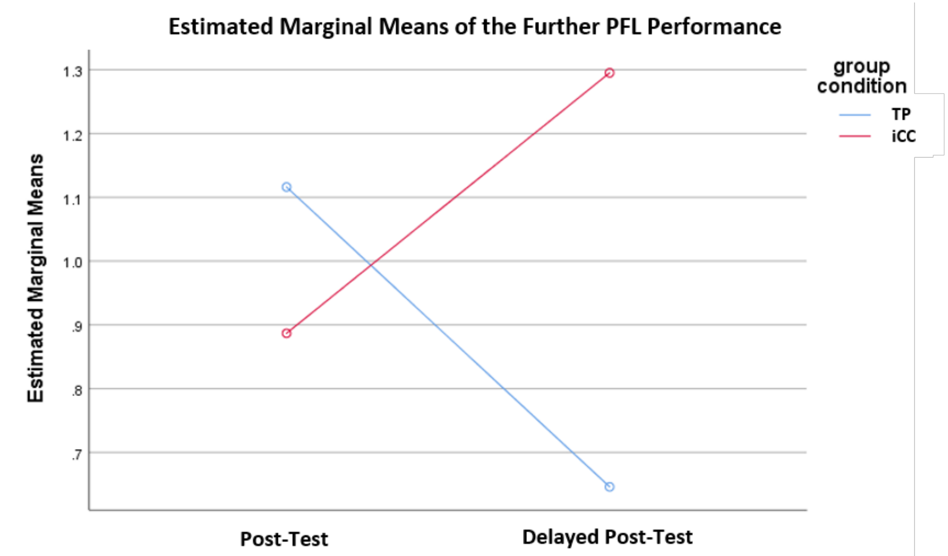
4.5.2 Further PFL Performance

Assumption of Normality and Homogeneity, and Sphericity

The Q-Q plots (see Appendix G) with the studentized residual to check the normality of residual with further PFL from the post-test and the delayed post-test indicate that the assumption of normality was violated, indicating that the the data has more extreme values than the data with normal distribution as the points curve off in the extremeties. The Box's M test indicates that the equality assumption was not violated (p = .370). Lastly, Levene's test showed that the assumption of homogeneity was not violated for further PFL problem performance 1 combination, with F(1, 77) = .20, p = .654. However, the homogeneity assumption was violated for further PFL problem performance 2 combination and with F(1, 77) = 5.15, p = .026.

GLM Repeated Measure Analysis

The results indicate that there was no significant decrease over time for participants in either group regarding further PFL problem performance, with Wilk's lambda = .99, F(1, 76) = 1.15, p = .287, η_p^2 = .02. On the other hand, there was a significant interaction effect between group condition and time, with Wilk's lambda = .89, F(1, 76) = 9.22, p = .003, η_p^2 = .11. The test of between- subject effect indicates that there was no significant group effect, with F(1, 76) = 1.06, p = .305, η_p^2 = .01. In summary, the results indicate a significant cross-over interaction effect. Below is the plot of the marginal mean of further PFL performance from post-test to delayed post-test after being adjusted for the covariate (pre-requisite knowledge check test score) which shows the change over time.



*Covariate appearing in the model is evaluated at pre-requisite knowledge check test score = .39

Figure 4.5.: Plot of group comparison on the marginal mean of PFL performance from the delayed post-test

5. DISCUSSION AND CONCLUSION

5.1 Discussion

In the current study, the relationship between the types of instructional strategy, motivational level, and performance on the direct application, PFL target problems and further PFL target problems was investigated. The participants were assigned to either the iCC or TP group and solved the assigned tasks individually. Their motivation score was also collected at the beginning of the study. The delayed post-test was conducted seven days after Session 1 to determine if there was any remaining effect of the instruction on the participants.

Bayesian one-way ANCOVA analyses were conducted to determine if there was a statistically significant difference between the iCC group and TP group on direct application performance, PFL performance and further PFL performance from the post-test and delayed post-test respectively, while controlling for the average pre-requisite knowledge check test score and/or motivational level variable(s). The results indicated that there was the moderate evidence to support that the group condition had a significant effect on direct application, PFL performance from the post-test while controlling for the average pre-requisite knowledge check test score and motivational level. The results also indicated that there was from moderate to strong evidence that supports that group condition had the significant effect on the PFL performance from the delayed post-test, further PFL performance from the post-test, and the further PFL performance from the delayed post-test, while controlling for the average pre-requisite knowledge check test score only. In other words, motivational level was a significant covariate only for the direct application performance and the PFL performance from the post-test.

In order to investigate the amount of the change in participants performance on PFL and further PFL problems, the GLM repeated measure analyses were conducted. The results showed two findings. First, there was a cross-over interaction effect regarding PFL performance. That is, there was a significant interaction effect between time and group condition. As the plot indicates, the mean across the time for each group might not be significantly different. However, the amount of change in PFL performance per group was significantly different. The analysis of further PFL performance also indicated the cross-over interaction effect over time, although neither time nor group condition alone had a significant effect. The results indicate that the iCC instructional strategy was more effective in maintaining the ability of near transfer than the TP strategy. It is possible that the questions provided to the iCC group facilitated preparing to *learn* to solve problems that required more than directly apply the prior knowledge. The effect remained even after the time passed.

In conclusion, the findings indicate that the iCC group did not outperform TP group in direct application performance and the PFL performance from the post-test. In addition, the motivational level was not an effective moderator between the instructional condition and the PFL performance. Lastly, the iCC group did not outperform the TP group in PFL performance or further PFL performance except in case of the further PFL performance from the delayed post-test (see Table 4.17). There was, however, a significant cross-over interaction effect between time and group condition for the PFL performance and further PFL performance, which indicates the remaining effect of the iCC instructional strategy over time.

There are a few limitations in this study. First, the external validity of the participants might not be high due to the sampling method. That is, the participants were chosen from only one university based on participants' volunteering using convenience sampling, the external validity might be low, thus, the external validity of the findings of this study could be low as well. In addition, it is possible that the reason that the motivational level alone was not an effective moderator is that participants might not have engaged as deeply with the experimental sessions because they were a short-term event. Previous studies regarding motivation address this phenomenon (Jang, 2008; Vansteenkiste et al., 2018). Another possible reason for the limited effect of motivational level to PFL variable might be due to the lab study setting because students do not expect to receive an academic grade or any negative consequences, other than receiving monetary compensation for their performance. Finally, the small sample size of this study might have also affected the results of the Bayesian analyses for the t-test and ANCOVA models especially the prior was not informative.

The implications of the results lead the potential future studies toward a couple of directions. First, the types of content that cause negative transfer when associated with the chain rule could be more thoroughly investigated. The author found numerous cases of potential negative transfer in participants' answers on test items (see Appendix H). For example, there was a participant who was confused with partial derivatives and implicit differentiation. Thus, it might be meaningful to investigate the types of content that trigger negative transfer regarding the chain rule. Second, follow-up studies that are associated with the iCC instructional strategy with types of feedback, frequency of feedback, and the timing of each feedback could be investigated. The participants who were in the iCC group only received one hint during Session 1, and it might have not been helpful. As the accuracy rate shows, the participants in the iCC group showed only 62% out of 100%. Thus, it is possible that the effect of varying types and frequency of feedback associated with the ICC strategy on the performance of transfer could be explored to better help participants learn with the ICC strategy. Finally, this current study only included students who need to take Calculus 1 to attain their degree from STEM majors. There are students in non-STEM majors, such as Economics or Accounting, who also need to take calculus courses. Thus, a follow-up study could include students in non-STEM majors who need to take calculus courses as well to improve the diversity of the sample.

A recent Washington Post article pointed out that nearly 50 accredited colleges and universities dropped SAT/ACT admissions requirement between 2018 and 2019 due to the lack of information that the standardized tests could provide about the students' academic performance (Strauss, 2019). The standardized test scores failed to students' performance in college. This trend is not surprising, as researchers also recognized the limitations of standardized tests, as well as the importance of developing curriculum to better train students transfer ability. As discussed in the literature review of this study, it is necessary to develop a new type of assessment that can measure students' transfer ability, which might better predict students' performance in the future. Therefore, developing a new type of assessment (a.k.a. Preparation for Future Learning assessment) to measure the transfer ability effectively should be more researchers' major area of focus in the domain of learning transfer.

5.2 Conclusion

The goal of this study was to find if ICC strategy was more effective regarding transfer performance than TP strategy; and, if motivation would be shown to be a moderator between the instruction condition and the PFL target problem performance. A total of 81 (ICC: 40, TP: 41) students in STEM majors who need to take a Calculus 1 course to attain their degree were recruited to participate in the study. There were two sessions total to find if there would be any difference in their performance on the immediate post-test and delayed post-test. The results from Bayesian ANCOVA analyses indicated there was moderate evidence that supports that the group condition had a significant effect on the direct application performance and the PFL performance during the first session while controlling for the motivational level and the average pre-requisite knowledge check test score. Participants' motivation was not an effective moderator between the instructional condition and the PFL problem performance. The GLM repeated measure analyses indicated that the effect of iCC instructional strategy remained longer than that of the TP strategy. Therefore, there was a cross-over interaction effect between the instructional condition and time in answering the chain rule definition and the usage, and performance on PFL, and further PFL problems. The implication of this study is that repeating tests might be more effective to improve performance on transfer tasks. In addition, the effect of iCC instructional strategy remained longer than that of the TP strategy even after the learning ended. Finally, the findings of the study might give the evidence that supports the importance of a new type of assessment to measure students' transfer ability.

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A. CALCULUS CURRICULUM IN THE STATE OF INDIANA

CALCULUS	
LIMITS AND CONTINUITY	C.LC.1: Understand the concept of limit and estimate limits from graphs and tables of values.
	C.LC.2: Find limits by substitution.
	C.LC.3: Find limits of sums, differences, products, and quotients.
	C.LC.4: Find limits of rational functions that are undefined at a point.
	C.LC.5: Find limits at infinity.
	C.LC.6: Decide when a limit is infinite and use limits involving infinity to describe asymptotic behavior. Find special limits $\lim_{x \to 0} \frac{\sin x}{x}$
	C.LC.7: Find one-sided limits.
	C.LC.8: Understand continuity in terms of limits.
	C.LC.9: Decide if a function is continuous at a point.
	C.LC.10: Find the types of discontinuities of a function.
	C.LC.11: Understand and use the Intermediate Value Theorem on a function over a closed interval.
	C.LC.12: Understand and apply the Extreme Value Theorem: If f(x) is continuous over a closed interval, then f has a maximum and a minimum on the interval.
DIFFERENTIATION	C.D.1: Understand the concept of derivative geometrically, numerically, and analytically, and interpret the derivative as a rate of change.
	C.D.2: State, understand, and apply the definition of derivative.
	C.D.3: Find the derivatives of functions, including algebraic, trigonometric, logarithmic, and exponential functions.
	C.D.4: Find the derivatives of sums, products, and quotients.
	C.D.5: Find the derivatives of composite functions, using the chain rule.
	C.D.6: Find the derivatives of implicitly-defined functions.
	C.D.7: Find the derivatives of inverse functions.
	C.D.8: Find second derivatives and derivatives of higher order.
	C.D.9: Find derivatives using logarithmic differentiation.
	C.D.10: Understand and apply the relationship between differentiability and continuity.
	C.D.11: Understand and apply the Mean Value Theorem.
APPLICATION OF DERIVATIVES	C.AD.1: Find the slope of a curve at a point, including points at which there are vertical tangents and no tangents.
	C.AD.2: Find a tangent line to a curve at a point and a local linear approximation.
	C.AD.3: Decide where functions are decreasing and increasing. Understand the relationship between the increasing and decreasing behavior of f and the sign of f'.
	C.AD.4: Solve real-world and other mathematical problems finding local and absolute maximum and minimum points with and without technology.
	C.AD.5: Analyze real-world problems modeled by curves, including the notions of monotonicity and concavity with and without technology.
	C.AD.6: Find points of inflection of functions. Understand the relationship between the concavity of f and the sign of f''. Understand points of inflection as places where concavity changes.
	C.AD.7: Use first and second derivatives to help sketch graphs modeling real-world and other mathematical problems with and without technology. Compare the corresponding characteristics of the graphs of f, f', and f''.
	C.AD.8: Use implicit differentiation to find the derivative of an inverse function.
	C.AD.9: Solve optimization real-world problems with and without technology.
	C.AD.10: Find average and instantaneous rates of change. Understand the instantaneous rate of change as the limit of the average rate of change. Interpret a derivative as a rate of change in applications, including distance, velocity, and acceleration.
	C.AD.11: Find the velocity and acceleration of a particle moving in a straight line.
	C.AD.12: Model rates of change, including related rates problems.

INTEGRALS	C.I.1: Use rectangle approximations to find approximate values of integrals.
	C.I.2: Calculate the values of Riemann Sums over equal subdivisions using left, right, and midpoint evaluation points.
	C.I.3: Interpret a definite integral as a limit of Riemann Sums.
	C.I.4: Understand the Fundamental Theorem of Calculus: Interpret a definite integral of the rate of change of a quantity over an interval as the change of the quantity over the interval, that is $\int_a^b f'(x)dx = f(b) - f(a)$
	C.I.5: Use the Fundamental Theorem of Calculus to evaluate definite and indefinite integrals and to represent particular antiderivatives. Perform analytical and graphical analysis of functions so defined.
	C.I.6: Understand and use these properties of definite integrals. $\int_a^b [f(x)+g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$ $\int_a^b k\square f(x)dx = k\int_a^b f(x)dx$ $\int_a^a f(x)dx = 0$ $\int_a^b f(x)dx = -\int_b^a f(x)dx$ $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$ $If\ f(x) \le g(x)\ on\ [a,b],\ then \int_a^b f(x)dx \le \int_a^b g(x)dx$
	C.I.7: Understand and use integration by substitution (or change of variable) to find values of integrals.
	C.I.8: Understand and use Riemann Sums, the Trapezoidal Rule, and technology to approximate definite integrals of functions represented algebraically, geometrically, and by tables of values.
APPLICATIONS OF INTEGRALS	C.AI.1: Find specific antiderivatives using initial conditions, including finding velocity functions from acceleration functions, finding position functions from velocity functions, and applications to motion along a line.
	C.AI.2: Solve separable differential equations and use them in modeling real-world problems with and without technology.
	C.AI.3: Solve differential equations of the form y' = ky as applied to growth and decay problems.
	C.AI.4: Use definite integrals to find the area between a curve and the x-axis, or between two curves.
	C.AI.5: Use definite integrals to find the average value of a function over a closed interval.
	C.AI.6: Use definite integrals to find the volume of a solid with known cross-sectional area.
	C.AI.7: Apply integration to model and solve (with and without technology) real-world problems in physics, biology, economics, etc., using the integral as a rate of change to give accumulated change and using the method of setting up an approximating Riemann Sum and representing its limit as a definite integral.

Figure A.1.: Calculus standards by Indiana Academic Standards, the Department of Education, 2014, updated in 2017. Retrieved from https://www.doe.in.gov/sites/default/files/standards/mathematics/calculus-standards.pdf

(*Indiana Academic Standards*, 2014)

B. LIST OF UNDERGRADUATE STEM MAJORS AT THE CHOSEN UNIVERSITY THAT REQUIRES CALCULUS 1 COURSE TO ATTAIN THE DEGREE

*majors that belong to both or multiple colleges are counted as one major, majors that only first-year students can be in temporarily were also listed.

<**College of Agriculture**>

1. Agricultural Economics
2. Agricultural Systems Management
3. Agronomy
4. Animal Sciences
5. Applied Meteorology and Climatology
6. Aquatic Sciences
7. Biochemistry
8. Biological Engineering
9. Crop Sciences
10. Environmental Studies (only for the First-year students)
11. Farm Management
12. Food Science
13. Forestry
14. Natural Resources and Environmental Science
15. Plant Genetics, Breedings, and Biotechnology
16. Plant Science

17. Plant Studies (only for the First-year students)
18. Pre-Agricultural & Biological Engineering
19. Pre-Veterinary Medicine (only for Doctor of Veterinary Medicine program preparation for undergraduate students)
20. Soil and Water Sciences
21. Sustainable Biomaterials-Process and Product Design
22. Wildlife

<**College of Engineering**>

1. Aeronautical & Astronautical Engineering
2. Agricultural Engineering
3. Biomedical Engineering
4. Chemical Engineering
5. Civil Engineering
6. Computer Engineering
7. Construction Engineering
8. Electrical Engineering
9. First-Year Engineering
10. Environmental and Ecological Engineering
11. Environmental and Natural Resources Engineering
12. Industrial Engineering
13. Interdisciplinary Engineering Studies
14. Materials Engineering
15. Mechanical Engineering
16. Multidisciplinary Engineering

17. Nuclear Engineering

<College of Management>

1. Industrial Management
2. Strategy and Organizational Management
3. General Management
4. Supply Chain, Information and Analytics

<College of Science>

1. Actuarial Science
2. Atmospheric Science/ Meteorology
3. Biology
4. Cell, Molecular, and Developmental Biology
5. Chemistry
6. Computer Science
7. Data Science
8. Ecology, Evolution, and Environmental Sciences
9. Environmental Geoscience
10. Genetics
11. Geology and Geophysics
12. Health and Disease
13. Interdisciplinary Science
14. Mathematics
15. Microbiology
16. Neurobiology and Physiology
17. Physics

18. Planetary Sciences
19. Pre-environmental Studies
20. Science
21. Statistics

<College of Technology>

1. Aeronautical Engineering Technology
2. Aerospace Financial Analysis
3. Airline Management and Operations
4. Airport Management and Operations
5. Animation
6. Audio Engineering Technology
7. Automation and Systems Integration Engineering Technology
8. Aviation Management
9. Building Information Modeling
10. Computer and Information Technology
11. Construction Management Technology
12. Cybersecurity
13. Data Visualization
14. Electrical Engineering Technology
15. Flight (Professional Flight Technology)
16. Game Development and Design
17. Industrial Engineering Technology
18. Mechanical Engineering Technology
19. Robotics Engineering Technology

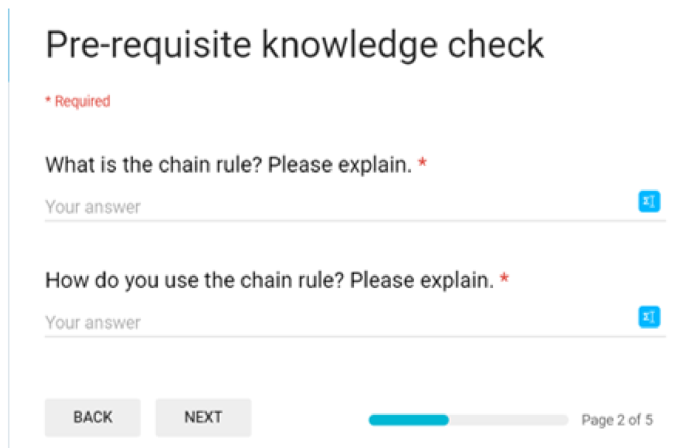
20. Supply Chain Management Technology
21. Systems Analysis and Design
22. Unmanned Aerial Systems
23. Virtual Product Integration
24. Visual Effects Compositing
25. Web Programming and Design

C. MATH ITEMS

*Items are all open-ended.

<Pre-Requisite Check Test Items>

1. What is the chain rule? Please explain.
2. How do you use the chain rule? Please explain.

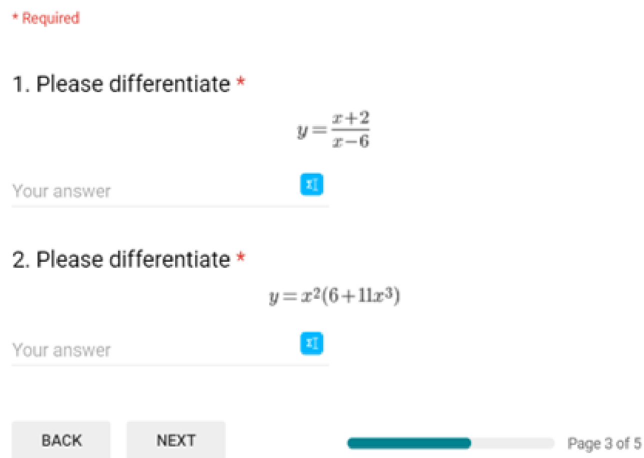


3. (Q1) Please differentiate:

$$y = \frac{x+2}{x-6}$$

4. (Q2) Please differentiate:

$$y = x^2(6 + 11x^3)$$



* Required

1. Please differentiate *

$$y=\frac{x+2}{x-6}$$

Your answer

2. Please differentiate *

$$y=x^2(6+11x^3)$$

Your answer

BACK NEXT Page 3 of 5

5. (Q3) Please solve x when $x \in [0, 2\pi)$:

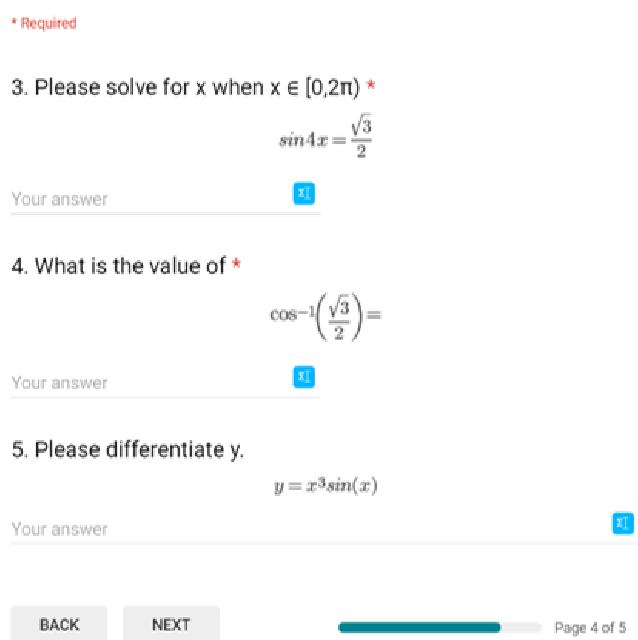
$$sin4x = \frac{\sqrt{3}}{2}$$

6. (Q4) What is the value of

$$cos^{-1}\left(\frac{\sqrt{3}}{2}\right) =$$

7. (Q5) Please differentiate y.

$$y = x^3 sin(x)$$



* Required

3. Please solve for x when x ∈ [0,2π) *

$$sin4x=\frac{\sqrt{3}}{2}$$

Your answer

4. What is the value of *

$$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)=$$

Your answer

5. Please differentiate y.

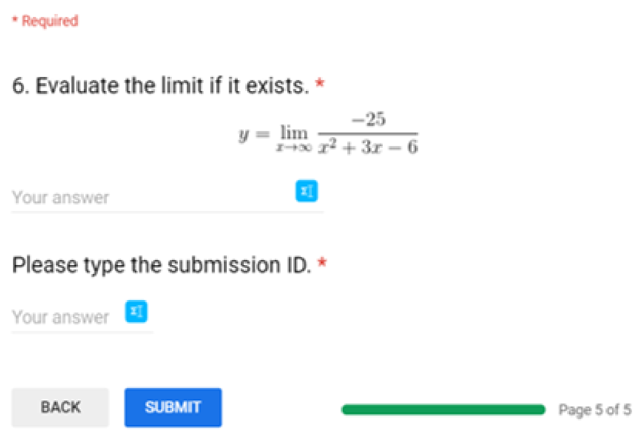
$$y=x^3sin(x)$$

Your answer

BACK NEXT Page 4 of 5

8. (Q6) Evaluate the limit if it exists.

$$y = \lim_{x \to \infty} \frac{-25}{x^2 + 3x - 6}$$



<iCC and TP Task Items>

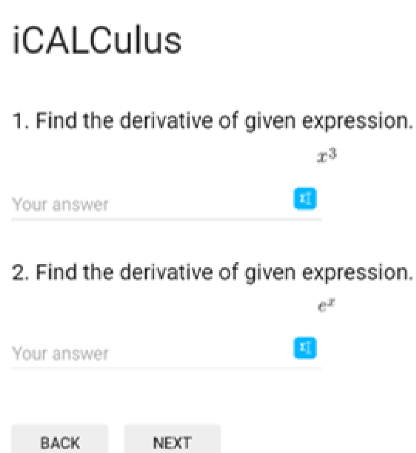
<u>iCC Task Items</u>

1. Find the derivative of given expression.

$$e^x$$

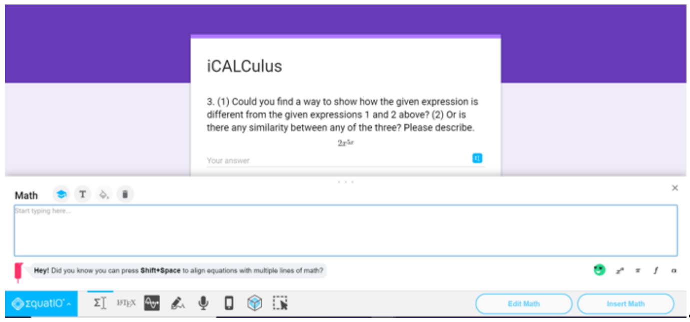
2. Find the derivative of given expression.

$$2x^{5x}$$



3. Could you find a way to show how the given expression is different from the given expressions 1 and 2 above? Or is there any similarity between any of the three? Please describe.

$$2x^{5x}$$



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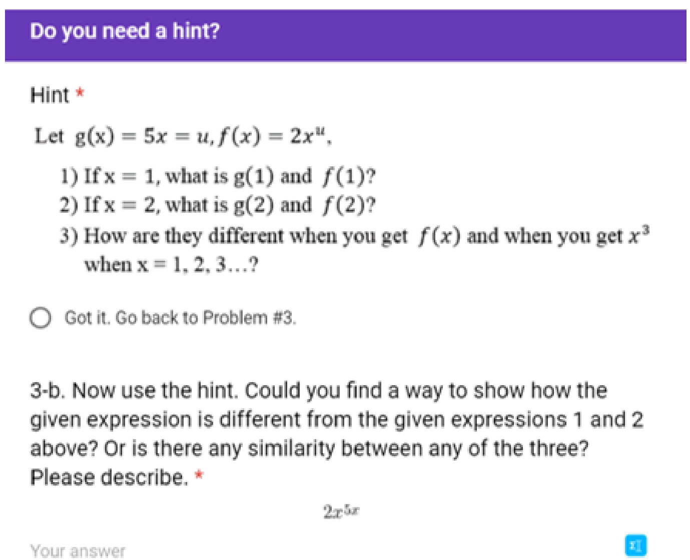
4. (3-b) Now use hint 1. Could you find a way to show how the given expression is different from the given expressions 1 and 2 above? Or is there any similarity between any of the three? Please describe.



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TP Task Item

Find a derivative of y.

$$y = 4x^{6x}$$

Worked_Example

* Required

Example

*

Find a derivative of y.

$y = 2x^{5x}$

Sol)

$$\frac{dy}{dx} = \frac{d}{dx}2x^{5x} = \frac{d}{dx}2x^{5x\ln(x)} = 2\frac{d}{dx}e^{5x\ln(x)}$$

Outer function is $y = f(u) = e^u$, Inner function $u = g(x) = 5x\ln(x)$

Using the chain rule,

$$\frac{dy}{dx} = 2\frac{d}{du}(e^u)\frac{d}{dx}(5x\ln(x))$$

1. Differentiate the outer function, $\frac{d}{du}(e^u) = e^u$

2. Differentiate the inner function, $\frac{d}{dx}(5x\ln(x)) = 5(\ln(x) + 1)$

$$\therefore \frac{dy}{dx} = 2e^{5x\ln(x)} \cdot 5(\ln(x) + 1) = 10x^{5x}(\ln(x) + 1)$$

○ Got it.

<Chain Rule Lesson Content>

What is the chain rule?

The Chain Rule

If g is differentiable at x and f is differentiable at g(x), then the composite function $F = f \circ g$ defined by $F(x) = f(g(x))$ is differentiable at x and F' is given by the product

$$F'(x) = f'(g(x)) \cdot g'(x)$$

In Leibniz notation, if $y = f(u)$ and $u = g(x)$ are both differentiable functions, then

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

Stewart, J. (2011). Calculus: Early Transcedentals. Cengage Learning.

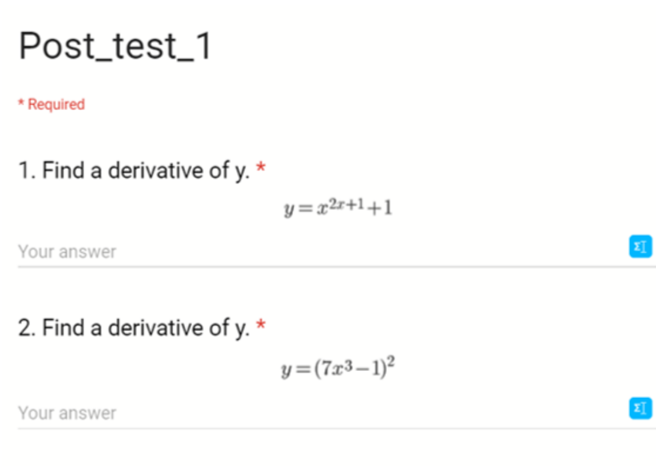
<Post-Test Items>

1. Find a derivative of y.

$$y = x^{2x+1} + 1$$

2. Find a derivative of y.

$$y = (7x^3 - 1)^2$$



Post_test_1

* Required

1. Find a derivative of y. *

$$y=x^{2x+1}+1$$

Your answer

2. Find a derivative of y. *

$$y=(7x^3-1)^2$$

Your answer

.

3. Find a derivative of y.

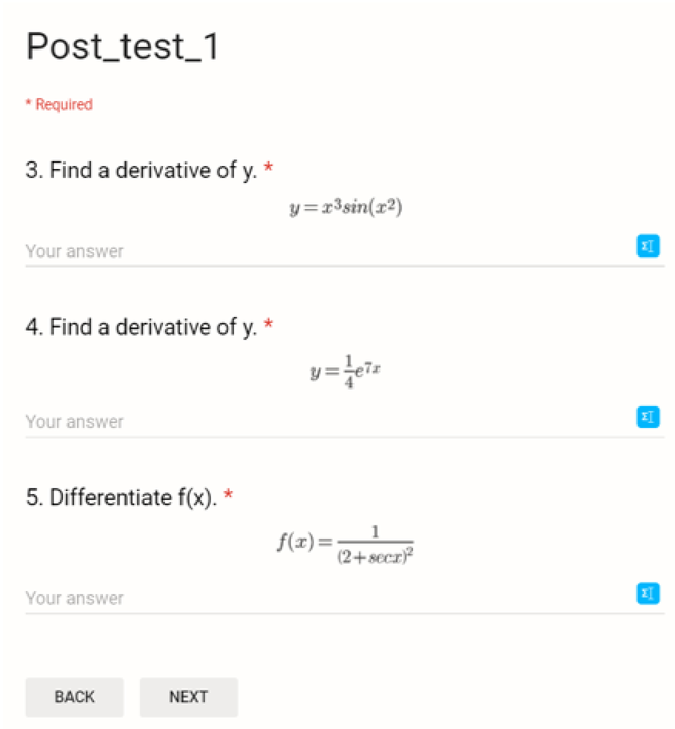
$$y = x^3 sin(x^2)$$

4. Find a derivative of y.

$$y = \frac{1}{4}e^{7x}$$

5. Differentiate *f(x)*.

$$f(x) = \frac{1}{(2 + sec(x))^2}$$



Post_test_1

* Required

3. Find a derivative of y. *

$$y = x^3 sin(x^2)$$

Your answer

4. Find a derivative of y. *

$$y = \frac{1}{4} e^{7x}$$

Your answer

5. Differentiate f(x). *

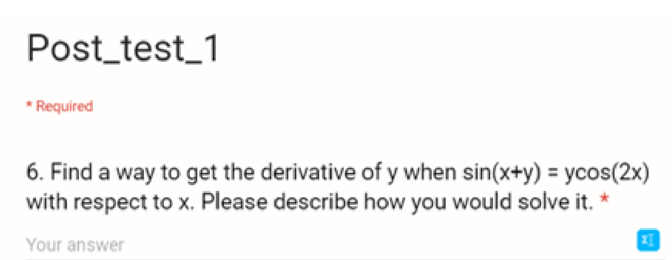
$$f(x) = \frac{1}{(2 + secx)^2}$$

Your answer

BACK NEXT

.

6. Find a way to get the derivative of y when $sin(x + y) = ycos(2x)$ with respect to x. Please describe how you would solve it.



Post_test_1

* Required

6. Find a way to get the derivative of y when sin(x+y) = ycos(2x) with respect to x. Please describe how you would solve it. *

Your answer

.

<Delayed Post-Test Items>

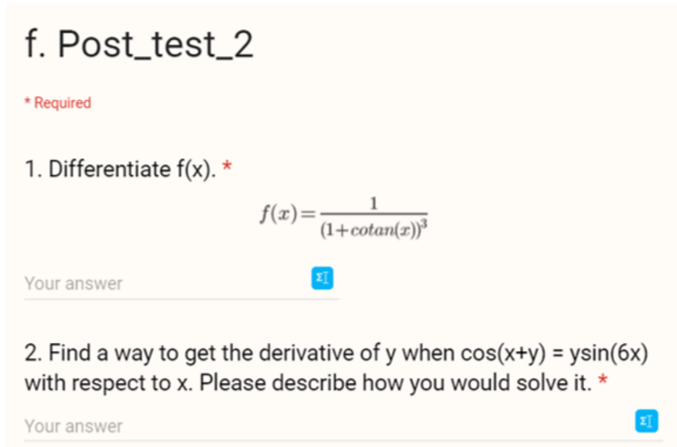
1. What is the chain rule? Please explain.

2. How do you use the chain rule? Please explain.

3. (Q1) Differentiate $f(x)$.

$$f(x) = \frac{1}{(1 + cot(x))^3}$$

4. (Q2) Find a way to get the derivative of y when $cos(x + y) = ysin(6x)$ with respect to **x**. Please describe how you would solve it.



f. Post_test_2

* Required

1. Differentiate f(x). *

$$f(x) = \frac{1}{(1+cotan(x))^3}$$

Your answer

2. Find a way to get the derivative of y when cos(x+y) = ysin(6x) with respect to x. Please describe how you would solve it. *

Your answer

.

D. CODEBOOK

Question	***(complete miss)***	***(near miss)***	***(correct)***
What is the chain rule? Key words: *differentiation for composite function *a compound function that consists of inner function and outer function ***participants often are confused with the chain rule and the product rule or differentiation by parts. Please be aware.**	- No response - wrong definition (i.e., mixed up with another mathematical rule, i.e., integration by parts) - very vague description and does not capture the specific characteristics of the chain rule at all (i.e., it is used for specific calculation) -if mentions the multiple functions **multiplied together**, which is related to the product rule, NOT the chain rule.	- give a correct example but not fully describe when to rule is used - mention that the function is composite (or compound) but does not mention inner function (or interior function) and outer function) or exterior function) specifically. -If a participant **only** mentions "complicated function" or "first part... the second part of the function", it is not the perfectly correct answer.	- describes when to use the rule correctly - give a correct example - mention how the compound function is different from simple function regarding differentiation (infer the "contrast"). *the description should fulfill at least **two** of these
How do you use the chain rule? Key words: *Derivative of inner function and then outer function *u substitution ***participants often are confused with the chain rule and the product rule or differentiation by parts. Please be aware.**	-No response -wrong example and wrong description	-give a correct example but does not specifically mention the compound function which consists of inner function and outer function. -give a correct description, but give a wrong example (not the composite function)	- give a correct example - mention how the compound function is different from simple function regarding differentiation (infer the "contrast"). -If a participant only wrote a formula such as $\frac{dy}{dx}f(g) = f'(g) \cdot g'$, it is a correct answer.
Q1 (Answer: $-\frac{8}{(x-6)^2}$)	- No response - completely wrong answer	-almost close to the answer but did not put the negative sign -if a participant put 4 instead of 8 as a numerator -if a participant did not simplify the numerator part, i.e. (x-6)-(x+2), it is not a perfectly correct answer. - if a participant did not use the fraction sign but used ÷ sign, it is not a perfectly correct answer.	-correct answer but just different form

Q2 (Answer: 12x + $55x^4$)	- No response - completely wrong answer	- one of the terms is correct but not the other one due to some calculation error, i.e., $36x^5$+$55x^4$ or $55x^4 + 6x$ -didn't simplify fully, i.e., $2x(6+11x^3)+33x^4$	- correct answer but the different form
Q3 (Answer: $\frac{\pi}{12}+\frac{\pi n}{2}, \frac{\pi}{6}+\frac{\pi n}{2}, n =$ 0,1,2,3 or $\frac{\pi}{12}, \frac{\pi}{6}, \frac{7\pi}{12}, \frac{4\pi}{6}, \frac{13\pi}{12}, \frac{7\pi}{6}, \frac{19\pi}{12}, \frac{10\pi}{6}$) *Participants should know that the period of sine functions are different when sinx and sin 4x, therefore, the number of answers are 8 when x ∈ $[0, 2\pi)$. Should understand period and amplitude of trigonometric function.	- No response - none of the answers is correct	- only gave one or two of correct answer(s), i.e., $\frac{\pi}{12}, \frac{\pi}{6}$ -gave more than one answers and only one of them is correct	- gave 8 correct answers or the correct answer with the correct number of n - gave the majority (> 50% of 8 correct answers) of correct answers
Q4 (Answer: $\frac{\pi}{6}$) *In order for arccos(y) to have values, the x ∈ [0, π] and the participants should know this.	- No response - completely wrong answer, i.e., wrong form and wrong value	- if a participant put 30 but did not put ° - if a participant put two answers and only one is correct -if a participant put two answers and one of them is wrong, it is near miss. -If a participant put an answer such as $\frac{\pi}{6}$+2πx (x=1, 2, 3, ….)	- correct answer but the different form, i.e., 30°
Q5 (Answer: $3x^2 \sin(x) + x^3 \cos(x)$)	-No response -does not know how to use the product rule to differentiate, i.e., x^3 *sinx + sinx * 3x, then it is a completely wrong answer.	-almost correct but put negative sign for $x^3 \cos(x)$, i.e. $3x^2 \sin(x) - x^3 \cos(x)$)	-correct answer
Q6 (Answer: 0) *participants should notice that it is ∞, meaning they should think both sides to get the answer	-No response -completely wrong answer	-, i.e., converges -if a participant put two answers considering when +∞ and −∞ and one is correct (limit = 0) but not the other one	-correct answer

Figure D.1.: Codes for the pre-requisite knowledge check test items

- Rough paper usage: For TP 01, he put the the answer for Q1 in the rough paper only. Some participants did not use EquatI0 to type the answer, and in that case, check the rough paper in case the participant wrote the answer it down just to understand the answer form more accurately.
- For pre-requisite knowledge check test Q1-6, the mastery level of the pre-calc is the key information that I am looking for, so the grading would be stricter.
- For the first chain rule open-ended questions, please see the guide.

Question	***(incomplete/complete miss)***		***(full completion/correct)***
Q1	- No response		- Correct answer
Q2	- No response		- Correct answer
Question	***(incomplete/complete miss)***	***(near completion)***	***(full completion)***
Q3	- No response - "I don't know." - "unsure" -mentioned the only similarity for all three expressions, i.e., the only similarity is that all are functions -wrote something but very vague, i.e., there is a similarity.	- mention how each expression is different but vaguely, i.e., "There is a similarity between the two but the way they will be derived will still be different and will be derived in two parts."	- does mention chain rule and use U substitution -does mention inner/outer function - does not mention 'inner function' and 'outer function' specifically, but still describe that the outer function part changes as the inner function part changes. - if a participant could differentiate between simple function (Q1) and composite function (Q2, 3) and mentioned that these three are similar later, it is still a correct answer
Q3-b Keywords: *a compound function consists of inner function and outer function	- No response - "I don't know." - "unsure" -mentioned similarity for all three expressions, i.e., only similarity is that all are functions -wrote something but does not describe anything, i.e., there is a similarity. -wrote that there is a difference and give some description but completely wrong, i.e., "If x = 1, and then x = 2, then the answer is different such that the first answer is: 2*1^5 and the second is 2*1^50. So, the difference in the exponents is a multiple of 10 since 5 * 10 = 50"	- mention how each expression is different but vaguely, i.e., "There is a similarity between the two but the way they will be derived will still be different and will be derived in two parts."	- does mention chain rule and use U substitution - does mention differentiating a function that consists of inner/outer function - does not mention 'inner function' and 'outer function' specifically, but still describe that the outer function part changes as the inner function part changes. - if a participant could differentiate between simple function (Q1) and compound function (Q2, 3) and mentioned that these three are similar later, it is still a correct answer -if a participant's answer did not develop for Q3-b since participants think that they answered Q3 correctly already, **AND the answer for Q3 is correct, categorize it as "full completion."**

Figure D.2.: Codes for iCC task items

Question	***(incomplete/complete miss)***	***(not there yet)***	***(near miss)***	***(correct)***
Worked example (Answer: $24x^{6x}(\ln(x)+1)$	-no response -completely wrong	-the participant knows how to do it roughly but couldn't finish the calculation.	- simple calculation error, i.e. $20x^{6x}(\ln(x)+1)$	-correct answer -if an answer is just a different form, it is a correct answer

Figure D.3.: Codes for TP task item

<table>
<tr><th>Question</th><th>(complete miss)</th><th>(not there yet)</th><th>(near miss)</th><th>(correct)</th></tr>
<tr><td>Q1
(Answer: $x^{2x+1}(\frac{2x+1}{x}+$
$2\ln(x))$)</td><td>-No response
-completely wrong answer i.e. $2x^{2x+1}$ or $(2x+1)\,x^{2x}+1$</td><td>-did use ln and/or e which infers that the participant knows the chain rule to differentiate but did not quite know how to find a derivative fully, i.e., $(2x+1)\cdot x^{2x+1}(\ln(x)+1)$ or $2x\left(\frac{d}{dy}\right)e^{2x}$
-did use ln and the chain rule but did not remove y away at the last stage properly, i.e., y(2ln(x) + 2 + (1/x))</td><td>-very similar to the correct answer but put one of the terms which is wrong, i.e., $x^{(2x+1)\cdot\ln(x)}\left(2+\frac{1}{x}+2\ln(x)\right)$</td><td>-correct answer
-correct answer but just a different form</td></tr>
<tr><th>Question</th><th>(complete miss)</th><th colspan="2">(near miss)</th><th>(correct)</th></tr>
<tr><td>Q2 (Answer: $294x^5-42x^2$)</td><td>-No response
-used the chain rule incorrectly</td><td colspan="2">-very similar to the correct answer but simple calculation error, i.e. 294 * x ^ 5 - 52 * x ^ 2
-did not fully simplify, i.e. $(49\cdot 6)\,x^5-\left(14\cdot 3x^2\right)$</td><td>-correct answer
-correct answer but just a different form, i.e. $42x^2\left(7x^3-1\right)$, or $42\times x^2\times\left(7x^3-1\right)$, or $21x^2(14x^3-2)$
-if the answer is simplified as the multiplication of polynomials, it is correct.</td></tr>
<tr><td>Q3 (Answer: $3x^2\sin(x^2)+2x^4\cos(x^2)$)</td><td>-No response
-completely wrong</td><td colspan="2">-simple calculation error
-did not fully simplify</td><td>-correct answer
-correct answer but just a different form</td></tr>
<tr><td></td><td>-just did simple differentiation incorrectly, i.e., $\cos\left(x^2\right)\cdot 3x^2$</td><td colspan="2">-the terms of the answer are the same form, and one of the terms is wrong due to simple calculation error, i.e. $3x^2\sin(x^2)+2x^5\cos(x^2)$</td><td></td></tr>
<tr><th>Question</th><th>(complete miss)</th><th>(Not there yet)</th><th>(near miss)</th><th>(correct)</th></tr>
<tr><td>Q4 (Answer: $\frac{7}{4}e^{7x}$)</td><td>-No response
-just used the simple differentiation to get the answer, i.e. (1/4)*7x*e^7x + 6x</td><td>-did use the ln, which infers that the participant understands that the expression is the composite function but did not know how to differentiate fully, i.e. $\left(\frac{\ln 7x}{28}\right)$
-did not find the derivative at the last stage of calculation correctly, i.e., $\frac{7}{4}\ln(7x)$, or $\frac{x}{4}e^{7x}$</td><td>-Simple calculation error, i.e., $\frac{1}{28}e^{7x}$</td><td>-correct answer</td></tr>
</table>

Q5 (Answer: $-\frac{2\sec(x)\tan(x)}{(2+\sec(x))^3}$)	-No response -completely wrong answer, including that the participant does not know that the derivative of sec(x) is sec(x)tan(x), i.e., $2(2+\sec x)^{-3}\cdot(\cos x)^{-2}$ -if a participant knows that the derivative of sec(x) is sec(x)tan(x) but cannot differentiate at all, it is a complete miss, i.e., $-2\cdot(2+\sec x)\cdot\sec x\cdot\tan x$	-1)does understand that the given function is a composite function and 2)did attempt to use the chain rule and 3)does know that the derivative of sec(*x*) is sec(*x*)tan(*x*) **BUT** did not fully know how to find the derivative correctly, i.e. $\frac{-2(2+\sec(x))(\sec(x)\tan(x))}{(2+\sec(x))^4}$ -if the answer has the similar form of the correct answer but two parts are wrong -if an answer is the same except that the participant could not get the derivative of the sec(x)	-simple calculation error -very similar to the correct answer but one of the terms is wrong, i.e. $-\frac{2\sec(x)\tan(x)}{(2+\sec(x))^4}$ -did not put the negative sign in front -if the form is very similar to the correct answer and the correct derivative of sec(x), but incorrect with one of the powers **AND** the negative sign	-correct answer -correct answer but just a different form
Q6 (Answer is $\frac{-2y\sin(2x)-\cos(x+y)}{\cos(x+y)-\cos(2x)}$, but focus on the participants' **explanation**) Keywords/phrases: *chain rule (u substitution) *$\frac{dy}{dx}$ = y' correctly *difference between the functions that consist of x variable only vs. the functions that consist of x and y variables *differentiating y implicitly with respect to x	-no response -"I don't know." - describe vaguely and does not infer any implicit differentiation or the chain rule **AND** do not give any answer. -describe vaguely **AND** the answer is completely wrong -describing differentiating trigonometric function only	- mention "implicit differentiation" only -mention "chain rule" only -if mentioning substitution but did not develop the description further to applying the chain rule, i.e., "I think doing this would involve moving the cos(2x) over to the left side by dividing and beginning to derive from there. However, because there is a y in the sine function, I think some form of substitution would have to be done in order to move forward. After this substitution though, the quotient rule could be used to solve for the derivative of y." -mention the chain rule and the substitution, but does not describe correctly, i.e. sinx + siny = ycos2x, let u=2x, sinx+siny=ycosu, try to solve it by the chain rule.	-if 1) the description explains how to differentiate (i.e., using $\frac{dy}{dx}$) inferring the chain rule or implicit differentiation but not fully accurate, **AND** 2) the derivative is close to the correct one i.e., "You must take the derivative with respect to x of both sides. This lets you find $\frac{dy}{dx}$ and understanding that $\frac{dy}{dx}$ = 1. You then solve the equation for $\frac{dy}{dx}$, which gives you: $\frac{dy}{dx}=\frac{-2y\sin(2x)-\cos(x+y)}{1-\cos(2x)}$."	-mention that the chain rule should be used for the function and there are y and y' so you would have to move y to the other side -mention the chain rule and describe it correctly -treat y as the implicit function of x -just put the answer only **AND** the answer is correct -just put "implicit differentiation" and put the answer **AND** the answer is correct -if the participant put the description and the answer, and the description is correct **BUT** the answer is close to the correct answer, i.e., $\frac{-2y\sin(2x)-\cos(x+y)}{1-\cos(2x)}$, then categorize it to "correct."

Figure D.4.: Codes for the post-test items

Question	Complete miss	Not there yet	Near miss	correct
Q1 (Answer: $\frac{3csc^2(x)}{(\cot(x)+1)^4}$)	-No response -completely wrong answer, including that the participant does not know that the derivative of cot(x) is -$csc^2(x)$	-1)does understand that the given function is a composite function and 2)did attempt to use the chain rule and 3)does know that the derivative of cot(x) is -$csc^2(x)$ **BUT** did not know fully how to find the derivative correctly -if the answer has the similar form of the correct answer but two parts are wrong -if an answer is the same except that the participant could not get the derivative of the cot(x)	-simple calculation error - very similar to the correct answer but one of the terms is wrong -if a participant put 3 in the denominator -if the form is very similar to the correct answer and the correct derivative of cot(x), but incorrect with one of the powers **AND** the negative sign	-correct answer -correct answer but just a different form
Q2 (*Answer:$-\frac{6y\cos(6x)+\sin(x+y)}{\sin(6x)+\sin(x+y)}$, but focus on the participants' explanation) Keywords: *chain rule solving with respect to y (does not have to specifically mention implicit differentiation)	-No response -"I don't know." -mentioned something, but vague and the process described is wrong -just mention trigonometric differentiation	-can include the process that the participant described to solve the derivative of the question	if 1) the description explains how to differentiate (i.e., using $\frac{dy}{dx}$) inferring the chain rule or implicit differentiation but not fully accurate, **AND** 2) the derivative is close to the correct one	-explicit description of implicit differentiation and the chain rule -explicit description and the answer which is close to the correct answer -no description but the answer is close to the answer, i.e. only one of the terms is incorrect -vague description and the answer are close to the answer -if the description is correct and the derivative is close to the correct answer.

Figure D.5.: Codes for the delayed post-test items

Definition level (more focused)	Example level	Code type
1. No definition	No example	COMPLETE MISS
	Completely wrong	COMPLETE MISS
	accurate	CORRECT
2. Completely wrong definition -mentioning integration by parts	No example	COMPLETE MISS
	Completely wrong	COMPLETE MISS
	accurate	NEAR MISS (not much likely to happen, so please check if the definition is completely wrong or if the example is accurate to make sure)
3. Vague definition i.e., -just wrote that it is used for complicated function, did not use any keywords (refer to the code table on p. 1)	No example	NEAR MISS
	Completely wrong	COMPLETE MISS
	accurate	CORRECT
4. Correct definition	No example	CORRECT
	Completely wrong	NEAR MISS (not much likely to happen, so please check if the definition is correct to make sure)
	accurate	CORRECT

Figure D.6.: Guide for "What is the chain rule?"

Description level (more focused)	Example level	Code type
1. No description i.e. it is for special calculation, I don't know	No example	COMPLETE MISS
	Completely wrong	COMPLETE MISS
	accurate	CORRECT
2. Completely wrong description i.e. mentioning simple differentiation	No answer	COMPLETE MISS
	Completely wrong	COMPLETE MISS
	accurate	NEAR MISS (not much likely to happen, so please check if the description is completely wrong or if the example is accurate to make sure)
3. Vague description i.e. can track the process and can include in the calculation process when you (coder) solve the composite function using the chain rule but does not use the key words (please refer to the code table on p. 1)	No example	NEAR MISS
	Completely wrong	COMPLETE MISS
	accurate	NEAR MISS
4. Correct description	No example	CORRECT
	Completely wrong	NEAR MISS (not much likely to happen, so please check if the description is correct to make sure)
	accurate	CORRECT

Figure D.7.: Guide for “How do you use the chain rule?”

Description level	Answer level	Code type
5. No description	No derivative	COMPLETE MISS
	Completely wrong derivative	COMPLETE MISS
	Close to the correct derivative (simple calculation error or one of the terms is incorrect)	NEAR MISS
	Correct derivative	CORRECT
6. Completely wrong description i.e., -just mention differentiating trigonometric differentiation -just mention simple differentiation only	No derivative	COMPLETE MISS
	Completely wrong derivative	COMPLETE MISS
	Close to the correct derivative (simple calculation error or one of the terms is incorrect)	NOT THERE YET (The likelihood of this category is low, therefore, re-check if the description IS actually completely wrong to make sure)
	Correct derivative	NOT THERE YET (The likelihood of this category is low, therefore, re-check if the description IS actually completely wrong to make sure)
7. Vague description i.e., -NOT describing the chain rule, u substitution, implicit differentiation specifically - just wrote "chain rule" or "implicit differentiation" only -if you can still include the process that the participant described when you (coder) solve the question even though the participant did not mention the chain rule or implicit differentiation explicitly	No derivative/ barely close to the correct derivative	NOT THERE YET
	Completely wrong derivative	COMPLETE MISS
	Close to the correct derivative (simple calculation error or one of the terms is incorrect)	NEAR MISS
	Correct derivative	CORRECT
8. A clear description of implicit differentiation inferring the chain rule	No derivative	CORRECT
	Completely wrong derivative	NOT THERE YET (The likelihood of this category is low, therefore, re-check if the description IS actually clear to make sure)
	Close to the correct derivative (simple calculation error or one of the terms is incorrect)	CORRECT
	Correct derivative	CORRECT

Figure D.8.: Guide for Post-Test Q6 and the Delayed Post-Test Q2

E. PLOTS FROM ITEM RESPONSE MODELS FOR THE ITEMS

*Dashed lines are the information curves and the colored lines are the trace lines of each category.

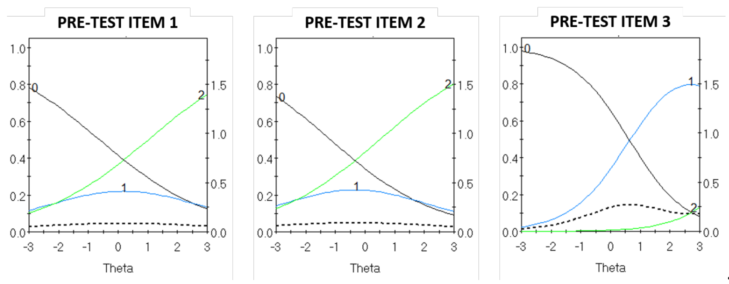


Figure E.1.: Plots of items with trace lines and information curves of the pre-requisite knowledge check test item 1, 2, 3

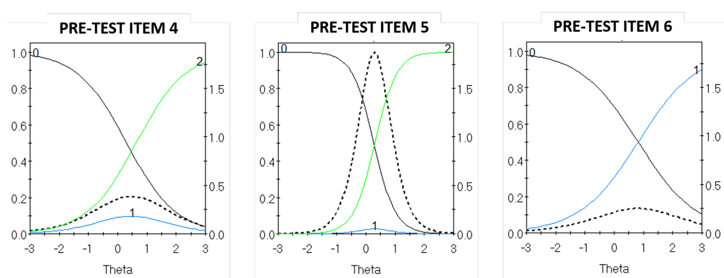


Figure E.2.: Plots of items with trace lines and information curves of the pre-requisite knowledge check test item 4, 5, 6

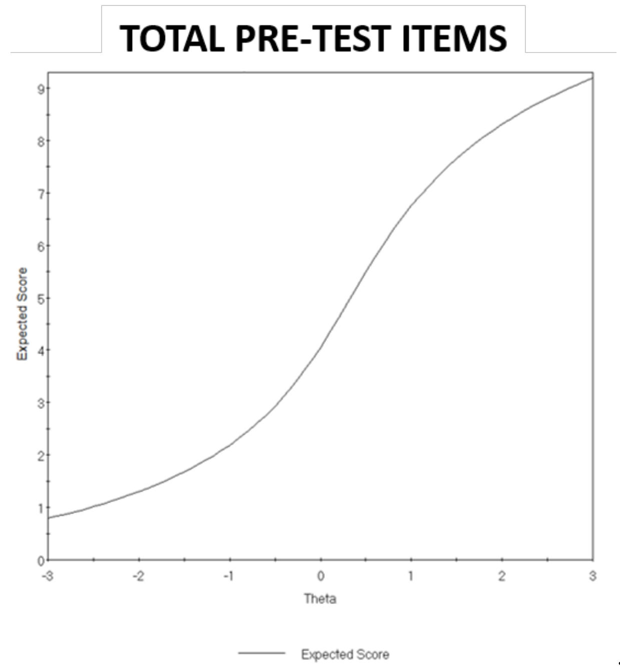


Figure E.3.: Test characteristic curve for pre-requisite knowledge check test items

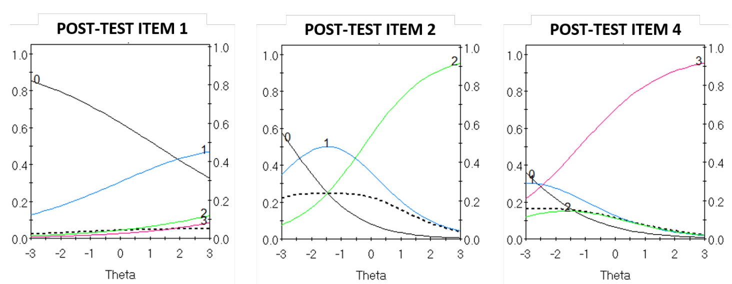


Figure E.4.: Plots of items with trace lines and information curves of the post-test item 1, 2, 4

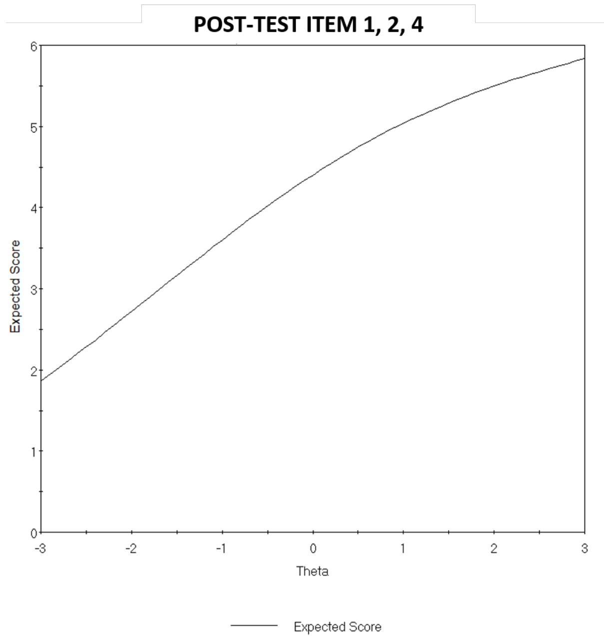


Figure E.5.: Test characteristic curve for post-test item 1, 2, 4

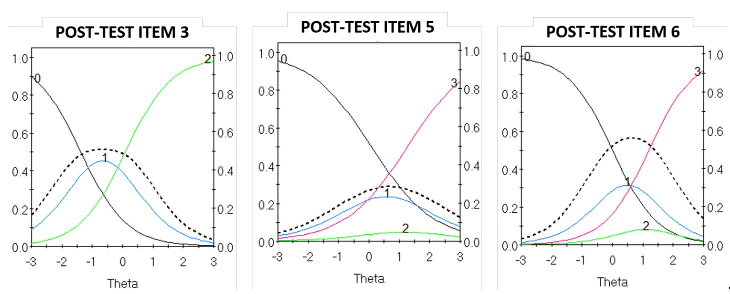


Figure E.6.: Plots of items with trace lines and information curves of the post-test item 3, 5, 6

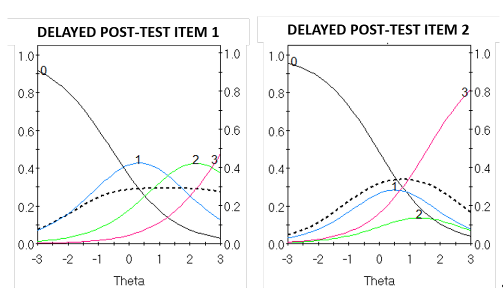


Figure E.7.: Plots of items with trace lines and information curves of the delayed post-test item 1, 2

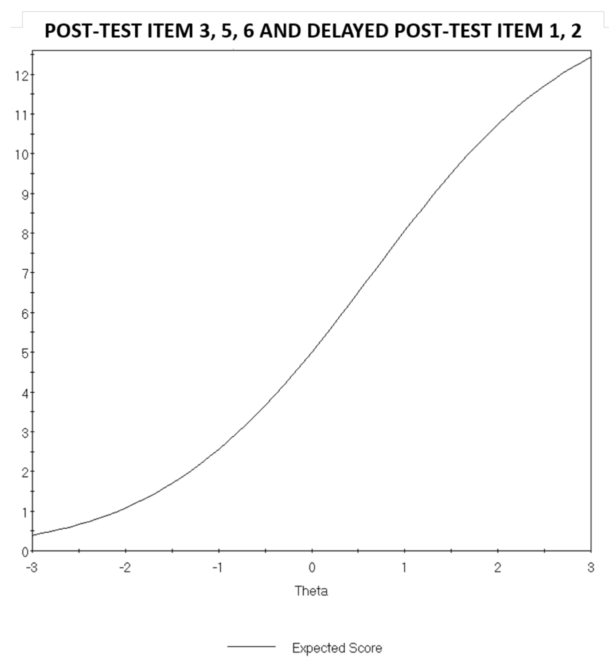


Figure E.8.: Test characteristic curve for post-test 3,5,6 and delayed post-test 1, 2

F. PLOTS OF LOG LIKELIHOOD, PRIOR DISTRIBUTION, AND POSTERIOR DISTRIBUTION OF MEAN DIFFERENCE FROM BAYESIAN INDEPENDENT SAMPLE T-TEST ANALYSES

*1: iCC group, 0: TP group

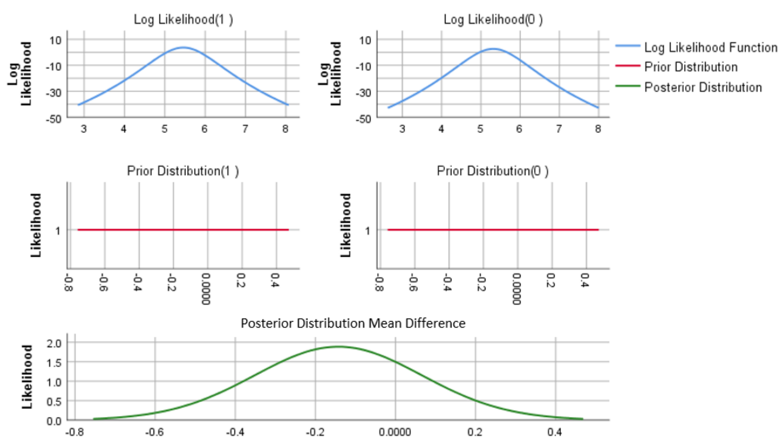


Figure F.1.: Motivation survey score

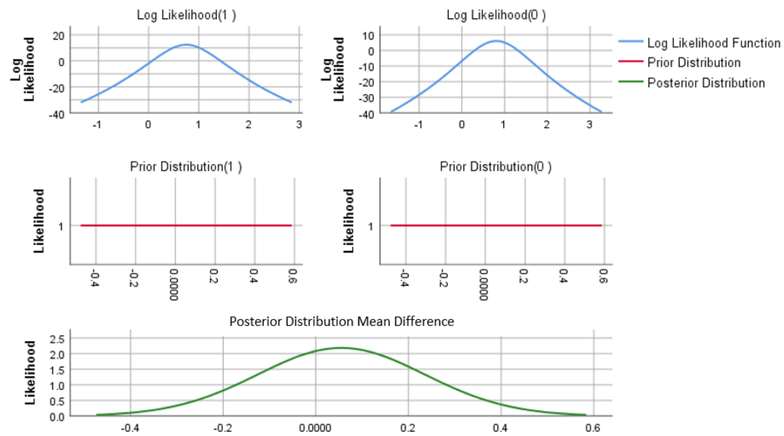


Figure F.2.: Pre-Requisite Knowledge Check Test Score for “What is the chain rule?”

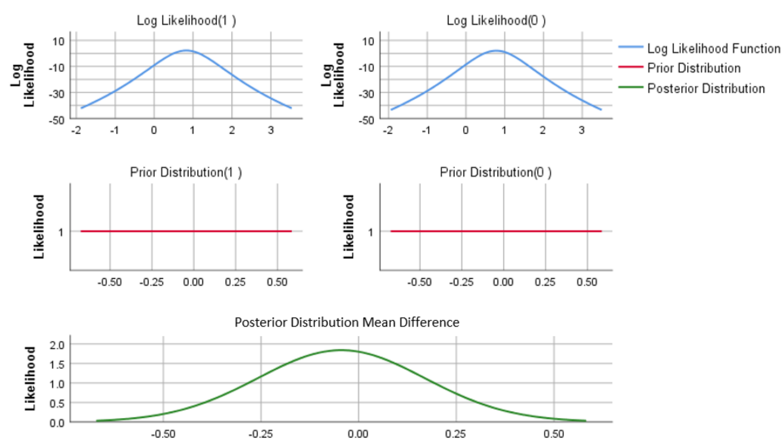


Figure F.3.: Pre-Requisite Knowledge Check Test Score for “How do you use the chain rule?”

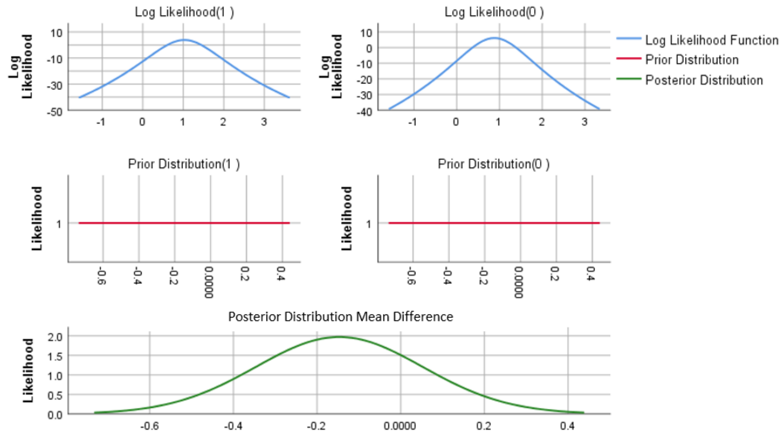


Figure F.4.: Pre-Requisite Knowledge Check Test Score for Q. 1

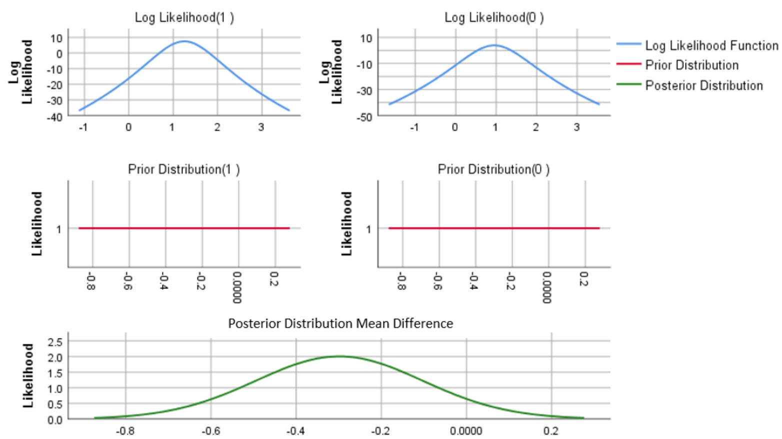


Figure F.5.: Pre-Requisite Knowledge Check Test Score for Q. 2

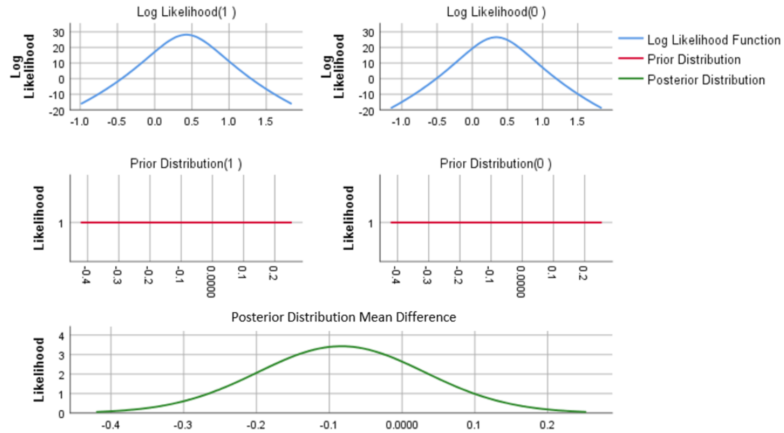


Figure F.6.: Pre-Requisite Knowledge Check Test Score for Q. 3

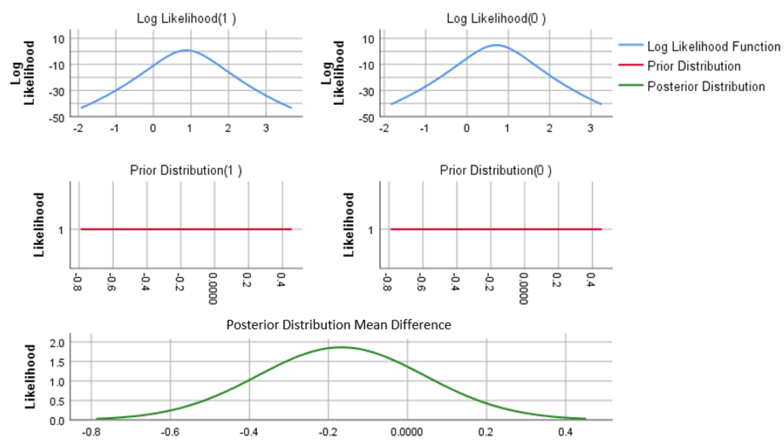


Figure F.7.: Pre-Requisite Knowledge Check Test Score for Q. 4

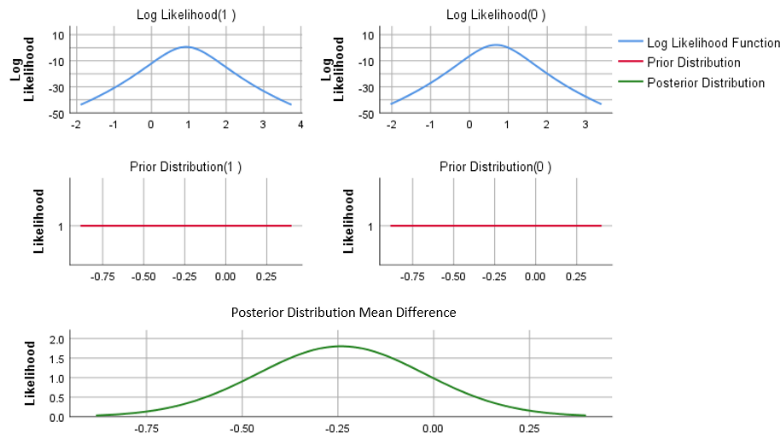


Figure F.8.: Pre-Requisite Knowledge Check Test Score for Q. 5

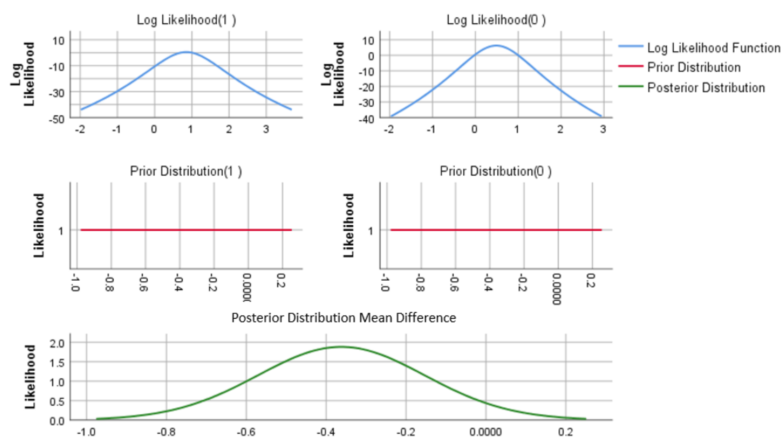


Figure F.9.: Pre-Requisite Knowledge Check Test Score for Q. 6

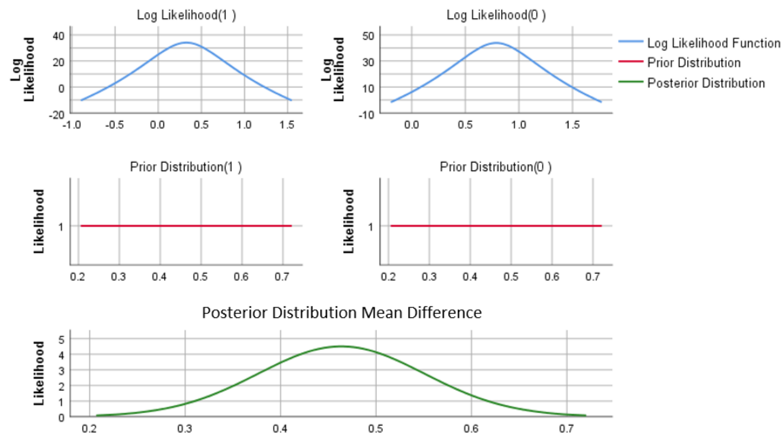


Figure F.10.: Task accuracy for iCC task Q. 3 vs. TP task

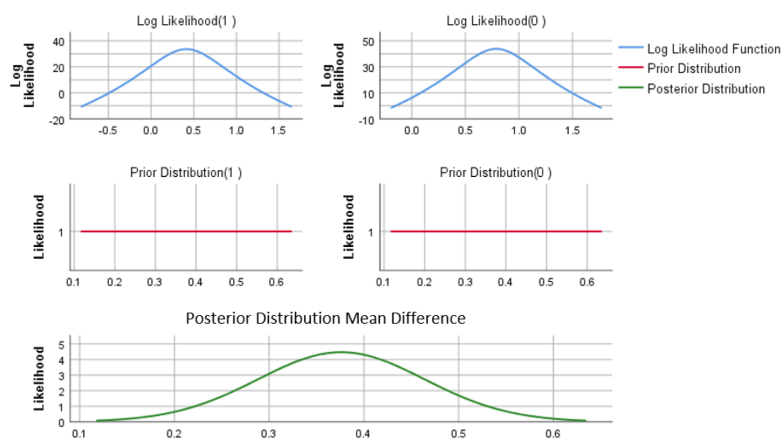


Figure F.11.: Task accuracy for iCC task Q. 3-b vs. TP task

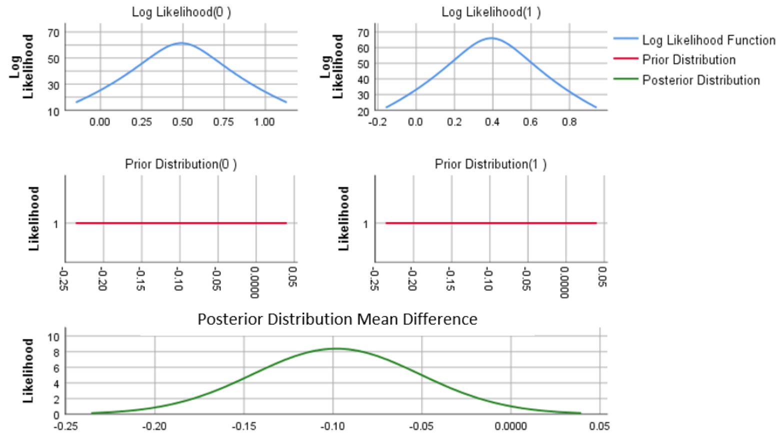


Figure F.12.: Direct application

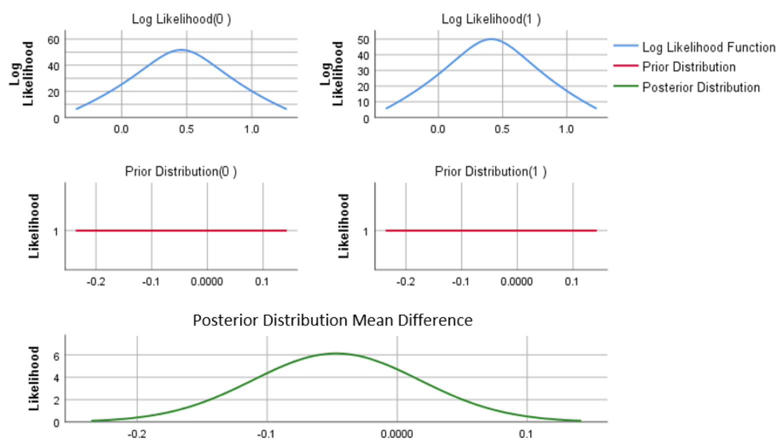


Figure F.13.: PFL Performance from the post-test

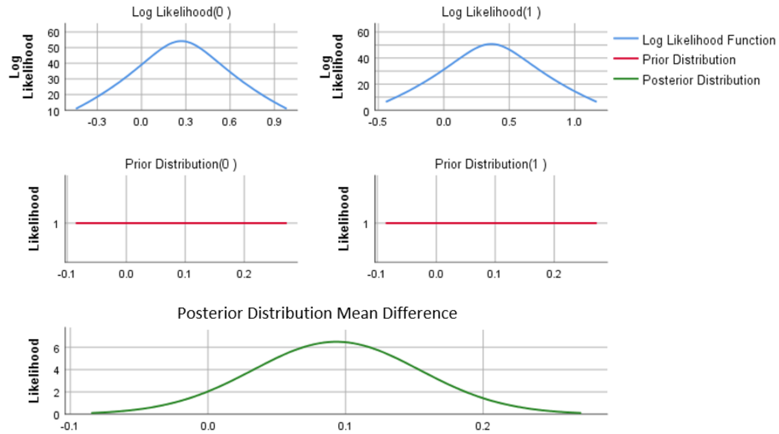


Figure F.14.: PFL performance from the delayed post-test

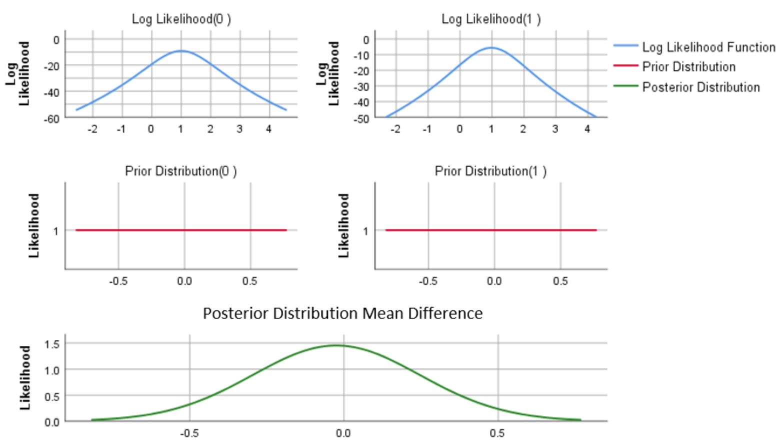


Figure F.15.: Further PFL performance from the post-test

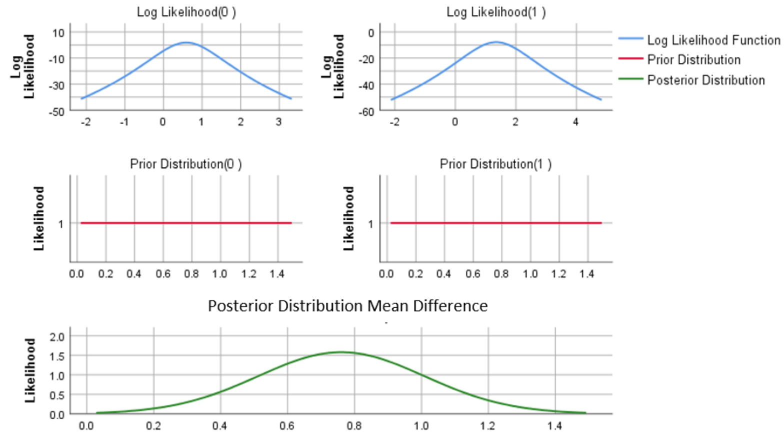


Figure F.16.: Further PFL performance from the delayed post-test

G. Q-Q PLOTS For Assumption of Normality Check for GLM Repeated Analyses

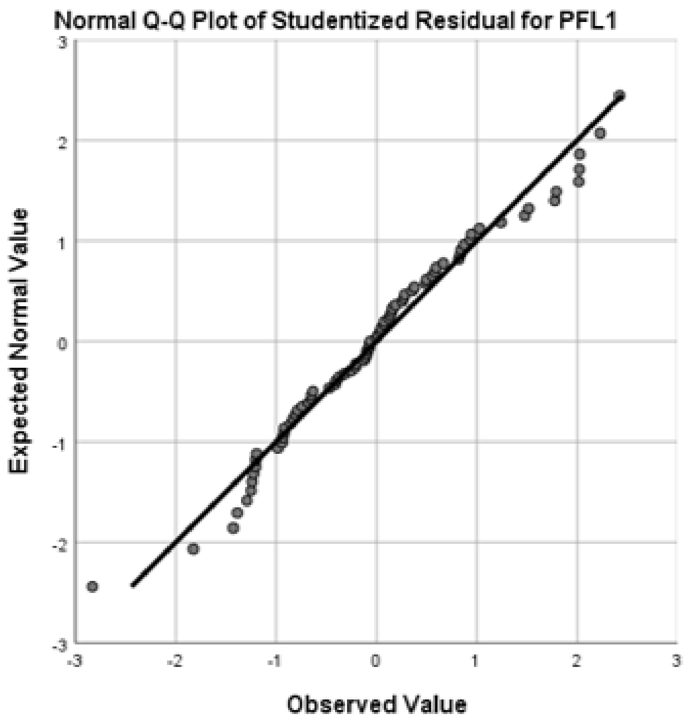


Figure G.1.: Scatter plot for normality check for the residual with PFL performance from the post-test

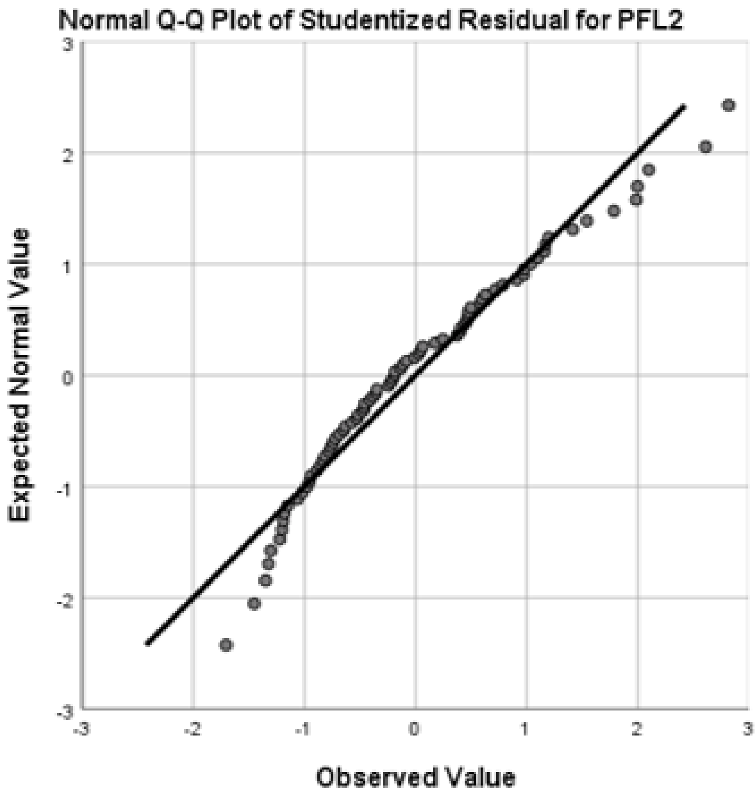


Figure G.2.: Scatter plot for normality check for the residual with PFL performance from the delayed post-test

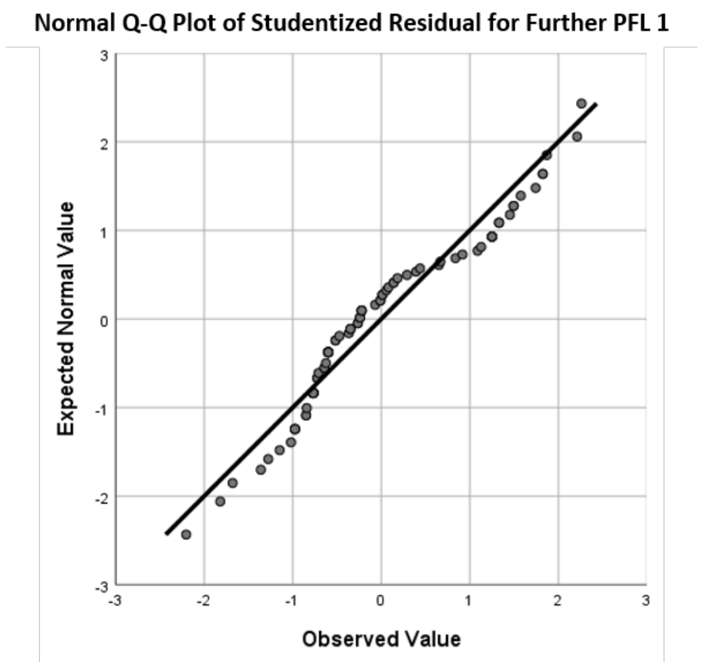


Figure G.3.: Scatter plot for normality check for the residual with further PFL performance from the post-test

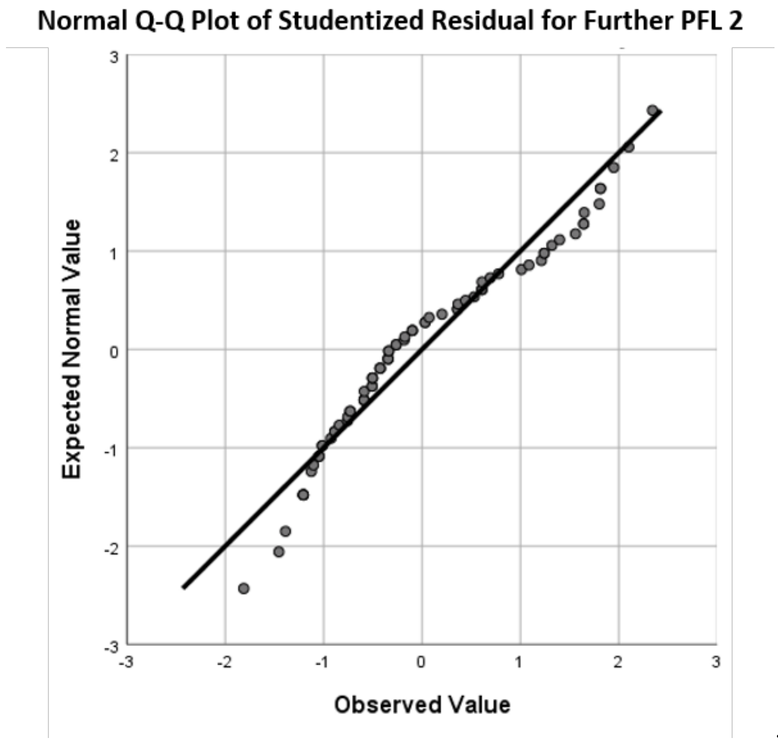


Figure G.4.: Scatter plot for normality check for the residual with further PFL performance from the delayed post-test

H. Examples of Negative Transfer

Post-Test Item 6

Find a way to get the derivative of y when $sin(x + y) = ycos(2x)$ with respect to x. Please describe how you would solve it.

Negative Transfer of Partial Derivative

*iCC group

> I think that the way to solve it would be to do a partial differential in terms of dy/dx. Then, isolate and solve for dy.

*TP group

> The best answer I can come up with here is to use partial derivatives. Basically, in partial derivatives, you take the derivative of functions with respect to one variable and hold the other variable as constant. If you take the partial derivative of both sides with respect, you can then solve for dy/dx, which is the derivative of y with respect to x. However, off the top of my head, I am not familiar with the trig rules enough to solve this problem.

Negative Transfer of Trigonometry Identity

*iCC group

> Split sin into sin(x)+sin(y) and do trig identity for the cos double angle. Then separate and differentiate.

Negative Transfer of Substitution Technique from Integration by Parts

*TP group

Participant A

> We take f(u) = ycos2x, f(v) = sin(x+y), then use chain rule to differentiate between them

Participant B

Transfer the cos(2x) to the other side, leaving y = (sin(x+y)/cos(2x)). Substitute in u for (x+y) and v for (2x), leaving y = (sinu/cosv)... I'm not sure how to go from there.

.