

DEVELOPMENT OF AN EDUCATIONAL TOOL FOR
DETERMINISTIC AND PROBABILISTIC SLOPE STABILITY ANALYSIS

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I dedicate this work to my Parents and my beloved Wife

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SYMBOLS

BSM	Bishop simplified method
c	Cohesion based on total stresses
c'	Cohesion based on effective stresses
CDF	Cumulative density function
COV	Coefficient of variation
$FORM$	First-Order Reliability Method
$FOSM$	First Order Second Moment
FS	Factor of safety
$LRFD$	Load and Resistance Factor Design
M	Margin of safety
MC	Monte Carlo Simulation
M_d	Driving moment inducing instability of slopes
M_r	Resisting moment against slope failure
N	Normal force on slice base of slopes
OMS	Ordinary method of slices
PDF	Probability density function
P_f	Probability of failure
q	Uniform surcharge load
Q	Load of gravity
R	Available shear strength
r_u	Pore water pressure ratio
SPM	Spencer Method
S_u	Undrained shear strength
T	Tangential force on slice base

W	Weight of soil (slice)
z_p	Depth below the piezoelectric line
α	Angle to the horizontal plane of slice base
β	Reliability index
ϕ	Friction angle
μ	Mean
σ	Standard deviation
σ_n	Normal stress based on total stresses
σ'_n	Normal stress based on effective stresses
ϕ	Angle of internal friction based on total stresses
ϕ	Angle of internal friction based on effective stresses
γ	Unit weight
γ_w	Unit weight of water
λ_h	Spatial variation in horizontal direction

ABSTRACT

Fernandes Leao, Thiago MSE, Purdue University, December 2019. Development of an Educational Tool for Deterministic and Probabilistic Slope Stability Analysis. Major Professor: Jiliang Li.

This research consists of the development of a new educational tool for calculations of 2D slope stability problems, named PNW-SLOPE. Slope stability has been considered one of the most important topics in geotechnical engineering for many years, so this is a subject which students should build a good background in the university. This program was created in Microsoft Excel with the aid of VBA (Visual Basic for Applications). The use of VBA allowed the creation of a good user interface, therefore those who are using the program can easily follow the instructions to create, analyze the model and check the results. Even though there are many commercial programs with the same application, this research presents a new alternative, more focused on educational purposes. PNW-SLOPE is divided in several modules. The first consists of the geometry definition of the slope. The second module consists of a deterministic slope stability analysis considering limit equilibrium method and the method of slices. The third module consists of a probability analysis considering Monte Carlo simulation. With these two options, users can compare both analysis and understand how important is the consideration of probability analysis in Geotechnical Engineering. This is a pertinent topic nowadays, since reliability analysis is increasingly being incorporated in standards and design codes throughout the world. An additional module was created for rock slope stability problems in which the failure results from sliding on a single planar surface dipping into the excavation. Several examples are presented to demonstrate some of the features of PNW-SLOPE

and results are verified with commercial programs such as Geostudio Slope/w and Rocscience Slide 2018.

1. INTRODUCTION

1.1 Overview

Evaluating the stability of slopes in soil is an important, interesting, and challenging topic in civil engineering. During the past 80 years, this subject has received extensive attention of researchers. The understanding of soil properties and analysis methods has improved considerably, leading to many methodologies which try to include properly all the parameters involved in this kind of problem. These advances and experience are essential for this relevant topic of geotechnical engineering and have helped a lot in the development of human society.

Besides the advance in the soil mechanics knowledge, computational tools also have advanced in the past decades, leading to more capacity to solve complex problems. One important software that has been used for at least 20 years for general calculations and data analysis is Microsoft Excel. It is one of the most important computational programs in engineering, with a wide range of applications. Moreover, one of the resources offered by this software, the VBA (Visual Basic Applications) gives an extra tool for development of more complex spreadsheets, with buttons and windows to help the use of the spreadsheet. Due to the reasons above, Microsoft Excel was the platform considered for development of PNW-SLOPE.

Another important subject that has increasingly been considered in geotechnical engineering is the inclusion of uncertainty in the formulation of the problems. Probability-based approaches for slope stability analysis have been a topic of research for about 50 years, but now they are more widespread. Designing standards and codes have tried to evolve and include reliability in every pertinent engineering situation. These facts gave students and professional an urge to improve their back-

ground in probability and statistics. It is important to know what the uncertainties are and what effect they will have on the results.

1.2 Statement of the Problem

Slope stability has received extensive attention of researchers for several years. Even today this topic remains one of the most challenging in geotechnical engineering with much work to be done yet. Even though rigorous methods have been created to deal with this problem in a more realistic way, classical methods continue to be widely used, both by researchers and engineers dealing with real situations. It is important therefore to understand their assumptions and limitations.

Reliability methods have found increasing use throughout geotechnical engineering, including the evaluation of slope stability. This is an important topic, sometimes not properly studied in the university. Students usually learn probability and statistics topics in the beginning of their studies. During the following years, only a few universities offer courses focused on application in civil engineering problems. The importance of reliability analysis in geotechnical engineering is one of the reasons for development of PNW-SLOPE.

Some commercial programs for geotechnical engineering already have incorporated resources to analyze slopes considering the uncertainty in the model. These programs offer advanced resources for reliability analysis, in which statistical parameters can be considered in the calculations. However, these features usually are not provided in educational versions, creating therefore a lack of tools specific for educational purposes.

1.3 Objectives and Scope of Work

The objectives of this research are:

- Provide a new educational tool for 2D slope stability analysis;

- Show how reliability methods could be applied in geotechnical engineering problems;
- Demonstrate how useful VBA programming language is in the development of spreadsheets for engineering applications.

To achieve the objectives mentioned above, the specific scope of work is enumerated as follows:

- Create a program in Microsoft Excel with the purpose to solve 2D slope stability analysis considering limit equilibrium method and method of slices for circular failure surfaces. The name of the application is PNW-SLOPE;
- Include a module specific for probabilistic slope stability analysis in which concepts of reliability theory can be applied for geotechnical engineering case studies. The approach adopted is the Monte Carlo simulation method;
- Create a module specific for deterministic and probabilistic analysis of rock slopes considering 2D planar failure case.

Even though many options of commercial and educational programs for slope stability calculations are available for use, the author considers important the creation of one more option, due to limitations of educational versions or lack of an easy user interface. It is also important to show how useful could be the application of advanced resources such as VBA for the development of spreadsheets in Microsoft Excel.

1.4 Significance

Problems associated with failures of natural and artificial slopes often pose formidable challenges in geotechnical engineering. Stability analysis is used in the construction of transportation facilities such as highways, railroads, airports, and canals; the development of natural resources such as surface mining, refuse disposal, and earth dams; as well as many other human activities involving building construction and

excavation. In general, this study is presenting a new tool for solving these kind of engineering problems through two different approaches: deterministic and probabilistic. While the deterministic method is more simplified and straightforward to solve, the probabilistic approach deals with the variability of parameters, such as materials properties. It is well accepted that uncertainties in geotechnical engineering design are unavoidable, therefore when they are considered, the associated risk can be quantified. With the deterministic analysis it is possible to calculate the factor of safety, and with probabilistic analysis the probability of failure is obtained. The Excel framework in which PNW-SLOPE is built is a familiar environment for professionals working in the Geotechnical Engineering community, which makes the tool simple and easy to use in various contexts.

PNW-SLOPE is a useful program for the civil engineering community. Namely, the proposed tool will help students, professors, researchers and geotechnical engineers. For students, PNW-SLOPE enables quick analysis of simple slope stability problems. Moreover, PNW-Slope introduces important concepts of probability and statistics applied to Geotechnical Engineering, a topic that is increasingly being incorporated in everyday practice of engineers, but not always properly studied when applied to engineering problems. For professors, it will be possible to apply this new software in their classes to teach students without the need of a commercial software. For researchers, PNW-SLOPE can serve as basis for the development of new modules that incorporate new features. Finally, geotechnical engineers working in industry can also use PNW-SLOPE as a tool for their initial slope designs. The examples presented here demonstrate the versatility and easiness of use of the proposed tool, making it relevant for a range of studies in the field of Geotechnical Engineering.

1.5 Thesis Outline

A total of five chapters are included in this thesis. The remaining chapters are organized as follows:

Chapter 2 presents the development of the first and second part of the program, which is related to geometry definition and deterministic calculation of slopes using limit equilibrium methods. In the literature review of this part, basic topics of slope stability are introduced. The development and applications of limit equilibrium method to solve slope stability analysis are described. Important resources offered by Microsoft Excel for development of civil engineering spreadsheets are also explained. At last, some examples are presented using the spreadsheet, with discussion of the results.

Chapter 3 presents the development of the third part of PNW-SLOPE. In this module, a new tool for probabilistic slope stability analysis is presented. This chapter explains basics concepts of probability and statistics, such as probability distribution and random variables. Some calculation models are presented, with emphasis on Monte Carlo simulation method, which is being considered in the spreadsheet. At last, some example are presented, with discussion of the results.

Chapter 4 presents an additional application for rock mechanics. This part is used for solving 2D planar failure of rocks with both deterministic and probabilistic approaches. This form of instability consists of a slip surface with a planar form following a major discontinuity.

Chapter 5 discusses the conclusions and recommendations for future studies. PNW-SLOPE can be enhanced by future students in many different ways to become a more complete program, and this chapter will describe some of these improvements.

Appendix A shows the complete reports generated by PNW-SLOPE for each of the illustrative examples presented in Chapters 2 and 3. Appendix B shows important parts of the coding of PNW-SLOPE.

2. DETERMINISTIC ANALYSIS OF SOIL SLOPES

This chapter presents the first and second parts of PNW-SLOPE. The first part consists of the geometry definition of the slope. Some of the parameters considered are the height and slope angle, number of layers, soil properties and pore water pressure. The critical slip surface can be defined or calculated automatically using optimization tools offered by Microsoft Excel. The second module consists of the deterministic slope stability analysis considering limit equilibrium method and the method of slices. Three methods are being considered: Ordinary Method of Slices, Bishop Simplified Method and Spencer Method, so their assumptions and results can be compared. It is possible to define regions with different number and width of slices. Total and effective stresses are being considered and can be selected separately for each soil layer. Detailed results are presented for different slope conditions. All results are validated with results from a commercial program.

2.1 Introduction

The stability condition of slopes is a subject of study and research in soil mechanics, geotechnical engineering and engineering geology. Some of the situations considered for this kind of analysis are earth and rock-fill dams, embankments, excavated slopes, and natural slopes in soil and rock.

There are at least two different approaches to deal with slope stability analysis, both widely used and well studied: limit equilibrium method and finite element method. Finite element method is based on solid mechanics by considering not only the equations of equilibrium but also those of compatibility, with an advantage of determine not only the factor of safety but also the displacements. However in this study only limit equilibrium method is considered, since this method is more sim-

plified and adequate to be implemented in a spreadsheet environment. Due to the large number of limit equilibrium methods available, it is neither possible nor desirable to review each of them, therefore in this study three different methods are being considered, each with different level of assumptions and simplifications.

The conventional approach consists of investigate the equilibrium of a mass of soil bounded below by an assumed potential slip surface and above by the surface of the slope [1]. The resulting shear stresses, induced along a potential or known failure surface, could exceed the shear strength of the soil and cause slope failure. The ratio of available shear strength to induced shear stress on a potential failure surface usually is referred as the factor of safety.

2.2 Literature Review

2.2.1 Slope Movements

Stability analysis is used in the construction of transportation facilities such as highways, railroads, airports, and canals; the development of natural resources such as surface mining, refuse disposal, and earth dams; as well as many other human activities involving building construction and excavation [2].

Slopes failures involve such a variety of processes and causative factors that they afford unlimited possibilities of classification. For instance, they can be divided according to the form of failures, the type of materials moved, the age, the stage of development, or the cause of movements.

The failure surface can be divided in three main types: circular, non-circular and composite (Fig.2.1). The failure shape will depend on the homogeneity of the materials in the slope. The non-circular failure surface may occur if there are weak layers or seams that start and end at or near the slope surface.

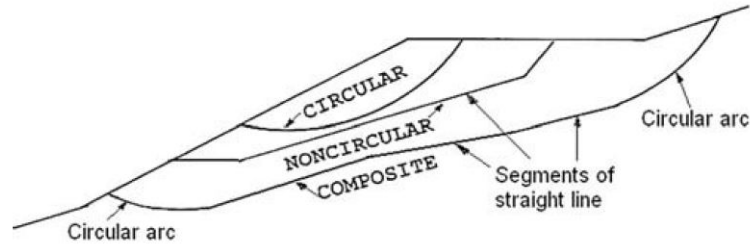


Fig. 2.1. Three types of failure surface [2]

2.2.2 Drained and Undrained Shear Strength

In the evaluation of shear strength of soils, two conditions can be considered in terms of the way how water move in or out soil when a load is applied: drained and undrained conditions. These concepts are of fundamental importance in analysis of soils, since they represent two different situations that may occur and must be properly considered. In drained condition, water flows into or out of a mass of soil as rapidly as the soil is loaded or unloaded. Because of this behavior, changes in load do not cause changes in pore pressure within the soil, or even if there is some initial change, it dissipates over time after loading. Drained condition does not mean that there is no water in the soil pores, it means that there are no excess pore water pressure. Undrained condition represents a soil in which there is no flow of water into or out in response to load changes. What happens in this situation is a change in pore water pressure during the load because the water is unable to move in or out of the soil as rapidly as the soil is being loaded or unloaded. Over time undrained condition changes to a drained condition if the load change stops and the pore pressures caused by the loading dissipate.

An analysis of drained conditions is performed using:

- Total unit weights
- Effective stress shear strength parameters
- Pore pressures determined from hydrostatic water levels or steady seepage analyses

An analysis of undrained conditions is performed using:

- Total unit weights
- Total stress shear strength parameters

2.2.3 Total and Effective Stress

Total stress is the sum of all forces, including those transmitted through inter-particle contacts and those transmitted through water pressures, divided by the total area. Total area includes both the area of voids and the area of solids. Effective stress represents the forces that are transmitted through particle contacts divided by the total area. It is equal to the total stress minus the water pressure. Total stress analysis are applicable only to undrained conditions, while effective stress analysis are applicable to drained conditions. The relation between total/effective stresses and shear stress is presented in Fig.2.2.

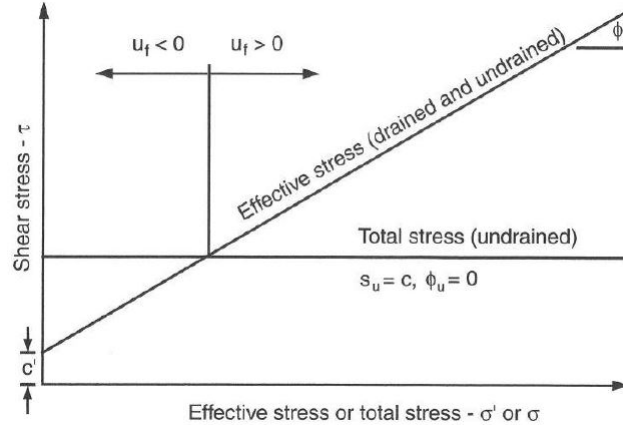


Fig. 2.2. Drained and undrained strength envelopes for saturated clay [3]

2.2.4 Short and Long-Term Analysis

The concepts of drained/undrained conditions and total/effective stresses are essential when doing short and long-term analysis of slopes. One example to show

this importance is for embankment construction which takes a short period of time. Two situations should be considered: right after the end of construction and after a longer period of time. During the short-term analysis, depending on the soil material, drainage can or cannot occur. If the embankment is composed by sand and the foundation is composed by clay, each part should be properly addressed. The embankment should be treated as drained and the foundation as undrained. Effective stress analysis is considered for the embankment and total stress analysis is considered for the foundation. On the other hand, for long-term analysis, both materials should be considered as drained, since drainage equilibrium has been reached.

2.2.5 Pore Water Pressure

Pore water pressure becomes relevant for the calculations when effective stresses are considered in the slope stability analysis. Various methods can be used to properly represent the effect of pore water pressure in the analysis. The two methods considered in this work are: piezometric line and pore pressure ratio.

A piezometric line can be used to estimate pore pressures within a seepage region. With a piezometric line, the pore water pressure is equal to the depth below the piezometric line multiplied by the unit weight of water [3]:

$$u = z_p \gamma_w \quad (2.1)$$

where

z_p is the depth below the piezoelectric line;

γ_w is the unit weight of water.

The other method presented here consists of consider a pore pressure ratio (μ_u) for each soil layer. The μ_u is defined as a ratio between the pore water pressure and the overburden pressure [2]:

$$r_u = \frac{u}{\gamma h} \quad (2.2)$$

where

γ is the unit weight of soil;

h is depth of soil between the ground surface and the failure surface.

When pore pressure ratio is considered, pore water pressure is equal to μ_u multiplied by the weight of soil above the failure surface:

$$u = r_u \gamma h \quad (2.3)$$

2.2.6 Limit Equilibrium Method

One of the most commonly used methods to solve slope stability problems is the limit equilibrium method. It consists of calculating the factor of safety for a specific problem. This factor of safety is the ratio between the shear strength and the shear stress. The factor of safety also can be represented in terms of forces or moments. For shear strength calculation, Mohr-Coulomb failure theory is considered by using the following equation:

$$s = c + \sigma_n \tan \phi \quad (2.4)$$

in which c = cohesion, σ_n = normal stress, and ϕ = angle of internal friction, based on total stresses. Both c and ϕ are known properties of the soil.

In terms of effective stress, the equation for Mohr-Coulomb becomes:

$$s = c' + \sigma'_n \tan \phi' \quad (2.5)$$

in which c' = cohesion, σ'_n = normal stress, and ϕ' = angle of internal friction, based on effective stresses.

Fig. 2.3 shows the significance of Eq. 2.5 for an elemental soil mass [4].

The limit equilibrium method is very simple to be applied for slope stability. First, the geometry needs to be defined, with an assumed failure surface. This critical slip surface should be defined by using some optimization scheme for searching for a

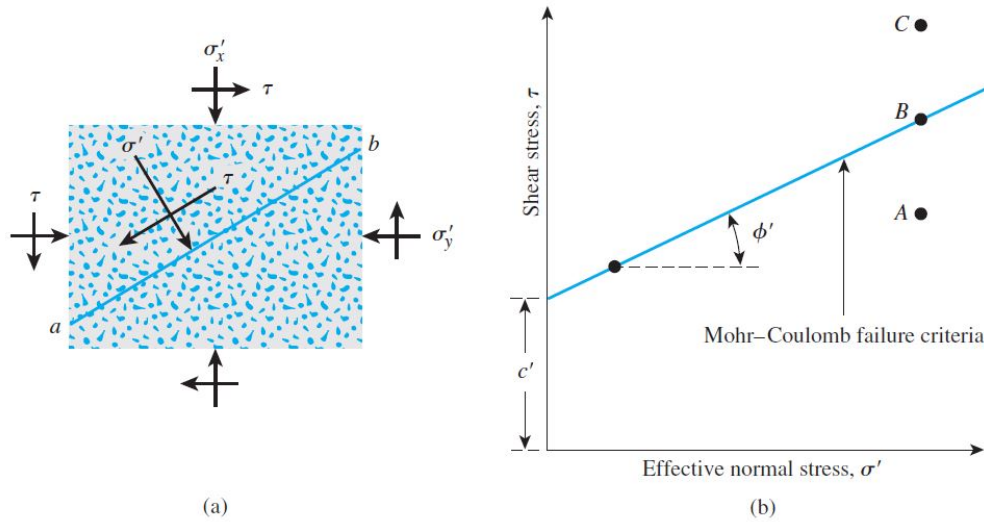


Fig. 2.3. Mohr-Coulomb failure criterion [4]

critical slip surface with the minimum factor of safety. Then, the stress along the failure surface is calculated using Mohr-Coulomb criterion. The first studies with this idea were presented by Fellenius [5] and Terzaghi [6]. The driving shear stress is calculated also for the failure surface, and then the factor of safety is obtained. It is considered only the static equilibrium in this kind of analysis, therefore equations for equilibrium of forces and moments need to be applied. When dealing with simple cases it is possible to solve the problem without any assumption. However in most situations this is not possible, requiring therefore some assumptions to be able to solve all equilibrium equations. Two cases in which the factor of safety can be calculated directly with the equilibrium equations are shown in Fig.2.4 and Fig.2.5. They are considered statically determinate problems.

Many limit equilibrium analysis procedures have been created, with different level of assumptions in the calculations. More complex methods tend to generate more reliable results, since they require less assumptions and simplifications. However they are more challenging to implement and require more computation time, including optimization tools to deal with the calculations.

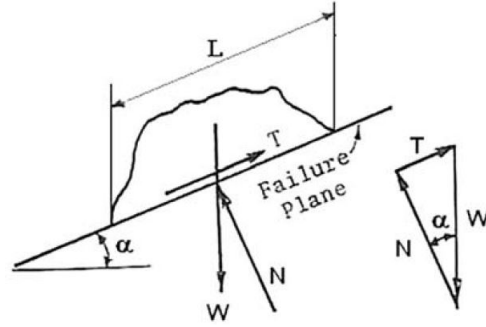


Fig. 2.4. Statically determinate plane failure [2]

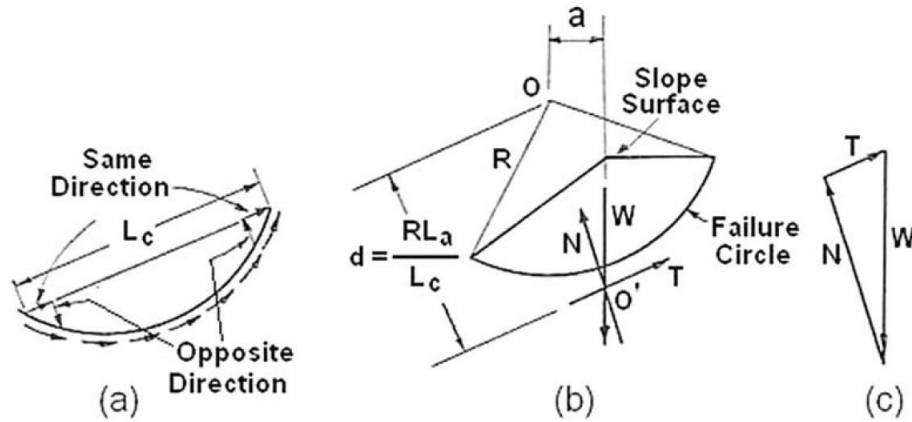


Fig. 2.5. Statically determinate cylindrical failure with $\phi = 0$ [2]

One of the most well-recognized approaches that uses limit equilibrium method is the method of slices, which is a relatively simple procedure to calculate the factor of safety. The soil mass above the slip surface is subdivided vertically into a number of parts, or slices, and each slice is analyzed individually (Fig.2.6).

Method of slices have different assumptions in the equilibrium of forces and moments. More simplified methods consider circular slip surfaces and only the overall moment equilibrium, with no inter-slice forces, while more complex methods consider non-circular slip surfaces and both overall equilibrium of moments and equilibrium of forces in each slice. Some of the classical limit equilibrium methods which applies method of slices were created by Bishop [7], Jambu [8], Morgenstern [9] and Spencer [10].

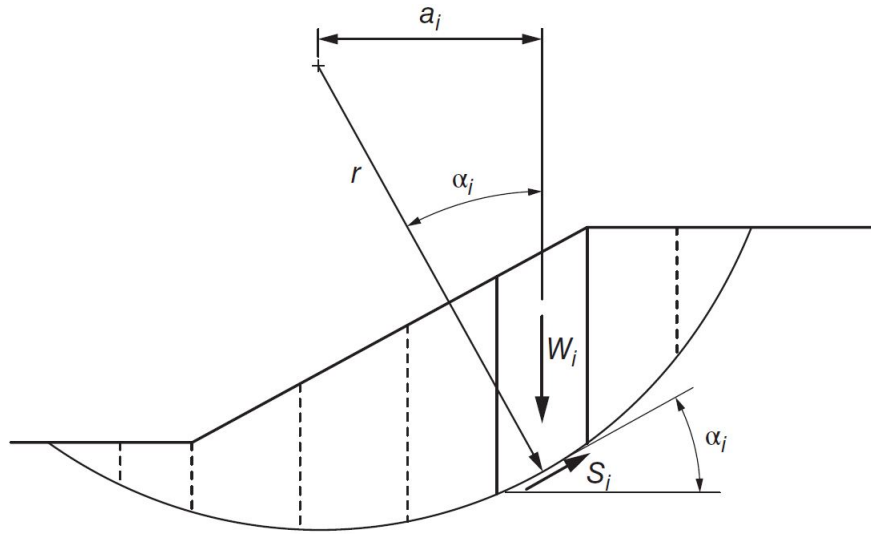


Fig. 2.6. Method of Slices

2.2.7 Ordinary Method of Slices

One of the most simplified method of slices is the Ordinary Method of Slices (OMS). This method has also been referred to as the “Swedish Method of Slices” and the Fellenius method [11]. The reason why this method is simple to solve is the assumption of no horizontal forces exist between the slices (Fig.2.7). Because of this assumption only overall moment equilibrium is considered.

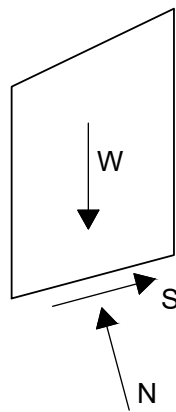


Fig. 2.7. Slice with forces considered in the Ordinary Method of Slices. [3]

The equations for factor of safety considering both total and effective stresses are presented below

Total stresses:

$$FS = \frac{\sum (c \Delta l + W \cos \alpha \tan \phi)}{\sum W \sin \alpha} \quad (2.6)$$

Effective stresses:

$$FS = \frac{\sum [c' \Delta l + (W \cos \alpha - u \Delta l) \tan \phi']}{\sum W \sin \alpha} \quad (2.7)$$

Duncan et al. [3] proved that the above equation for effective stresses can lead to unrealistically low, even negative, values for the effective stresses on the slip surface. Therefore they developed an alternative expression for effective stresses, which is being considered in this study and is presented in the following:

Effective stresses (Alternative):

$$FS = \frac{\sum [c' \Delta l + (W \cos \alpha - u \Delta l \cos^2 \alpha) \tan \phi']}{\sum W \sin \alpha} \quad (2.8)$$

2.2.8 Bishop Simplified Method

Another important method of slices is the Bishop Simplified Method (BSM). This method considers transmission of forces between the sides, however these forces are horizontal [7]. Due to this assumption there are no shear forces between the slices (Fig.2.8).

The equations for factor of safety considering both total and effective stresses are presented below:

Total stresses:

$$FS = \frac{\sum \left[\frac{c \Delta l \cos \alpha + W \tan \phi}{\cos \alpha + (\sin \alpha \tan \phi) / FS} \right]}{\sum W \sin \alpha} \quad (2.9)$$

Effective stresses:

$$FS = \frac{\sum \left[\frac{c' \Delta l \cos \alpha + (W - u \Delta l \cos \alpha) \tan \phi'}{\cos \alpha + (\sin \alpha \tan \phi') / FS} \right]}{\sum W \sin \alpha} \quad (2.10)$$

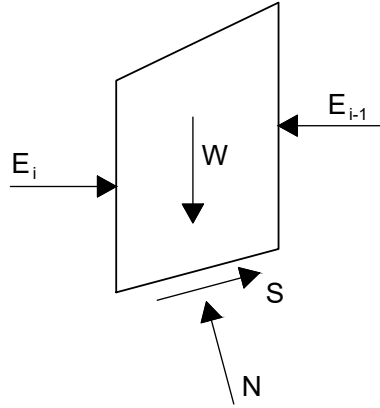


Fig. 2.8. Slice with forces considered in the Bishop Simplified Method. [3]

Factor of safety is presented in both sides of these equations, requiring therefore some iterations with adjustment of the factor of safe for each trial. The final value of FS is obtained when both sides of equation reach approximately the same value.

2.2.9 Spencer Method

The Spencer Method (SPM) is one of the most refined methods, since it satisfies all the equilibrium equations [10].

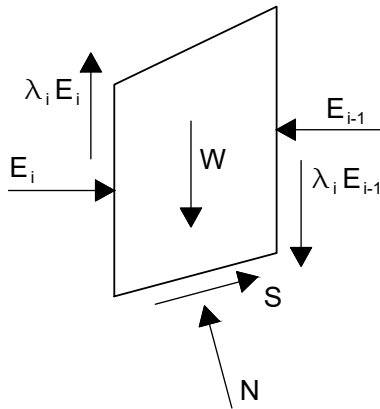


Fig. 2.9. Slice with forces considered in Spencer Method.

The equations considered in this work for Spencer Method were derived from Mohr-Coulomb criterion and equilibrium considerations [12]. Equations Eq.2.11 -

2.13 are used for calculation of forces in each slice. Equation Eq.2.14 shows the overall equilibrium of forces and Eq.2.15 shows the overall equilibrium of moments.

$$S_i = \frac{[c'_i \Delta l_i + (N_i - u_i \Delta l_i) \tan \phi'_i]}{FS} \quad (2.11)$$

$$E_i = E_{i-1} + N_i \sin \alpha_i - S_i \cos \alpha_i \quad (2.12)$$

$$N_i = \frac{\left[W_i - (\lambda_i - \lambda_{i-1}) E_{i-1} - \frac{1}{FS} (c'_i \Delta l_i - u_i \Delta l_i \tan \phi'_i) (\sin \alpha_i - \lambda_i \cos \alpha_i) \right]}{\left[\lambda_i \sin \alpha_i + \cos \alpha_i + \frac{1}{FS} \tan \phi'_i (\sin \alpha_i - \lambda_i \cos \alpha_i) \right]} \quad (2.13)$$

$$\sum [S_i \cos \alpha_i - N_i \sin \alpha_i] = 0 \quad (2.14)$$

$$\sum [(S_i \sin \alpha_i + N_i \cos \alpha_i - W_i) L_{xi} + (S_i \cos \alpha_i - N_i \sin \alpha_i) L_{yi}] - M_w = 0 \quad (2.15)$$

$$L_{xi} = 0.5(x_i + x_{i+1}) - x_c \quad (2.16)$$

$$L_{yi} = y_c - 0.5(y_i + y_{i+1}) \quad (2.17)$$

$$\lambda = \lambda' \sin \left[\frac{x_i - x_0}{x_n - x_0} \pi \right] \quad (2.18)$$

FS is obtained by using some optimization tool varying λ' and FS until reach the equilibrium of forces and moments.

2.2.10 Factor of Safety Criteria

It is important to consider uncertainty when evaluating the factor of safety of the slope. The greater the degree of uncertainty about shear strength and other conditions, and the greater the consequences of failure, the larger should be the required factor of safety [3].

Several studies suggest minimum factor of safety values to be considered depending on the condition of the slope. Table 2.1 presents minimum values depending both on cost and consequence of failure and uncertainty degree. Table 2.2 presents conventional, prudent practice for some types of slopes and conditions.

Table 2.1.
Recommended Minimum Values of Factor of Safety [3]

Cost and Consequence of Slope Failure	Uncertainty of Analysis Conditions	
	Small	Large
Cost of repair comparable to incremental cost to construct more conservatively designed slope	1.25	1.50
Cost of repair much greater than incremental cost to construct more conservatively designed slope	1.50	2.00

Table 2.2.
Factor of Safety Criteria [1]

Types of Slopes	Required Factors of Safety		
	For End of Construction	For Long-Term Steady Seepage	For Rapid Drawdown
Slopes of dams, levees, and dikes, and other embankment and excavation slopes	1.30	1.50	1.00 - 1.20

2.3 Development of PNW-SLOPE

2.3.1 Spreadsheets in Engineering

The efficiency in the use of spreadsheets for engineering problems arises from the fact that the spreadsheet already contains routines for data input, data or graphical output and a range of algebraic and logic functions and that there is no requirement for a programmer to rewrite these routines [13]. Some design routines are both laborious and time consuming because trial designs may have to be repeated a number of times before an acceptable solution is obtained. One way to deal with this is the use of additional features for optimization, such as the add in Solver in the case of Microsoft Excel.

Spreadsheets have been broadly used in engineering as teaching tools. Zanelidin and Ashar [14] presents an application of spreadsheets for teaching construction management concepts and applications. Zanelidin and El-Ariss [15] shows a broader ap-

plication for civil engineering topics in general. Wong and Barford [16] presents an application of teaching through an user-friendly Excel VBA programming in chemical engineering. The use of Excel with VBA as a teaching tool represents a favorable environment for students to learn programming skills, since they are already familiar with Excel [17].

It is also possible to find a great number of references presenting useful applications and examples of spreadsheets for a variety of scientific disciplines. Davies [13] presents a number of examples of spreadsheets used for solving structural engineering design problems. Billo [18] shows how to implement numerical methods in engineering using Microsoft Excel. Bourg [19] presents a more practical application for real-world problems. This large number of material with this purpose proves the importance of this subject nowadays.

Specifically for geotechnical slopes, some works have applied the use of spreadsheets for reliability analysis [20], [21], [22]. These works use matrix functions and Excel's Solver as optimization tool for calculations. Cao et al. [23] developed a tool for solving more efficiently Monte Carlo simulations at relatively small probability levels. Wang et al. [24] presented another application in geotechnical engineering to apply bayesian equivalent sample to obtain meaningful statistics and probability distributions of geotechnical properties from the amount of observation data during the site investigation.

2.3.2 Application of VBA Programming Language

In addition to the extensive list of worksheet functions and array of calculation tools for scientific and engineering calculations, Microsoft Excel also contains a programming language that allows users to create procedures, sometimes referred to as macros, that can perform even more advanced calculations or that can automate repetitive calculations. The name of this language is Visual Basic for Applications, or simply VBA.

VBA macros are usually referred to as procedures. They are written or recorded on a module sheet. A single module sheet can contain many procedures. There are two different kinds of procedures: Sub and Function procedures. The difference between a function and a sub is that a function can return a value while this is not possible in a sub. Sub procedures are used to perform some actions.

VBA editor is the part of Microsoft Excel in which all the work using this VBA can be done. From the Developer tab, on the Code panel, there is a button to access this editor 'Visual Basic button'. Fig.2.10 presents the initial interface of VBA editor.

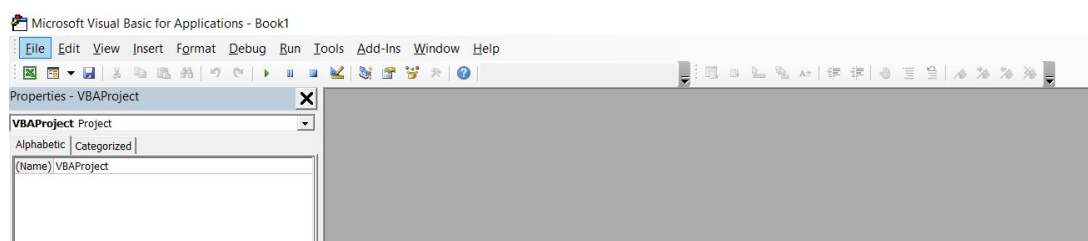


Fig. 2.10. VBA Editor

Some useful applications of VBA are presented below. They were emphasized here due to their importance for the spreadsheet presented in this work. For a more complete description of other VBA resources, it is recommended the use of a more specialized book [18].

- **Toolbox:** The toolbox is a floating window that allows the addition of controls to a Userform. Some useful controls are click buttons, option buttons and text boxes (Fig. 2.11).
- **Create, modify, delete values and formulas in cells:** It is possible to create routines to work with cell values. Fig.2.12 shows part of the code which cell values are being assigned and also formulas.
- **Create functions:** A Function procedure is a VBA code that performs calculations and returns a value (or an array of values). The return values have the following rules: The data type of the returned value must be declared in the

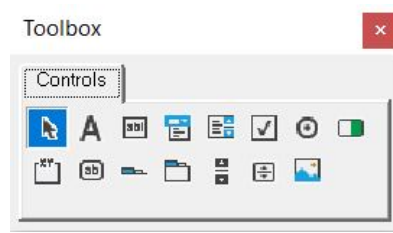


Fig. 2.11. Toolbox

Function header and the value to be returned must be assigned to a variable having the same name as the Function.

- Create an interactive chart: It is possible to create and change chart properties with command buttons (Fig.2.13).
- Set up all parameters of the Add-in Solver present at Excel: The integration of Solver in the routines is important, since it is possible to run automatically this resource for optimization calculations. In this case study, Solve is being used for calculation of the critical slip surface of the slope for different method of analysis. The objective, variable cells to be changed and constraints are defined automatically in the VBA code.
- Generate reports in .pdf format: It is possible to generate a .pdf file for specific cells (Fig.2.14).
- Record macros: The macro recorder records all the steps in Visual Basic for Applications (VBA) code. It is an easier way to write the code, since every task done during the recording is registered.

```

'Setting spreadsheet
Range("BI18").Formula = "=SUM(BI21:BI121)"
Range("BP18").Formula = "=SUM(BP21:BP121)"
Range("BQ18").Formula = "=SUM(BQ21:BQ121)"
Range("D18").Formula = "=MIN(C20:C121)"
Range("BJ18").Formula = "=SUM(BJ21:BJ121)"
Range("BU18").Formula = "=SUM(BU21:BU121)"

Cells(3, 84) = Start.Name_Project
Cells(4, 84) = Start.Author
Cells(5, 84) = Start.Date_Proj

```

Fig. 2.12. Part of the code assigning formulas and values to cells

```

If Det1.Cons_ppw.Value = True And Det1.Option_piez.Value = True Then

    Set srsNew = cht.Chart.SeriesCollection.NewSeries
    With srsNew
        .Name = "Piez"
        .Values = ActiveSheet.Range(Cells(27, 8), Cells(27, 13))
        .XValues = ActiveSheet.Range(Cells(26, 8), Cells(26, 13))
        .Format.Line.Weight = 3
        .Format.Line.ForeColor.RGB = RGB(0, 0, 204)
        .MarkerStyle = xlMarkerStyleNone
        .Format.Line.DashStyle = msoLineDashDotDot
    End With

End If

```

Fig. 2.13. Part of the code dealing with chart generation

```

myFile = Application.GetSaveAsFilename _
    (InitialFileName:=strPathFile, _
    FileFilter:="PDF Files (*.pdf), *.pdf", _
    Title:="Select Folder and FileName to save")

'export to PDF if a folder was selected
If myFile <> "False" Then
    Range(Cells(1, 79), Cells(20 + Deterministic.N_Slices.Value, 105)).ExportAsFixedFormat _
        Type:=xlTypePDF, _
        Filename:=myFile, _
        Quality:=xlQualityStandard, _
        IncludeDocProperties:=True, _
        IgnorePrintAreas:=False, _
        OpenAfterPublish:=True
    MsgBox "PDF file has been created: " _
        & vbCrLf _
        & myFile
End If

```

Fig. 2.14. Generate PDF files for specific cells

2.3.3 Userforms

This part presents the main windows of PNW-SLOPE. Fig.2.15 shows the initial screen after the program is open. It is possible to define the name of the project, authors and date. Each of the modules can be accessed through this screen. The button 'Define geometry' opens the window presented in Fig.2.16, while button 'Deterministic Analysis' opens the window showed in Fig.2.17.

After the project is already defined and analyzed, it can be saved by clicking on the 'Save Project' button. This button opens an option for the user choose the destination and name of the file. This file, which has the extension ".pnw" contains all the information created by the model. This file can be read by clicking on the 'Load Project' button. Another way to save the analysis is simply rename the Excel file with the name of the project, since the spreadsheet always keep the last results.

Purdue University Northwest - PNW/SLOPE

PURDUE
UNIVERSITY
NORTHWEST

PNW-SLOPE - Deterministic and Probabilistic Slope Stability Analysis

Save Project Load Project Deterministic Load Project Probabilistic

Project Name: Ch3_Example1d
Authors: Thiago
Date: 9/3/2019

Define geometry Deterministic Analysis Probabilistic Analysis

Exit

Fig. 2.15. PNW-SLOPE - Initial screen

Define geometry

Step 1: Choose Units

☒ Metric
 ☐ Imperial

Coordinates: Metric m Imperial ft
 Cohesion: kN/m² psf
 Unit weight: kN/m³ pcf
 Surcharge: kN/m lb/ft

Step 2: Define Geometry

☐ Option 1: Height and Slope Angle
☐ Option 2: Height and Horizontal
☒ Option 3: Define surface by points

Number of Points: 4

Point	X	Y
Point 1:	-5	0
Point 2:	0	0
Point 3:	10	5
Point 4:	15	5

Step 3: Define Layers

Number of Layers: 2

Layer	Yt	Yb	c	phi	U.W.
Layer 1	5	0	10	10	17.64
Layer 2	0	-5	8	5	18

Step 4: Critical Slip Surface

☐ Calculate
☒ Specific

Xc	2
Yc	11
r	13

Adjust Grid

Y (top)	10	Spacing (x)	1
Y (bottom)	-5	Spacing (y)	1
X (left)	-5		
X (right)	15		

☒ Adjust Manually

Fig. 2.16. PNW-SLOPE - Geometry definition

Deterministic Analysis

Step 1: Define Type of Analysis

☐ Total Stresses
☒ Effective Stresses
☐ Choose for each layer (T or E)

Step 2: Define Slices

☐ Uniform Width
☒ Variable Width

Number of Zones: 3

Generate slices

Right to left

From	To	N
Xmax	10	5
	0	10
Xmin		5

Geometry

Point	X	Y
Point 1:	-5	0
Point 2:	0	0
Point 3:	10	5
Point 4:	15	5
Point 5:		
Point 6:		
Point 7:		
Point 8:		
Point 9:		
Point 10:		

Step 3: Run Analysis

Results

OMS

Xc	2.000
Yc	11.000
r	13.000
F	1.043

BMS

Xc	2.000
Yc	11.000
r	13.000
F	1.093

SPM

Xc	2.000
Yc	11.000
r	13.000
F	1.093

Image

☒ OMS - Ordinary Method of Slices
☐ BSM - Bishop Simplified Method
☐ SPM - Spencer Method

Fig. 2.17. PNW-SLOPE - Deterministic Analysis

2.3.4 Flowcharts

An efficient way to represent an algorithm by using flowcharts. Flowcharts show the step by step of some procedure as boxes of different types, and then connecting the boxes with arrows. Some of the flowcharts considered in this research are presented in the following.

Flowcharts in Fig.2.18 - 2.20 present the step by step of input data by the user. One of the features of this software is to help students solving 2D slopes, therefore it is important to define clearly each step in the problem.

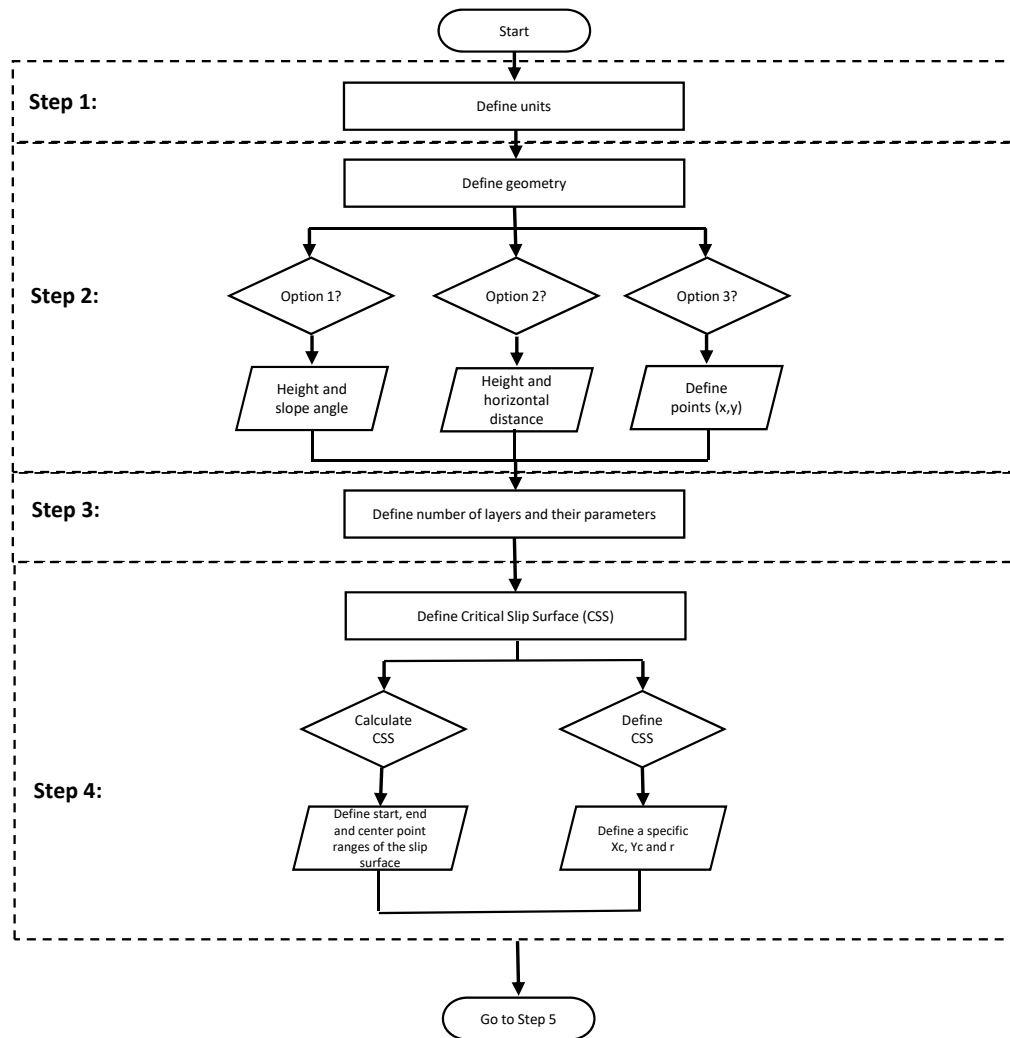


Fig. 2.18. Flowchart for geometric input parameters - Part 1

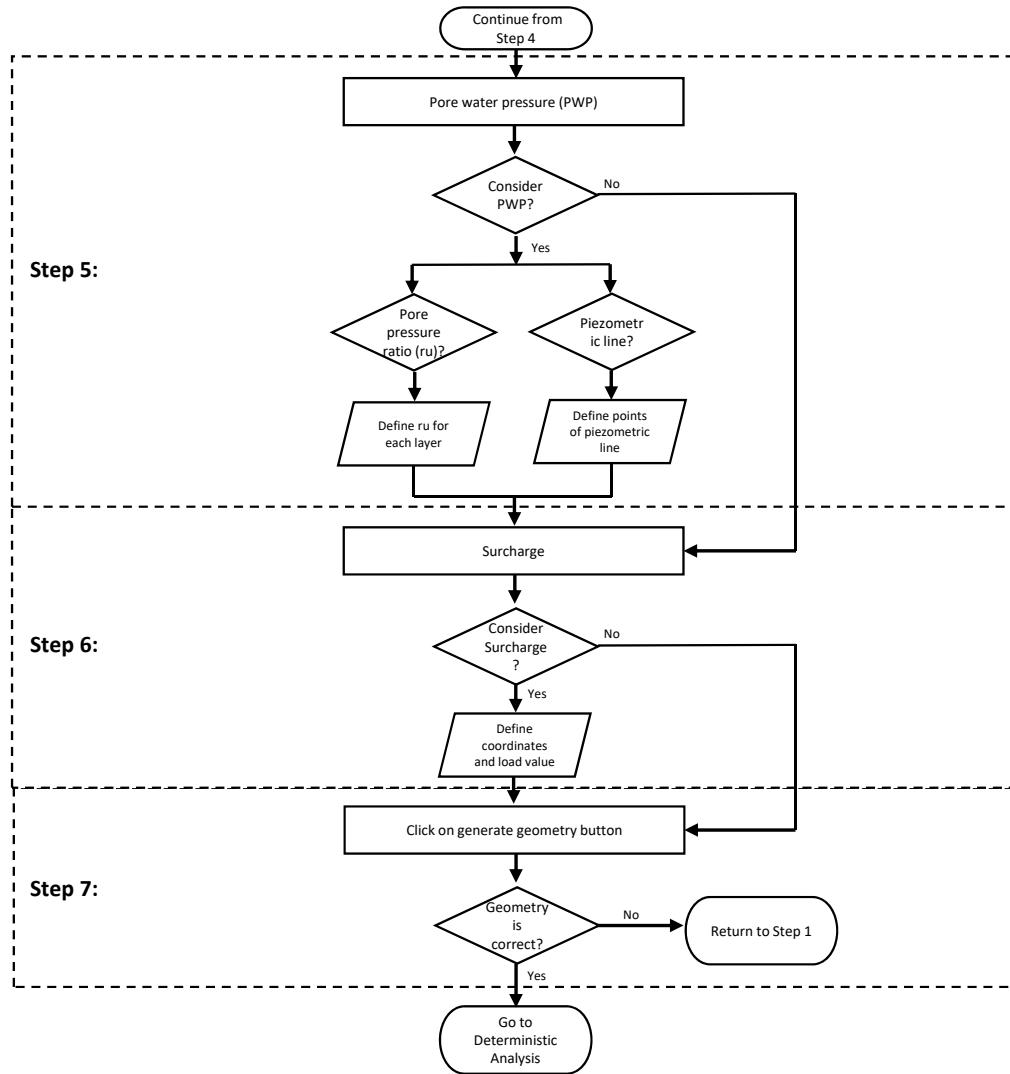


Fig. 2.19. Flowchart for geometric input parameters - Part 2

Flowchart in Fig.2.21 presents the analysis considered in the spreadsheet. All the input data inserted by the user is read and applied in the formulation of the problem. The code create specific sheets for each method considered. Each slice is represented in one row. The solution for OMS is found straightforward with the sum of resisting and driving forces in each slice. For BMS and SPM it is necessary some iterative method of optimization to find the factor of safety.

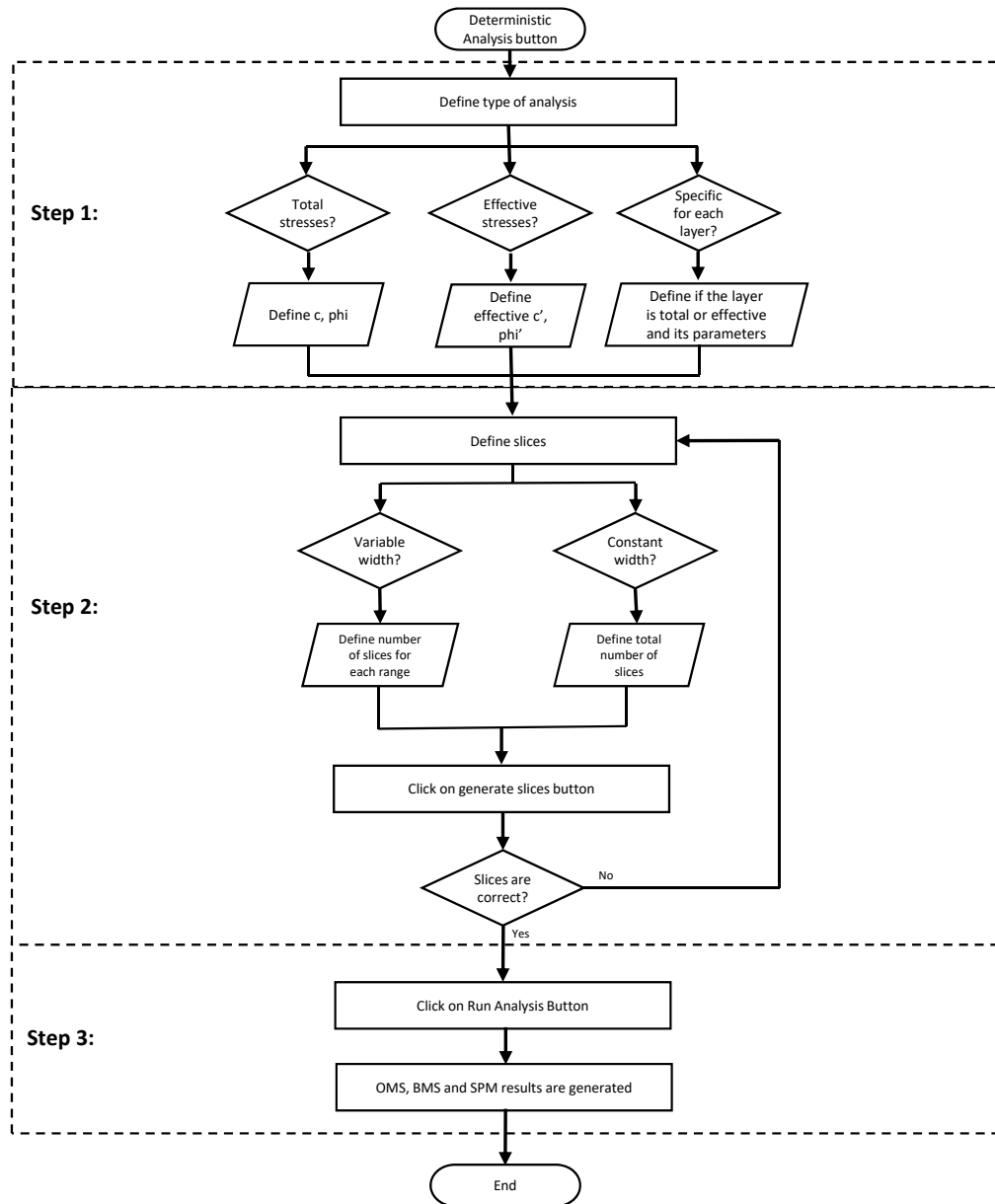


Fig. 2.20. Flowchart for analysis input parameters

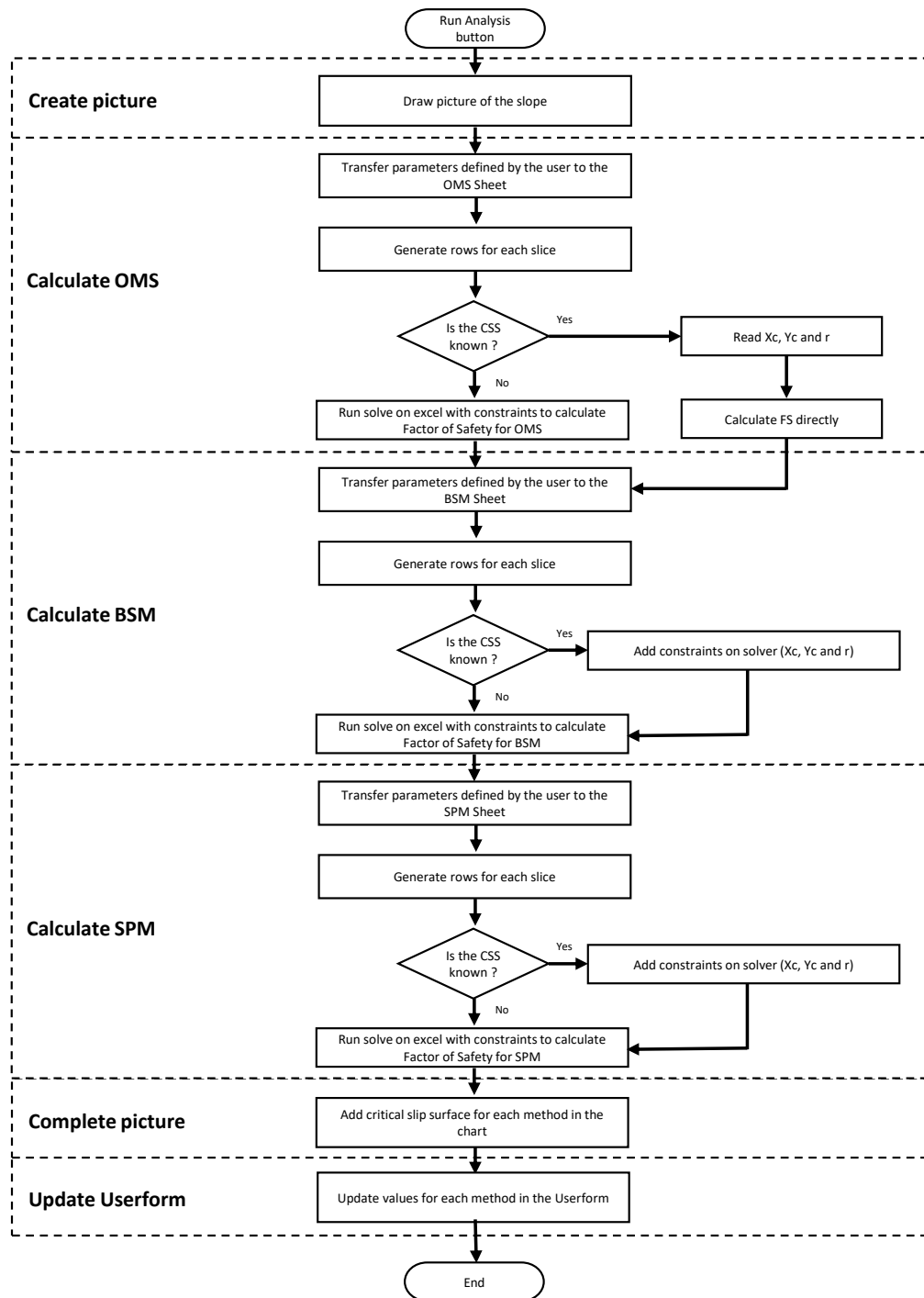


Fig. 2.21. Flowchart for analysis procedure

2.4 Illustrative Examples

In this section, several examples are presented to illustrate the use of PNW-SLOPE. The three limit equilibrium methods presented in this work are considered for each situation. The complete reports created by PNW-SLOPE are presented in Appendix A. Each example was verified and validated with results from SLOPE/W [25].

2.4.1 Example 2.1 - Short term stability

The first example represents a case for stability at the end of construction for an embankment. This problem was adapted from US Army Corps Engineering Manual [1] and is shown in Fig.2.22. Because the soils in this case are do not drain during construction, undrained shear strength is considered (c and ϕ in terms of total stresses). The critical slip surface is already defined. No water pressure is being considered, neither external loads. All units are in imperial system.

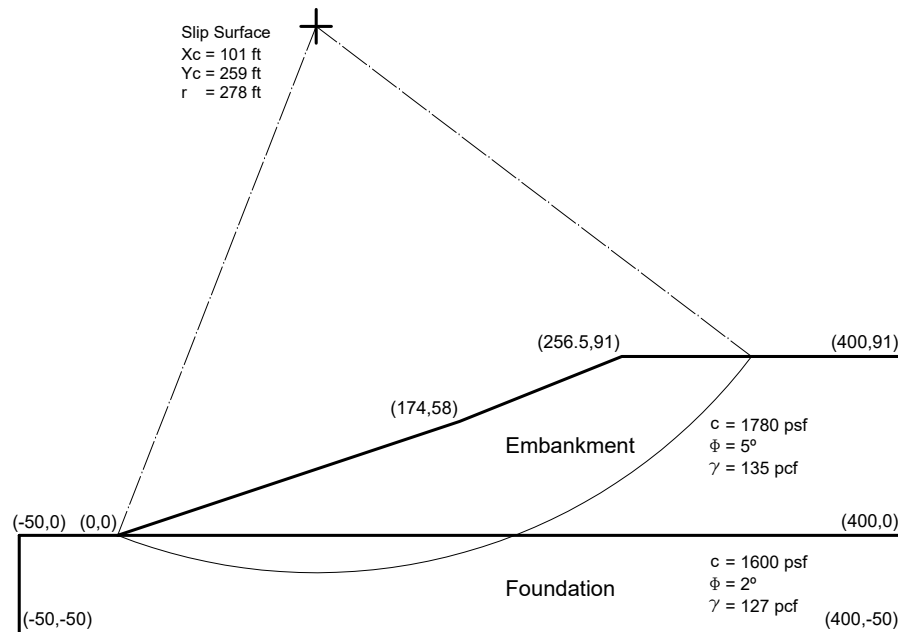


Fig. 2.22. Example 2.1 - Geometry

2.4.2 Example 2.1 - Results

The results generated by PNW-SLOPE for Example 2.1 are presented below. Fig.2.23 shows the slices considered in the analysis. Table 2.3 compares the results for each method considered in the spreadsheet with the results from SLOPE/W.

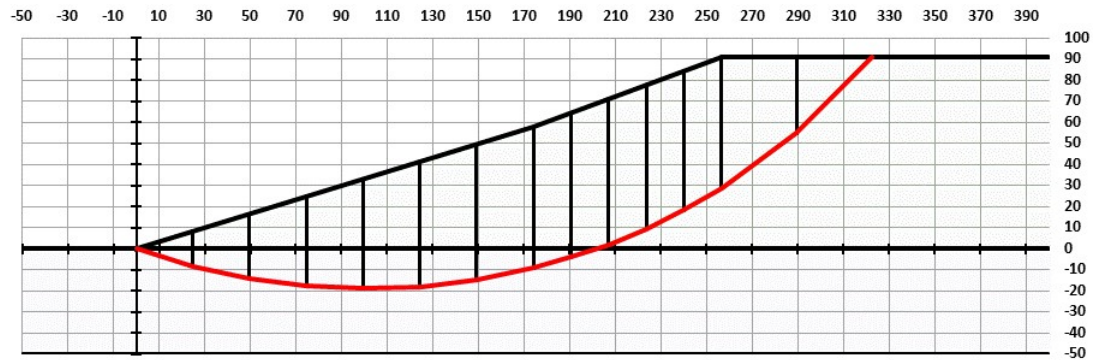


Fig. 2.23. Example 2.1 - Slices considered

Table 2.3.
Example 2.1 - Results

Program	FS(OMS)	FS(BSM)	FS(SPM)
PNW-SLOPE	1.311	1.330	1.327
SLOPE/W	1.311	1.331	1.327

For this example, the results for the three methods considered were approximately the same. The factor of safety for OMS was the lowest between the three methods, which is consistent with the tendency of more simplified methods lead to more conservative results. In this case both total and effective analysis would lead to the same results, since there is no pore water pressure in the analysis. Results generated by PNW-SLOPE are approximately the same of SLOPE/W.

2.4.3 Example 2.2 - Long term stability

The second example represents a case for long term stability for an embankment. This problem was adapted from US Army Corps Engineering Manual [1] and is shown in Fig.2.24. For steady-state seepage conditions, drained shear strength parameters are considered for all soils (c' and ϕ'). Pore water pressures are characterized by the piezometric line indicated in the figure. The critical slip surface is already defined. All units are in imperial system.

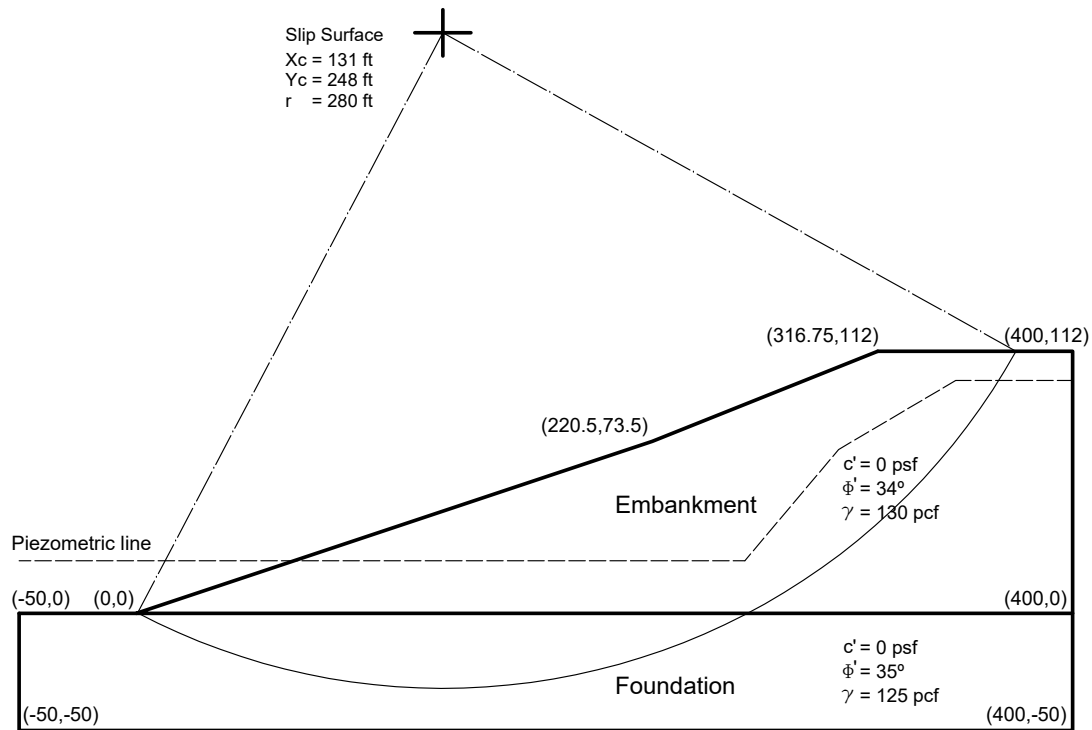


Fig. 2.24. Example 2.2 - Geometry

2.4.4 Example 2.2 - Results

The results generated by PNW-SLOPE for Example 2.2 are presented below. Fig.2.25 shows the slices considered in the analysis. Table 2.4 compares the results between PNW-SLOPE and SLOPE/W.

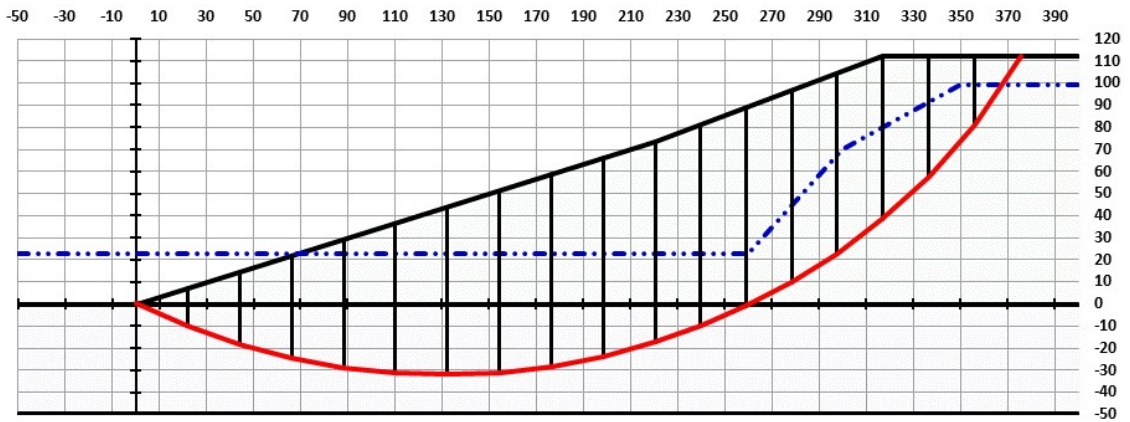


Fig. 2.25. Example 2.2 - Slices generated

Table 2.4.
Example 2.2 - Results

Program	FS(OMS)	FS(BSM)	FS(SPM)
PNW-SLOPE	1.777	1.941	1.989
SLOPE/W	1.796	1.985	1.999

For this example, the results for the BSM and SPM methods were approximately the same. In this case OMS does not represent a realistic approach, as the results are showing. Results generated by PNW-SLOPE were validated by SLOPE/W.

2.4.5 Example 2.3 - Homogeneous soil

This example was taken from Fredlund [26] and represents a homogeneous slope with three separate water conditions:

- a Dry;
- b R_u defined pore pressure;
- c Pore pressures defined using a water table.

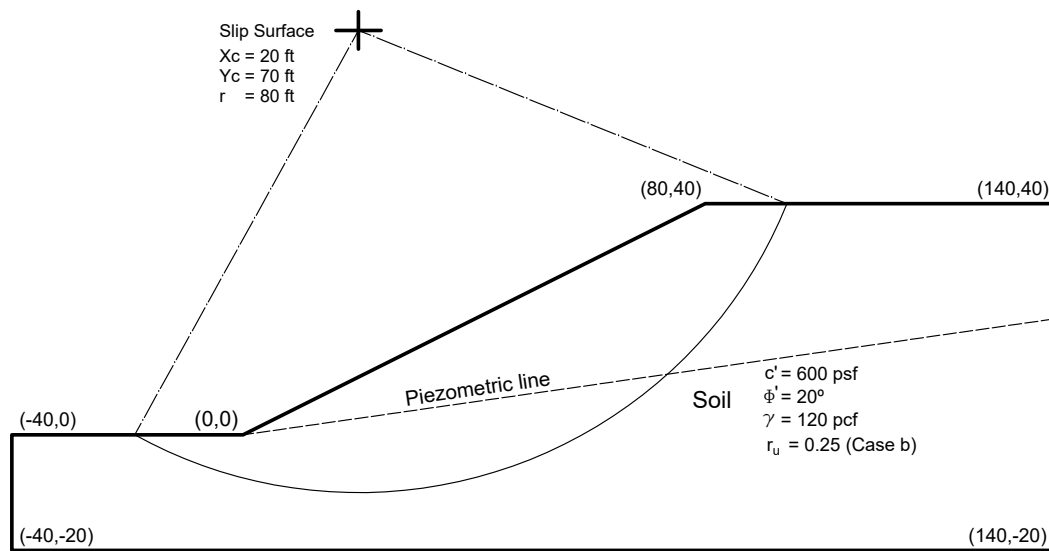


Fig. 2.26. Example 2.3 - Geometry

2.4.6 Example 2.3 - Results

The three situations presented in Example 2.3 were analyzed at PNW-SLOPE and the results are showed below. Fig.2.27 shows the slices considered in the analysis. Table 2.5 compares the results between PNW-SLOPE and SLOPE/W for each method.

In this example it was possible to understand the effect of pore water pressure in the stability of the slope. Shear strength decreases when pore water pressure and

effective stresses are considered. This fact could be demonstrated with the reduction of the factor of safety when pore water pressure was considered in the analysis. Results for BSM and SPM were approximately the same, and OMS method led to more conservative results.

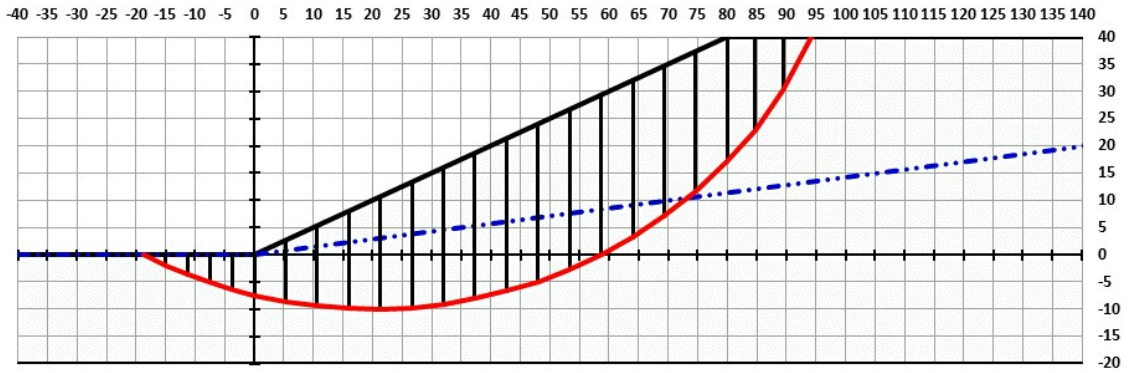


Fig. 2.27. Example 2.3 - Slices considered

Table 2.5.
Example 2.3 - Results

Case	Program	FS(OMS)	FS(BSM)	FS(SPM)
a	PNW-SLOPE	1.931	2.080	2.076
	SLOPE/W	1.926	2.077	2.075
b	PNW-SLOPE	1.688	1.763	1.770
	SLOPE/W	1.669	1.761	1.762
c	PNW-SLOPE	1.717	1.834	1.834
	SLOPE/W	1.713	1.830	1.830

2.4.7 Example 2.4 - Heterogeneous soil

This example is composed by a heterogeneous slope with three different layers. Each layer is represented by its undrained shear strengths (S_u). This problem was adapted from Low [27] and is shown in Fig.2.28. Metric unit system was considered in this case.

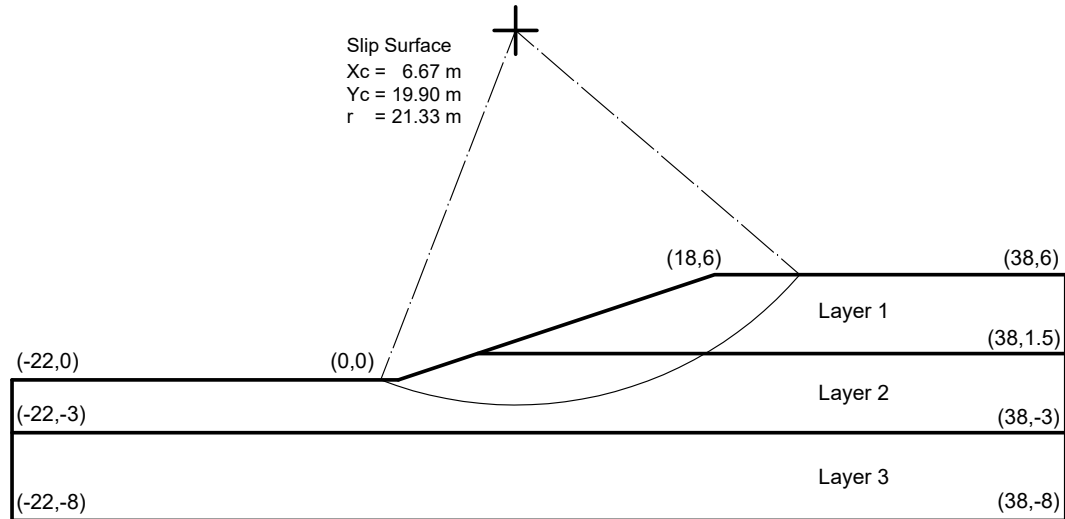


Fig. 2.28. Example 2.4 - Geometry

Table 2.6.
 Example 2.4 - Parameters

Layer	S_u (kN/m ²)	γ (kN/m ³)
Layer 1	30	18
Layer 2	20	18
Layer 3	150	18

2.4.8 Example 2.4 - Results

The results generated by PNW-SLOPE for Example 2.4 are presented below. In this case S_u was assigned as c in the program and $\phi = 0$. Fig.2.29 shows the slices considered. Table 2.7 compares the results between PNW-SLOPE and SLOPE/W for each method.

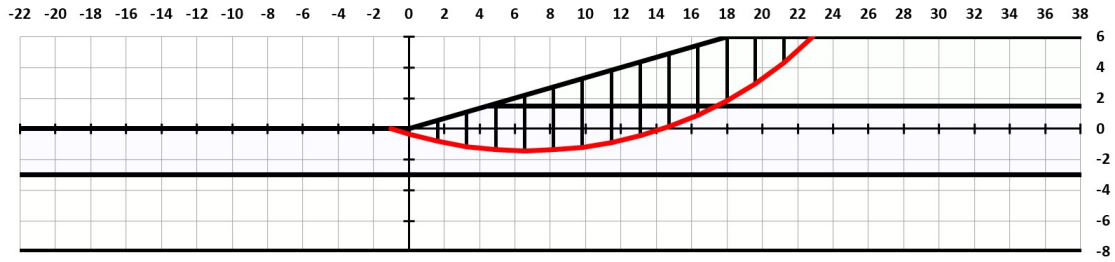


Fig. 2.29. Example 2.4 - Slices considered

Table 2.7.
Example 2.4 - Results

Program	FS(OMS)	FS(BSM)	FS(SPM)
PNW-SLOPE	1.743	1.743	1.743
SLOPE/W	1.756	1.756	1.756

For this example, the results for all the methods were approximately the same. Results generated by PNW-SLOPE were validated by SLOPE/W.

2.4.9 Example 2.5 - Homogeneous soil with different angles

This example is composed by a homogeneous soil slope with height of 10m. Three different angles are compared, corresponding to inclinations of 1:1, 1:1.5 and 1:2 (Fig.2.30). There is a firm stratum at 15m below the top of the slope. The critical slip surface is not defined and needs to be calculated. For this calculation, the ranges for initial and final of slip surface are presented in Table 2.8.

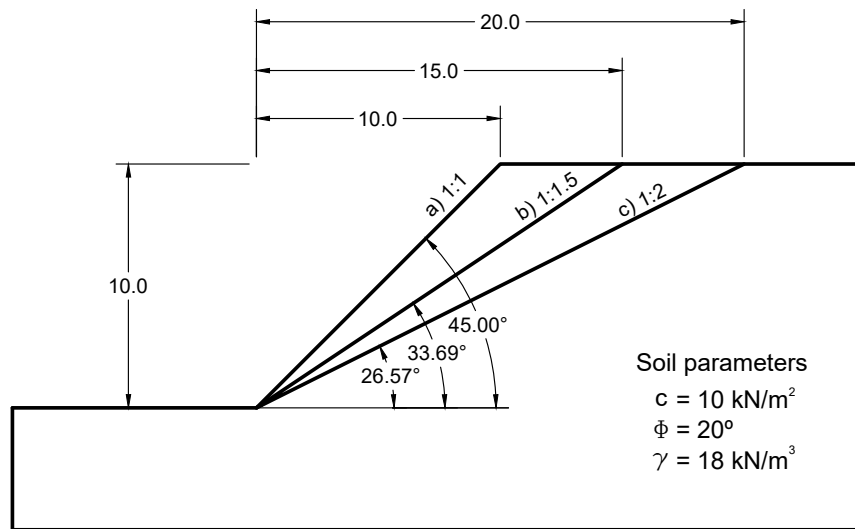


Fig. 2.30. Example 2.5 - Geometry

Table 2.8.
Example 2.5 - Critical slip surface parameters

		a	b	c
Toe (Left)	$X_i(\text{m})$	-5.0	-5.0	-5.0
	$X_f(\text{m})$	0.0	0.0	0.0
Crest (Right)	$X_i(\text{m})$	10.0	15.0	20.0
	$X_f(\text{m})$	15.0	20.0	25.0
Center	$Y_{\max}(\text{m})$	1.0	1.0	1.0
	$Y_{\min}(\text{m})$	-2.0	-2.0	-2.0

2.4.10 Example 2.5 - Results

The results for Example 2.5 are presented in Tables 2.9-2.11. The slices considered in the analysis are showed in Figs.2.31-2.33.

This example showed the comparison of three different slopes for the same condition. When the slope angle is reduced, the factor of safety increases. The difference between OMS and the other two methods increases when the slope angle decreases, becoming not reliable for small angles. In this problem it is possible to consider the criteria presented on Table 2.2 to evaluate the factor of safety. The minimum values are 1.30 for end of construction and 1.50 for long term stability. Considering BSM and SPM methods, the first situation with the angle of 45.00° would not be adequate in neither short or long term analysis. The second case with the angle of 33.69° would not be adequate for long term analysis only, while the third slope with the angle of 26.57° would be considered acceptable for both situations.

Table 2.9.
Example 2.5a - Results

Method	Program	Xc (m)	Yc (m)	r (m)	FS
OMS	PNW-SLOPE	1.086	10.987	11.080	0.942
	SLOPE/W	0.831	10.393	10.426	0.945
BSM	PNW-SLOPE	1.114	10.359	10.430	1.007
	SLOPE/W	0.831	10.393	10.426	1.002
SPM	PNW-SLOPE	0.931	12.085	12.214	1.013
	SLOPE/W	1.386	10.405	10.497	1.012

Table 2.10.
Example 2.5b - Results

Method	Program	Xc (m)	Yc (m)	r (m)	FS
OMS	PNW-SLOPE	2.583	15.557	15.641	1.130
	SLOPE/W	4.304	10.483	11.332	1.200
BMS	PNW-SLOPE	2.610	15.557	15.774	1.206
	SLOPE/W	4.304	10.483	11.332	1.334
SPM	PNW-SLOPE	2.676	15.878	16.101	1.202
	SLOPE/W	4.304	10.483	11.332	1.328

Table 2.11.
Example 2.5c - Results

Method	Program	Xc (m)	Yc (m)	r (m)	FS
OMS	PNW-SLOPE	5.127	18.390	19.092	1.341
	SLOPE/W	7.360	10.609	13.279	1.476
BMS	PNW-SLOPE	3.759	22.216	22.532	1.422
	SLOPE/W	7.210	10.580	12.803	1.714
SPM	PNW-SLOPE	3.752	22.170	22.485	1.419
	SLOPE/W	7.210	10.580	12.803	1.706

This analysis is important for a initial design for a slope in which the angle is not defined yet.

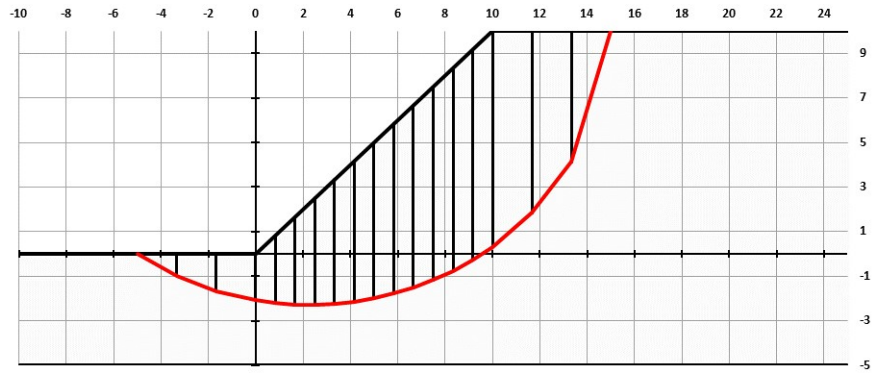


Fig. 2.31. Example 2.5a - Slices considered in PNW-SLOPE

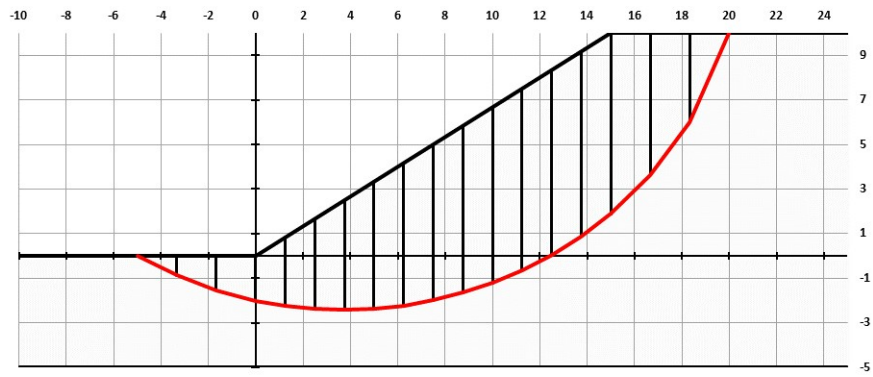


Fig. 2.32. Example 2.5b - Slices considered in PNW-SLOPE

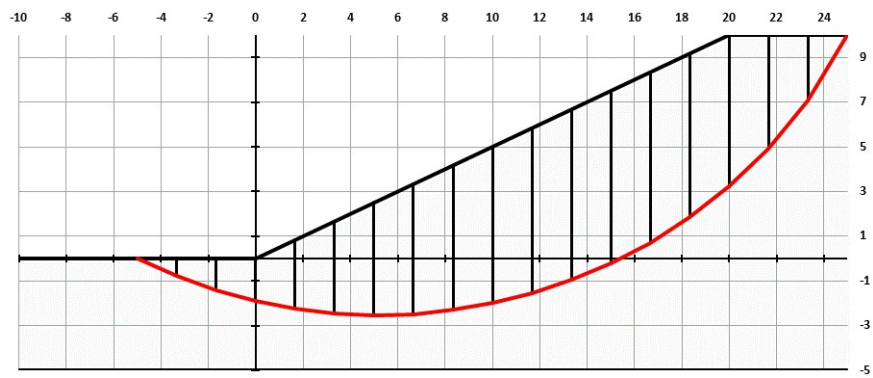


Fig. 2.33. Example 2.5c - Slices considered in PNW-SLOPE

2.4.11 Example 2.6 - Slope with more complex geometry

This example presents a more complex geometry, composed by three slopes with different inclinations and four different layers (Fig.2.34-2.35). Long term stability is considered. All parameters are showed in Tables 2.12-2.14. All units are in metric system.

Table 2.12.
Example 2.6 - Parameters

Layers	Top (m)	Bottom (m)	c (kN/m²)	Φ(°)	γ(kN/m³)
Layer 1	30.0	20.0	8.0	18.0	19.0
Layer 2	20.0	12.0	5.0	25.0	18.0
Layer 3	12.0	3.0	10.0	20.0	18.5
Layer 4	3.0	-15.0	8.0	18.0	19.0

Table 2.13.
Example 2.6 - Geometry

POINTS:	P1	P2	P3	P4	P5	P6
X (m)	-20.0	0.0	20.0	50.0	70.0	90.0
Y (m)	0.0	0.0	15.0	20.0	30.0	30.0

Table 2.14.
Example 2.6 - Critical Slip Surface

Xc (m)	Yc (m)	r (m)
26.0	42.0	55.4

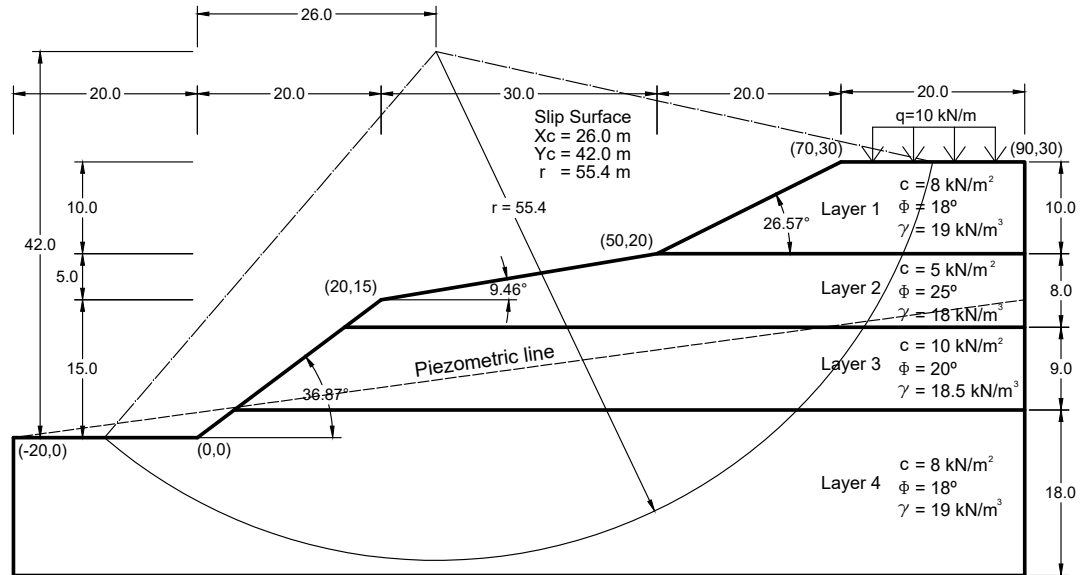


Fig. 2.34. Example 2.6 - Geometry

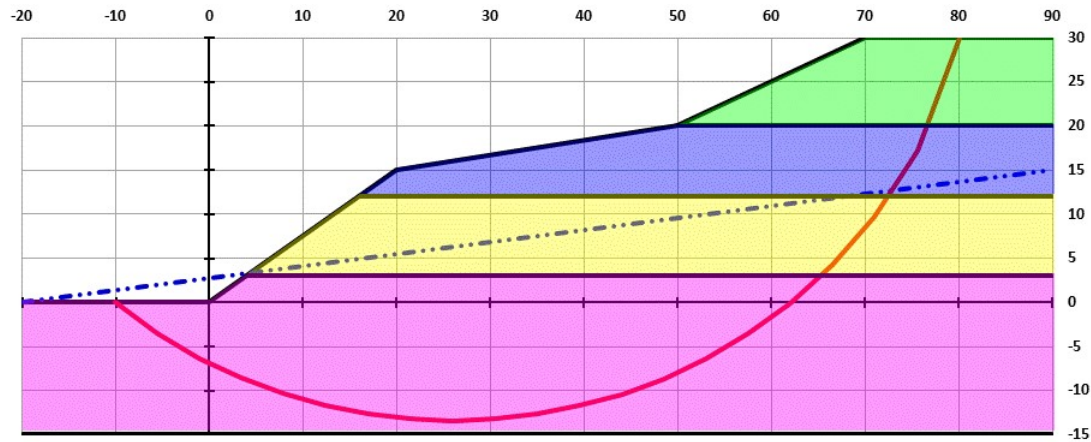


Fig. 2.35. Example 2.6 - Geometry created by PNW-SLOPE

2.4.12 Example 2.6 - Results

The results for Example 2.6 are presented in Table 2.15 and Fig. 2.36.

For this example, a more complex geometry was analyzed. External forces were considered, both from external loads on the top of the slope and water in region close to the toe, in which the water level was greater than the top of the slope. The results

for the BSM and SPM methods were approximately the same. The result for OMS was lower than the other two methods. In this case the slope would be unstable for OMS method, since the factor of safety is less than 1. However for the three methods this slope would not be adequate when considering the minimum values from table 2.2. Results generated by PNW-SLOPE were validated by SLOPE/W.

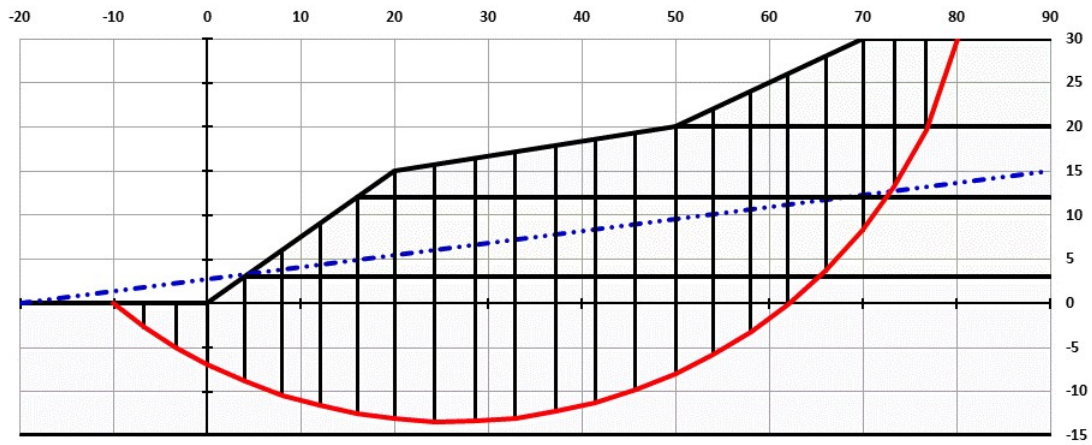


Fig. 2.36. Example 2.6 - Slices considered

Table 2.15.
Example 2.6 - Results

Program	FS(OMS)	FS(BSM)	FS(SPM)
PNW-SLOPE	0.964	1.160	1.152
SLOPE/W	0.967	1.163	1.162

2.5 Conclusion

The objective of this chapter was to present PNW-SLOPE, a new tool to perform 2D slope stability analysis. This part focused on the first two modules of the program: geometry definition and deterministic slope stability analysis. Some of the options offered by PNW-SLOPE were the inclusion of up to ten layers with different properties, consideration of pore pressure water and external forces. Several examples were studied, representing common slope situations. These examples demonstrated some of the capabilities of PNW-SLOPE, which include a graphic user interface for data entry and help buttons at each step briefly explaining the theory and how to use the program. Examples 2.1 and 2.2 presented an embankment analysis at two stages: right after the end of construction and after a long period of time. Depending on the situation and on the soil properties, the soil should be modeled with undrained or drained condition. It is important for the program to include both total and effective analysis options, since the drainage condition affect the type of analysis to be considered. Example 2.3 presented a slope under different conditions: dry, with pore water pressure represented by r_u coefficient and with piezometric line. The results showed a reduction in factor of safety when pore water pressure is being considered in the analysis. Example 2.4 presented a heterogeneous slope represented by undrained shear strengths. Example 2.5 presented a situation comparing the factor of safety for different slope angles. Example 2.6 presented a more complex geometry, with more layers and including both external loads and pore water pressure. All the examples were verified and validated with commercial programs for geotechnical engineering. The advantages when considering VBA programming language in Excel were demonstrated with PNW-SLOPE. VBA has an immense potential to increase the capacity of spreadsheets developed by students, and programming skills plays a fundamental role in this task for Civil Engineers as this work could demonstrate.

3. PROBABILISTIC ANALYSIS OF SOIL SLOPES

This chapter presents the third module of PNW-SLOPE. In addition to the geometry generation and deterministic analysis presented in the previous chapter, this part consists of a tool to perform probabilistic slope stability analysis. This module uses the geometry generated in the first module and allows the inclusion of statistical parameters to define the problem in a probabilistic approach. Reliability analysis and probabilistic methods increasingly have found application in geotechnical engineering in recent years. While in deterministic analysis the factor of safety for slope stability is the ratio of the sum of the resisting forces (or moments) to the sum of the driving forces (or moments), it is hard to evaluate how reliable are the results, since uncertainty is not being considered in the calculations. Probabilistic analysis consists of estimating the probability of failure by considering the variability of parameters. Direct reliability analysis is considered, with the use of analytical methods to obtain probability descriptions of the system. The approach considered in this study was the use of Monte Carlo simulations, a relatively simple technique to be implemented in a spreadsheet environment. Histograms are presented both in terms of factor and margin of safety. It is possible to check the influence of each parameter in the factor of safety by generating specific charts. Some examples are presented to show some of the useful applications of PNW-SLOPE for reliability analysis. Results are compared with commercial programs. It is expected with this educational tool a simple and intuitive way for students to understand the application of uncertainty in slope calculations. Moreover, the concepts reviewed in this study can be applied for reliability analysis of other geotechnical applications.

3.1 Introduction

The concept of probability and statistics has been introduced in Civil Engineering for at least 50 years. Ang and Tang [28] explain that uncertainty is part of engineering problems, since there is some variability in materials, calculations and methodologies. In all nature, it is almost impossible to estimate a behavior with one hundred percent of certain about every property. Therefore, a proper model to represent all the different randomness present in the study plays an essential role in the level of knowledge achieved. The concept of risk and probability brings equilibrium between cost and system performance, with the uncertainties properly addressed.

In geotechnical engineering, the application of uncertainty analysis is more challenging than in other fields, such as structural and mechanical engineering. The uncertainties in geotechnical engineering are largely inductive: starting from limited observations, judgment, knowledge of geology, and statistical reasoning are employed to infer the behavior of a poorly-defined universe [29]. The pioneers Casagrande [30] and Peck [31] started dealing with uncertainties in geological materials using the observational method. In this method, engineers make reasonable estimates of the parameters and the amounts by which they could deviate from the expected values. While uncertainty and reliability have a long history in geotechnical engineering, only in the last twenty years that several researches have made major advances in use more formal and rational approaches for reliability theory in geotechnical engineering [32].

Baecher and Christian [29] enumerates at least four reasons in which the geotechnical engineer today must be able to deal with reliability:

1. Regulatory and legal pressure force geotechnical engineers to provide answers about the reliability of their designs;
2. Management decisions on whether to proceed with a project course of action, how to finance it, and when to schedule it are increasingly based on statistical decision analysis;

3. Modern building codes are based on LRFD approaches, which are in turn based on reliability methods;
4. Reliability theory provides a rational way to deal with some historically vexed questions, such as how much confidence should the engineer place on a calculated factor of safety?

In order to develop appropriate input, the engineer must understand the nature of uncertainty and probability. Most geotechnical uncertainty reflects lack of knowledge, and probability based on the engineer's degree of belief comes closest to the profession's practical approach [32]. Therefore, there is an increasing need for this kind of knowledge in the new generation of engineers. The current challenges to the engineers and researches today are to make use of probabilistic methods in practice and to improve investigations and analyses so that each additional data point provides maximal information.

This chapter focus on helping students understand some of the basic concepts involved in reliability analysis applied to geotechnical engineering. Another goal is to compare deterministic and probabilistic analysis of slopes. In deterministic analysis, this uncertainty can be accounted for by applying judgment in using a factor of safety consistent with the variability/uncertainty in the data. That is, a high factor of safety would be used where the values of the parameters are not well known. In probabilistic approach this uncertainty is already being addressed in the analysis, since materials and parameters are defined in terms of means and variances. The use of probabilistic methods does not eliminate the problem with uncertainties, instead it provides a method that not ignores the fact that a result is uncertain, giving a consistent working method that deals with the uncertainties.

3.2 Literature Review

3.2.1 Basic Statistics

This section introduces basic theory related to statistics and probabilistic calculations.

Event Probability

The probability of a certain event to occur is by definition between zero and one. Probability of 0 means that the event will not happen, and probability of 1 means that it will happen. Probability can also be expressed in percentage (0% to 100%). The equation for probability is presented in the following

$$0 \leq P[A] \leq 1 \quad (3.1)$$

Where $P[A]$ denotes the probability of an event A .

The relation between two events can be described by the Venn-diagram. Figure 3.1 shows when two events are added.

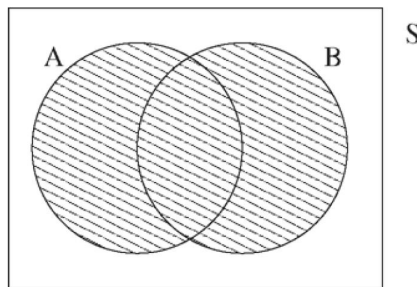


Fig. 3.1. Venn diagram illustrating the union $A \cup B$

The equation for addition of two events A and B is:

$$P[A \cup B] = P[A] + P[B] - P[A \cap B] \quad (3.2)$$

When one event influences the other, there is a conditional probability defined by equation:

$$[A | B] = \frac{P[A \cap B]}{P[B]} \quad (3.3)$$

Random Variable

Random variable is used to identify events so they can be treated numerical in calculations. Since most engineering problems are expressed in terms of numerical quantities, random variables are particularly appropriate.

Distributions

Several numbers of probability distributions are suitable to use when describing a geotechnical parameter. Which one to choose depends on the specific parameter and its nature. Two types of distributions often used in geotechnical engineering are described in the following

Normal Distribution

The normal distribution is the most used distribution [Fig.3.2]. It is also referred to as Gaussian distribution. The normal distribution is largely used today because sums of random variables tend to a normal distribution. This is proven by the central limit theorem [33].

Lognormal Distribution

The Lognormal distribution is a normal distribution in which the logarithm of the random variable is considered. A random variable which is log-normally distributed takes only positive real values (Fig.3.3). This is good for engineering problems which negative values are not expected, such as loads or soil cohesion.

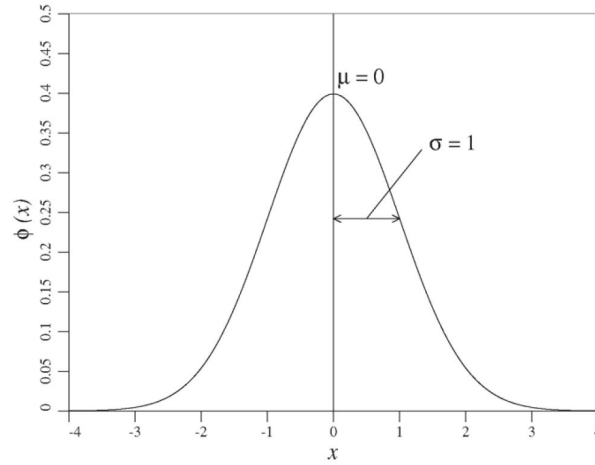


Fig. 3.2. Normal distribution

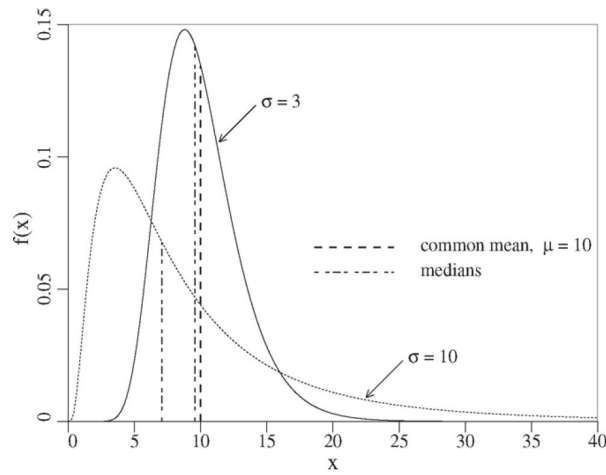


Fig. 3.3. Lognormal distribution

3.2.2 Uncertainty in Soil Properties

The input parameters to an analysis have to be collected from investigations, measurements and evaluations. This fact leads to various sources of uncertainties. Since uncertainty is unavoidable in engineering, it is important to know how to incorporate it adequately. Geotechnical engineers, like engineers in other disciplines, have developed several strategies for dealing with uncertainty. Some of them are described in the following [32]:

- Ignoring it.
- Being conservative. Rather than get involved in the details of how often undesirable things might happen and what their consequences might be, the engineer makes the structure or system so robust that it will resist anything.
- Using the observational method. The observational method has established itself as the preferred way for geotechnical engineers to deal with uncertainty in situations for which simple conservatism is unsatisfactory.
- Quantifying uncertainty. This is the purpose of reliability approaches.

Geo-materials (i.e. soils and rocks) are natural materials whose properties are affected by geological processes, such as weathering, erosion, and sedimentation [32]. The properties of geo-materials therefore exhibit natural variability and uncertainty. The uncertainty of geotechnical material parameters consists of the major effort in the estimation of statistical parameters of load and resisting forces to be considered in the reliability analysis.

Nadim [34] divided the uncertainties associated with a geotechnical problem into two categories, aleatory and epistemic uncertainties. Aleatory uncertainties are the natural randomness that is in a parameter, therefore they cannot be eliminated or reduced. Epistemic uncertainty is related to the knowledge of a parameter, therefore they are affected by measurement and model being considered. A more detailed classification is proposed by Christian et al. [35], which classified the uncertainty associated with soil properties into four categories: spatial soil variability (inherent soil variability), measurement error, statistical error in the mean and bias in measurement procedures (Fig.3.4).

The input to any reliability analysis includes descriptions of the relevant parameters describing physical properties, loads, and geometry and of their uncertainties. Usually these are in the form of means and variances or standard deviations or probabilities of occurrence. In probability and statistics, estimations of reasonable statistics and probability distributions of geotechnical properties require a large amount

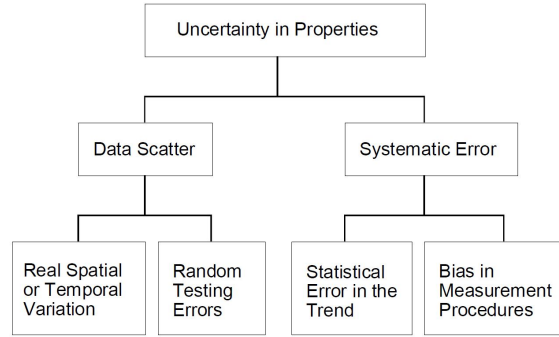


Fig. 3.4. Conceptual separation of uncertainty into its components for geotechnical applications [35].

of observation data from in situ (field) and/or laboratory tests. This estimate of geotechnical properties is presented in the next section.

3.2.3 Standard Deviations and Coefficients of Variation

If several tests are performed to measure a soil property, it will usually be found that there is scatter in the values measured. Standard deviation can be used to characterize such scatter. The greater the scatter, the larger the standard deviation. If a sufficient number of measurements have been made, the standard deviation can be computed using the formula [3]:

$$\sigma = \sqrt{\frac{1}{N-1} \sum_1^N (x - x_{av})^2} \quad (3.4)$$

where σ is the standard deviation, N is the number of measurements, x is a value of the measured variable, and x_{av} is the average value of x .

The coefficient of variation is the standard deviation divided by the expected value of a variable, which for practical purposes can be taken as the average:

$$COV = \frac{\sigma}{\mu} \quad (3.5)$$

where COV is the coefficient of variation and μ is the mean.

It is not possible to have a large number of measurements for every situation being studied. When this lack of data occurs, standard deviation and coefficient of variation cannot be calculated and must be estimated. Experience and judgement need to be used to estimate adequate values of standard deviation and coefficient of variation. Values of COV for various soil properties and in situ tests are shown in Table 3.1. These values may be of some use in estimating COVs for reliability analyses, but the values cover wide ranges, and this table provides only rough estimates of COV.

Table 3.1.
COV for Geotechnical Properties and in Situ Tests [3]

Property or in Situ Test	COV (%)	References
Unit weight (γ)	3 - 7	Harr (1987), Kulhawy (1992)
Buoyant unit weight (γ_b)	0 - 10	Lacasse and Nadim (1997), Duncan (2000)
Effective stress friction angle (ϕ')	2 - 13	Harr (1987), Kulhawy (1992), Duncan (2000)
Undrained shear strength (s_u)	13 - 40	Kulhawy (1992), Harr (1987), Lacasse and Nadim (1997)
Undrained strength ratio (s_u / σ'_v)	5 - 15	Lacasse and Nadim (1997), Duncan (2000)
Standard penetration test blow count (N)	15 - 45	Harr (1987), Kulhawy (1992)
Electric cone penetration test (q_c)	5 - 15	Kulhawy (1992)
Mechanical cone penetration test (q_c)	15 - 37	Harr (1987), Kulhawy (1992)
Dilatometer test tip resistance (q_D)	5 - 15	Kulhawy (1992)
Vane shear test undrained strength	10 - 20	Kulhawy (1992)

3.2.4 Reliability Analysis

Reliability-based analysis of slopes consists of consider the uncertainty of parameters, such as soil properties, in the calculations. The factor or margin of safety is therefore represented in a form of probability density function (PDF). Probability of failure is calculated from this PDF. For evaluation if this probability is acceptable, target levels of reliability need to be established. This task involves evaluation of the potential consequences of failure and the required investment. This target probabilities of failure usually are based on historical estimates of reliability for common civil engineering structures.

Christian [32] enumerates at least three categories of tools which can be considered in reliability analysis:

1. Direct reliability analysis;
2. Event trees, fault trees and influence diagrams;
3. Other statistical techniques.

This work considers only the first category. This approach consists of consider the uncertainties in properties, geometries, loads, water levels, etc to create an analytical model to obtain probabilistic descriptions of the behavior of a structure of system. The main goal is to find a probability of failure for the situation being studied and check if this probability is acceptable for the type and importance of the case.

In direct reliability analysis, there are at least two ways to represent the failure for some situation in terms of the available shear strength (R) and the shear stress required to support the load of gravity (Q). First is considering the factor of safety (FS) as

$$FS = R/Q \quad (3.6)$$

Another way is to consider the margin of safety as

$$M = R - Q \quad (3.7)$$

In the case where factor of safety is being considered, failure occurs when $FS \leq 1.0$. For margin of safety, failure occurs when $M \leq 0$. Therefore, the probability of failure (P_f) can be calculated both in terms of FS and M. Fig.3.5 shows the probability of failure obtained for margin of safety distribution, which is the area under the pdf lying to the left of the origin.

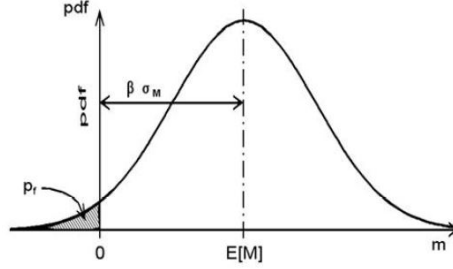


Fig. 3.5. PDF of margin of safety [32].

Another important parameter used is the reliability index β . When Q and R are normally distributed, the margin of safety is also normally distributed and the reliability index is calculated using the following equations [36]:

In terms of M :

$$\beta = \frac{\mu_M}{\sigma_M} \quad (3.8)$$

In terms of R and Q :

$$\beta = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_M^2 + \sigma_Q^2}} \quad (3.9)$$

When Q and R are log-normally distributed, the margin of safety is also log-normally distributed and the reliability index is calculated using the following equation:

$$\beta = \frac{\ln \left[\frac{\mu_R}{\mu_Q} \sqrt{\frac{1 + COV_Q^2}{1 + COV_R^2}} \right]}{\sqrt{\ln [(1 + COV_Q^2)(1 + COV_R^2)]}} \quad (3.10)$$

3.2.5 Computational Methods

Computational methods are used with the objective of compute β and P_f . They can be applied after the computational model, such as a method-of-slices slope stability program, and the statistical description of the material properties have been established. Four approaches are briefly described by Christian [36] in the following:

First Order Second Moment (FOSM)

The FOSM method starts by computing FS or M using the best estimates of the parameters and then employs a Taylor series expansion to estimate the variance of FS or M. It then computes β and P_f . Its advantages are that it is easy to use, requires relatively little computation, is easily programmed in table form, and reveals the relative contributions of the individual parameters. Its disadvantages are that the results can be sensitive to the starting values and for non-linear computational models the results can be misleading.

First Order Reliability Method (FORM)

FORM starts with the best estimates of the parameters and iterates to find the shortest distance from the dimensionless point representing the best estimates of the parameters to the dimensionless failure surface. Its advantages are that the results are independent of the starting point and represent a consistent definition of β and P_f . Its disadvantages are that it is computationally more difficult than FOSM and can be hard to implement. It is also difficult to calculate the probability of failure if the failure surface is curved. The Second Order Reliability Method (SORM) is an improvement to incorporate higher order failure terms.

Point Estimate Method (P-E)

These methods assume that the joint probability distributions of the parameters can be represented by a simpler surface, which can then be used to evaluate the parameters of FS or M. The P-E method can be shown to be a form of Gaussian quadrature. The advantages of these methods are their computational simplicity combined with accuracy for most situations. Their disadvantages are that they can become unwieldy when the number of variables is large and obtaining insight into the contributions of the individual parameters is more difficult than in FOSM or FORM.

Monte Carlo (MC) Simulation

MC simulation uses a random number generator to create large set of values of the uncertain parameters according to their prescribed probabilistic distributions and then computes the statistics of FS or M from the results. By using a random number generator all numbers have the same probability. Its major advantages are that it is relatively simple to implement and can be used for almost any model or parameters. Its disadvantages are that it can require a great deal of computation, it does not give much insight into the contributions of the individual parameters, and the subtleties of random number generation may lead to unexpected difficulties and errors. Most applications use some form of variance reduction scheme to reduce the computational effort.

3.2.6 Interpreting Results of Reliability Analysis

The last step in the probability analysis is to understand what the results mean. It is important to know how to evaluate the probability of failure, if it is acceptable for the situation being studied. It is important also to understand the effect of each parameter and their uncertainties in the results. Several studies provide charts and

tables suggesting the probability of failure to be considered for different situations [37], [38].

One way to represent the reliability of a slope is in terms of the expected performance level. US Army Corps of Engineers [37] provide a table showing the probability of failure for different performance levels, from hazardous to high level (Table 3.2). Another study presented by Whitman [38] gives a relationship between levels of annual probability of failure and the consequence of failure for different structures (Fig.3.6). In recent years, the $f-N$ and $F-N$ diagrams have proven to be useful tools for describing the meaning of probabilities and risks in the context of other risks with which society is familiar (Fig.3.7 and 3.8).

Table 3.2.
Relationship between β and P_f [37].

Reliability index	Probability of failure	Expected performance level
1.0	0.16	Hazardous
1.5	0.07	Unsatisfactory
2.0	0.023	Poor
2.5	0.006	Below average
3.0	0.001	Above average
4.0	0.00003	Good
5.0	0.0000003	High

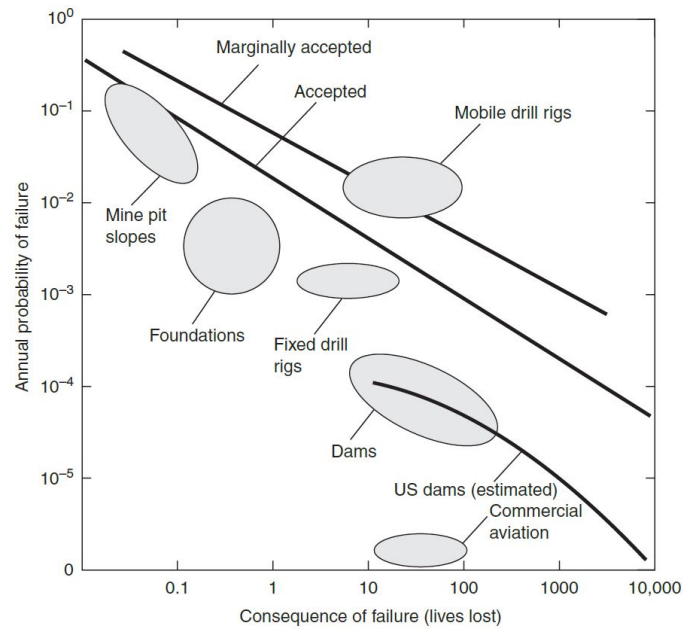


Fig. 3.6. Risks for selected engineering projects [38].

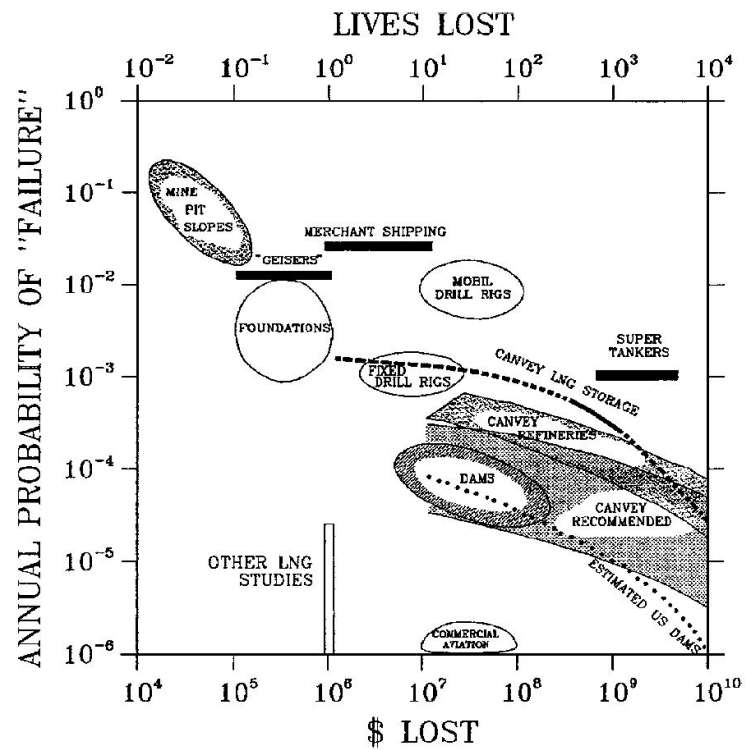


Fig. 3.7. One version of f-N plot annual risk cost or number of lives. In this plot both cost and lives are shown; it is customary to use one or the other rather than both on same plot [32].

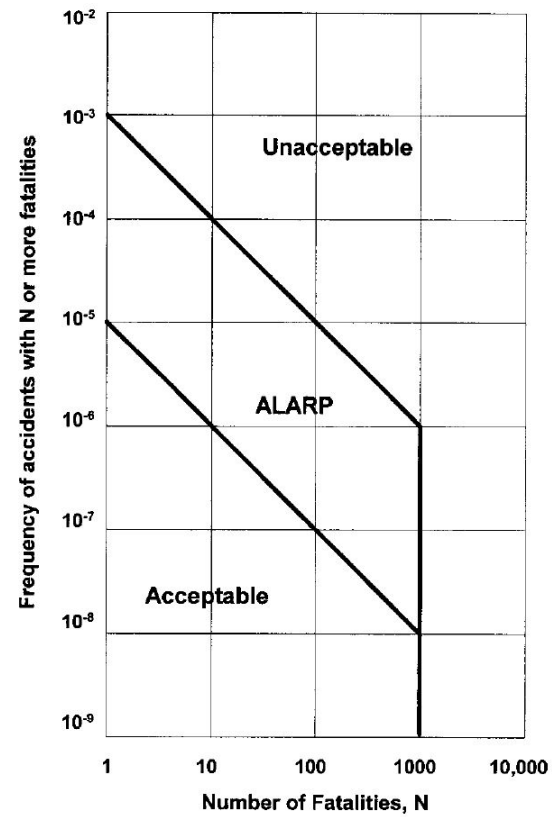


Fig. 3.8. F–N diagram adopted by Hong Kong Planning Department for planning purposes [32].

3.3 Development of PNW-SLOPE - Probabilistic

3.3.1 Probability Tools in Excel

Microsoft Excel offers some resources for probabilistic evaluation. The three functions listed below can be used as a VBA function in Excel.

- The Microsoft Excel RND function returns a random number that is greater than or equal to 0 and less than 1.
- The NORM.DIST function is one of the statistical functions. It is used to return the normal distribution for the specified mean and standard deviation.

The NORM.DIST function syntax is:

NORM.DIST(x , mean , standard-deviation , cumulative-flag)

where

x is the value you want to calculate the distribution for, any numeric value

mean is the arithmetic mean of the distribution, any numeric value

standard-deviation is the standard deviation of the distribution, a numeric value greater than 0

cumulative-flag is the form of the function, a logical value: TRUE or FALSE. If cumulative-flag is TRUE, the function will return the cumulative distribution function; if FALSE, it will return the probability mass function

- The LOGNORM.DIST function works similar to NORM.DIST, but lognormal distribution is considered instead of normal. This function returns the lognormal distribution of x, where $\ln(x)$ is normally distributed with parameters mean and standard deviation.

3.3.2 Userforms

The probabilistic module of PNW-SLOPE is described in this section. Fig.3.9 shows the screen for definition of the probabilistic parameters. It is possible to choose which parameters will be considered in the analysis in terms of probabilistic distributions. After the analysis is complete, the results can be visualized in the same screen.

Probabilistic Parameters

Units

☒ Metric ☐ Imperial

Coordinates: Metric: m Imperial: ft
Cohesion: kN/m² psf
Unit weight: kN/m³ pcf
Surcharge: kN/m lb/ft

Step 3: Analysis

Method: OMS ☐ Spatial variability
Iterations: 5000

Step 1: Define Probabilistic Properties for Geometry

☒ Y (top) ☐ Y (bot)
Det Det

Layer	Y (top) Det	Y (bot) Det
Layer 1	5	0
Layer 2	0	-5

Geometry

Factor of safety (Deterministic) 1.043

Step 2: Define Probabilistic Properties for Materials

☐ SD ☒ COV

☒ Cohesion ☒ Fric Ang ☒ Unit Weight

	Mean	SD	COV	Dist	
				Max	Min
Layer 1	10	1.000	0.1	20	0
Layer 2	8	0.800	0.1	20	0

	Mean	SD	COV	Dist	
				Max	Min
Layer 1	10	0.500	0.05	20	0
Layer 2	5	0.250	0.05	20	0

	Mean	SD	COV	Dist	
				Max	Min
Layer 1	17.64	0.176	0.01	19	17
Layer 2	18	0.180	0.01	19	17

Results

Probability of Failure (%) 21.2
Reliability Index (Normal) 0.802
Reliability Index (Lognormal) 0.787
Factor of safety (Mean) 1.043
Factor of safety (Max) 1.257
Factor of safety (Min) 0.822

References

Fig. 3.9. PNW-SLOPE - Probabilistic module

The convergence of Monte Carlo iterations can be generated by clicking on 'Convergence' button (Fig.3.10).

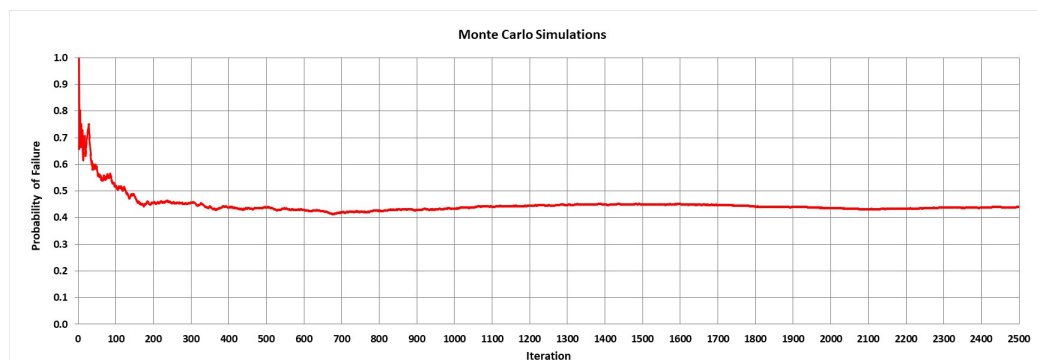


Fig. 3.10. PNW-SLOPE - Monte Carlo convergence

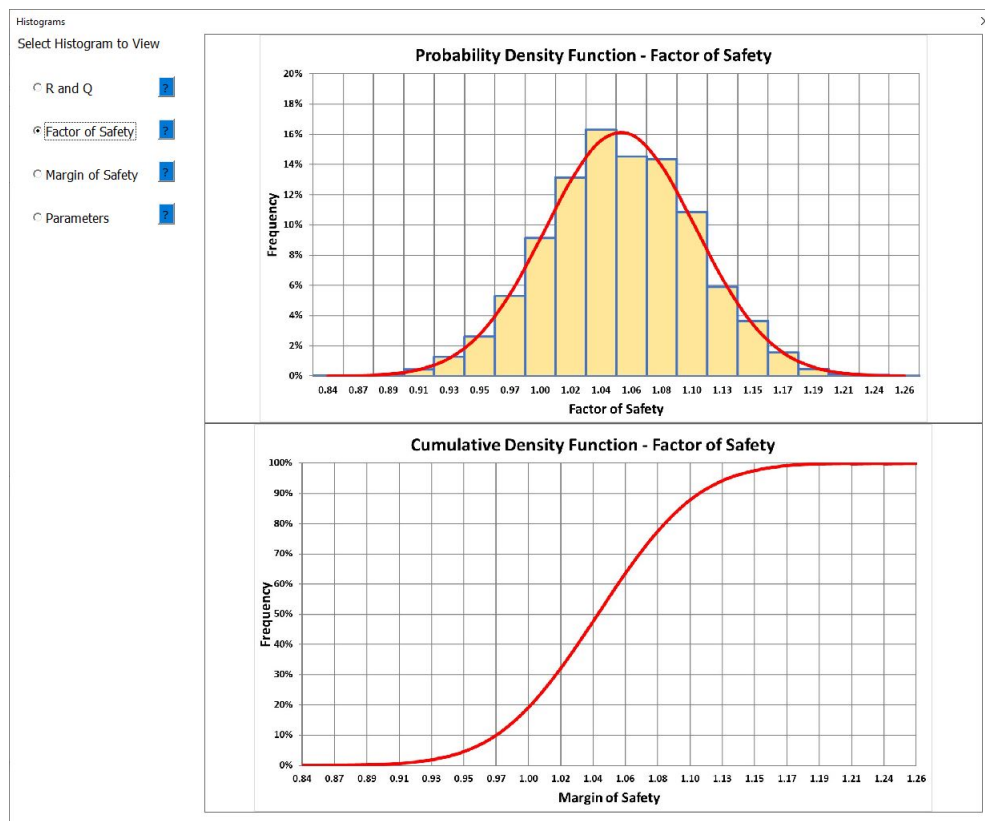


Fig. 3.11. PNW-SLOPE - Select Histograms

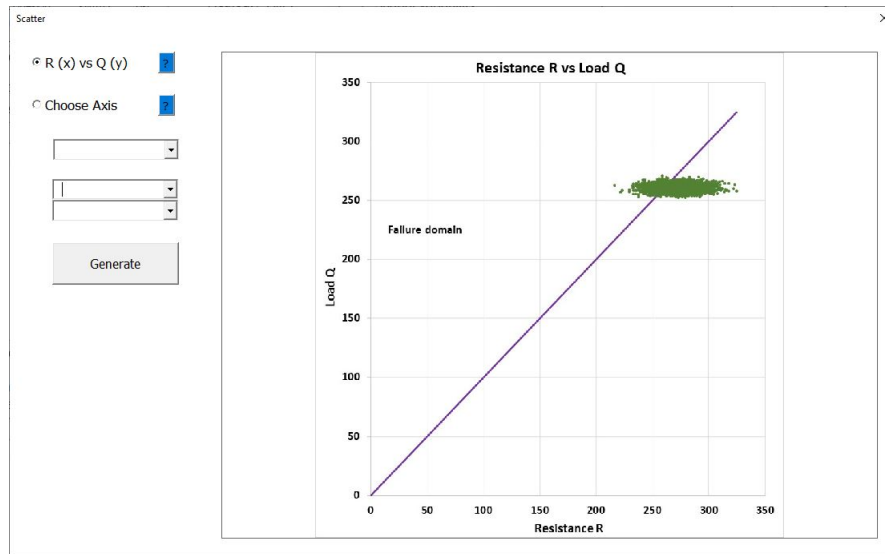


Fig. 3.12. PNW-SLOPE - Generate Scatter Chart - R vs Q

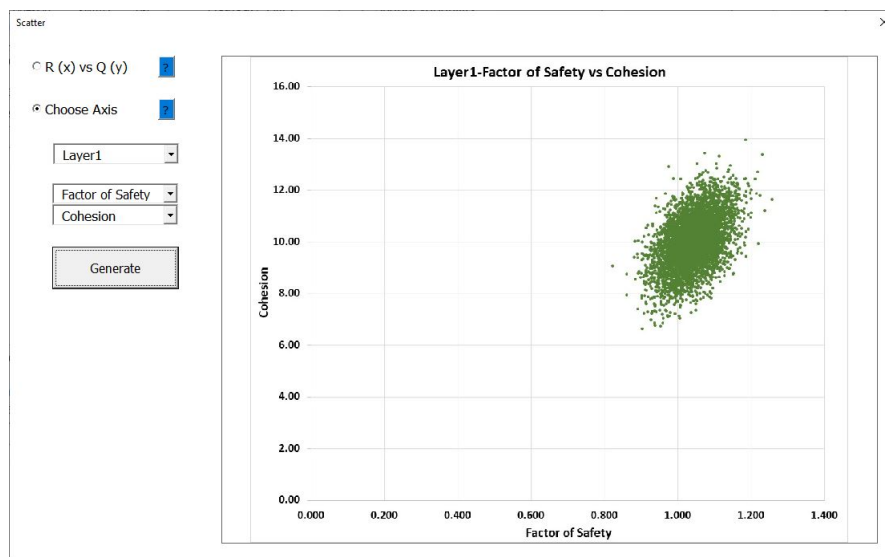


Fig. 3.13. PNW-SLOPE - Generate Scatter Chart - FS vs Cohesion

Additional results can be generated by clicking on the 'Histogram' and 'Scatter Charts' buttons (Figs.3.11-3.13).

3.3.3 Flowcharts

Flowchart in Fig.3.14 presents the step by step of input data by the user in the probabilistic module.

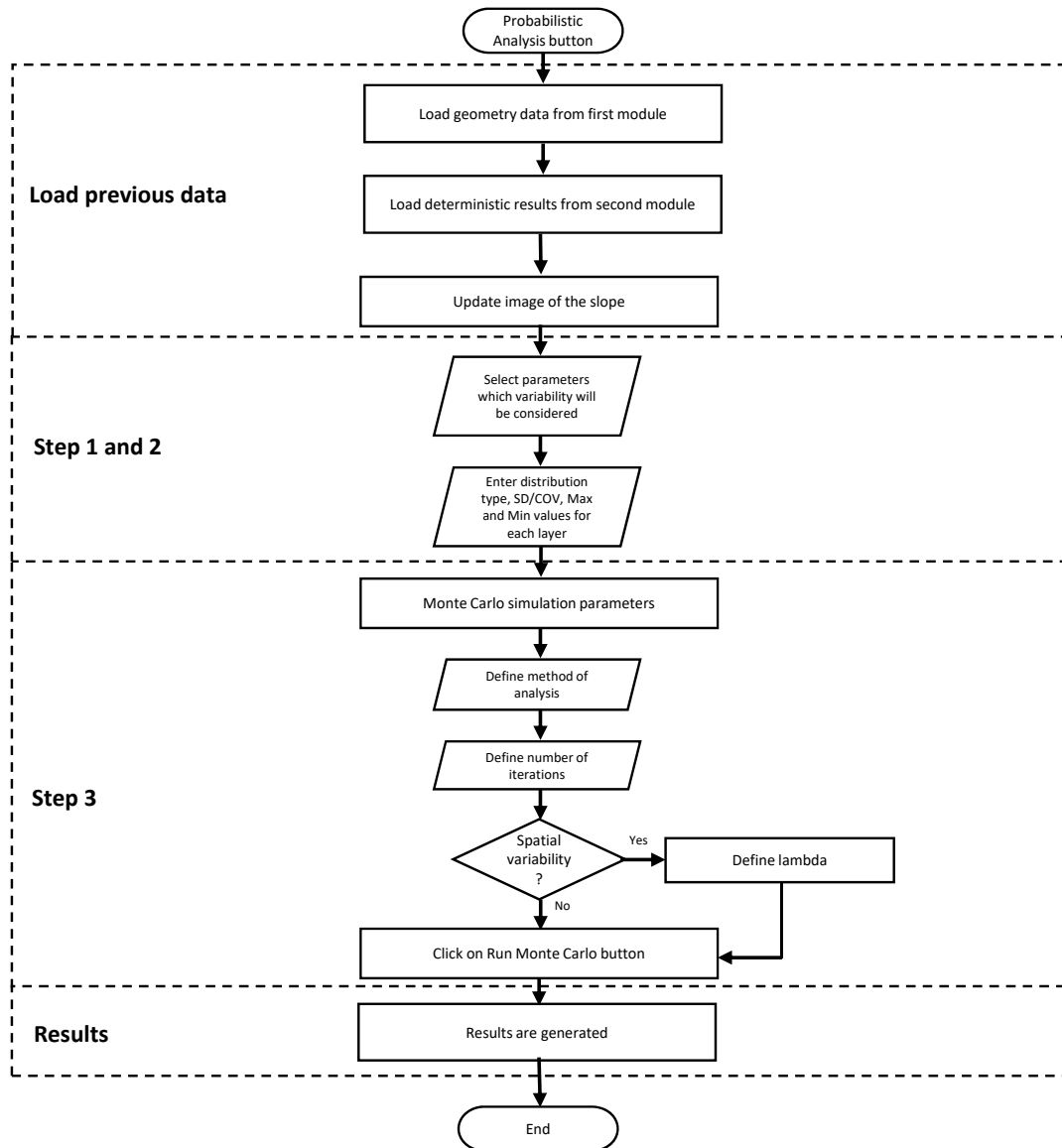


Fig. 3.14. Flowchart for probabilistic parameters

3.4 Illustrative Examples

In this section, four examples are given to illustrate the use of PNW-SLOPE considering probabilistic analysis. The complete reports created by PNW-SLOPE are presented in Appendix A. The results are validated with the commercial software SLIDE 2018 from Rocscience [39].

3.4.1 Example 3.1 - Heterogeneous soil with different COV

This example is composed by a heterogeneous slope with two different layers. Cohesion, internal friction angle and unit weight are represented in terms of their means and coefficient of variation. This problem was adapted from Malkawi [40] and is shown in Fig.3.15. The goal is to compare the results when different uncertainty levels are considered. The coefficient of variation (COV) is equally incremented between the four situations being studied (Tables 3.3 and 3.4). Normal distribution is considered for all parameters.

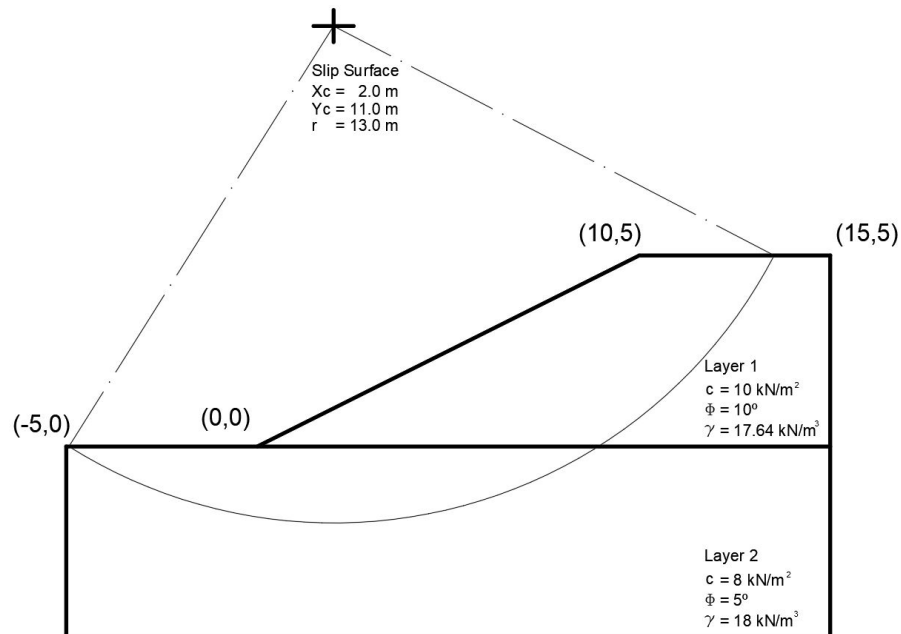


Fig. 3.15. Example 3.1 - Geometry

Table 3.3.
Example 3.1 - Parameters Layer 1

Soil parameter	Mean	Max	Min	Coefficient of Variation			
				a	b	c	d
c (kN/m ²)	10.00	20.00	0.00	0.10	0.20	0.30	0.40
Φ' (°)	10.00	20.00	0.00	0.05	0.10	0.15	0.20
γ (kN/m ³)	17.64	19.00	17.00	0.01	0.02	0.03	0.04

Table 3.4.
Example 3.1 - Parameters Layer 2

Soil parameter	Mean	Max	Min	Coefficient of Variation			
				a	b	c	d
c (kN/m ²)	8.00	20.00	0.00	0.10	0.20	0.30	0.40
Φ' (°)	5.00	20.00	0.00	0.05	0.10	0.15	0.20
γ (kN/m ³)	18.00	19.00	17.00	0.01	0.02	0.03	0.04

Slopes analyzed with probabilistic approach must be defined with estimate of the parameters. In this case the coefficient of variation (COV) is considered, but this problem could also be described in terms of the standard deviations (SD). The relation between COV and SD was described before by Eq.3.5.

3.4.2 Example 3.1 - Results

The results for probability of failure (P_f) and reliability index (β) are presented for each case (Table 3.5). Both parameters can indicate how reliable is the situation being studied. OMS was considered for deterministic analysis (Fig.3.16). The number of 5000 Monte Carlo samples was considered both in PNW-SLOPE and SLIDE 2018.

Table 3.5.
Example 3.1 - Results

Parameter	Program	a	b	c	d
Factor of Safety	PNW-SLOPE	1.043			
	SLIDE	1.046			
Probability of Failure (%)	PNW-SLOPE	21.20	35.54	39.58	43.08
	SLIDE	19.72	34.10	39.26	42.82
Reliability Index β (Normal)	PNW-SLOPE	0.802	0.363	0.244	0.187
	SLIDE	0.866	0.423	0.273	0.204
Reliability Index β (Lognormal)	PNW-SLOPE	0.787	0.319	0.175	0.097
	SLIDE	0.861	0.382	0.205	0.111

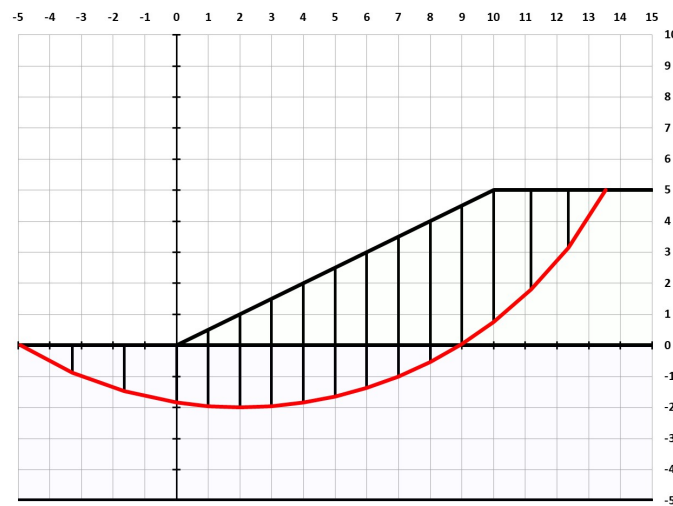


Fig. 3.16. Example 3.1 - Slices considered

The results indicated an increase in probability of failure when COV was proportionally incremented. The reason for this tendency was the growth in variability of the factor of safety when each parameter is varying more. This fact leads to an increase in the area of probability distribution with factor of safety lower than 1. Figs.3.17-3.18 clearly show this fact.

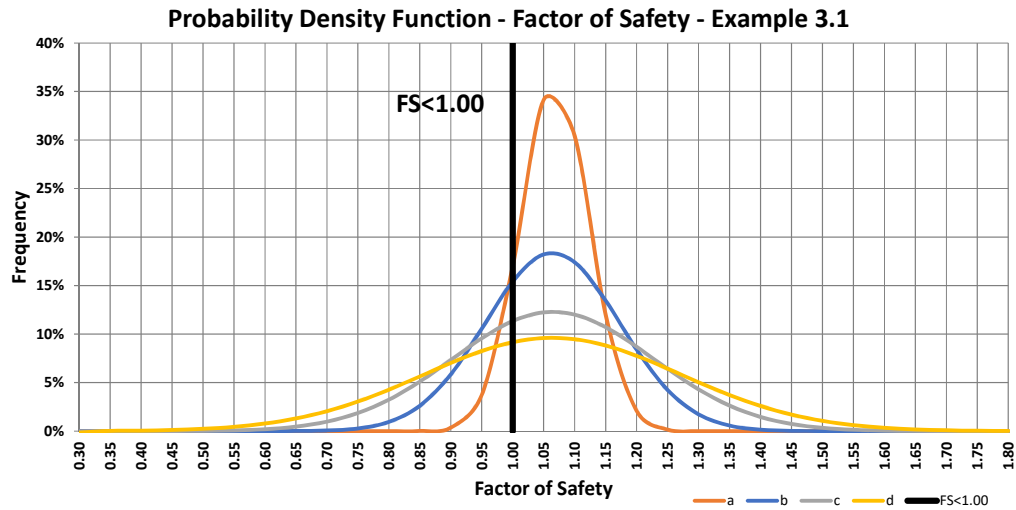


Fig. 3.17. Example 3.1 - PDF of FS - Cases a, b, c, d

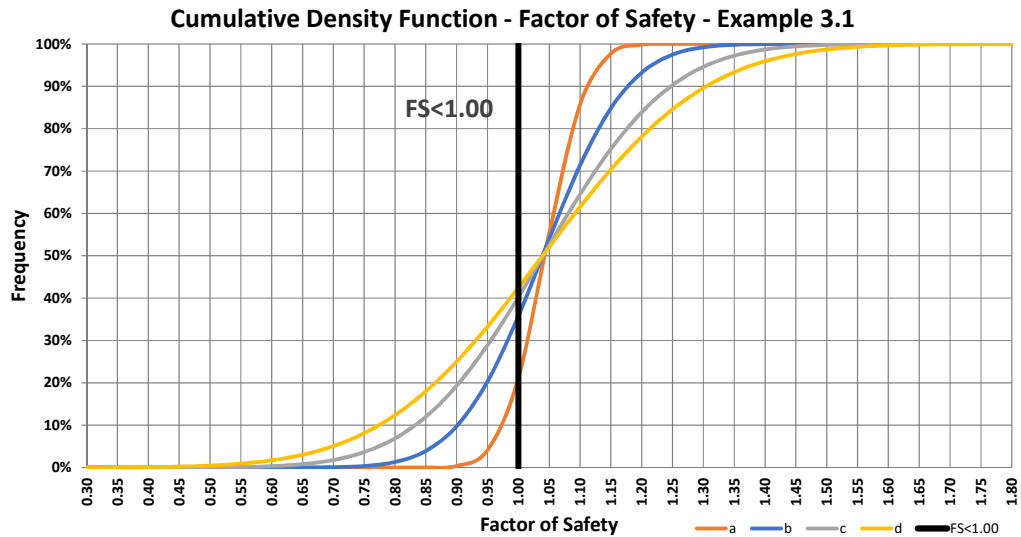


Fig. 3.18. Example 3.1 - CDF of FS - Cases a, b, c, d

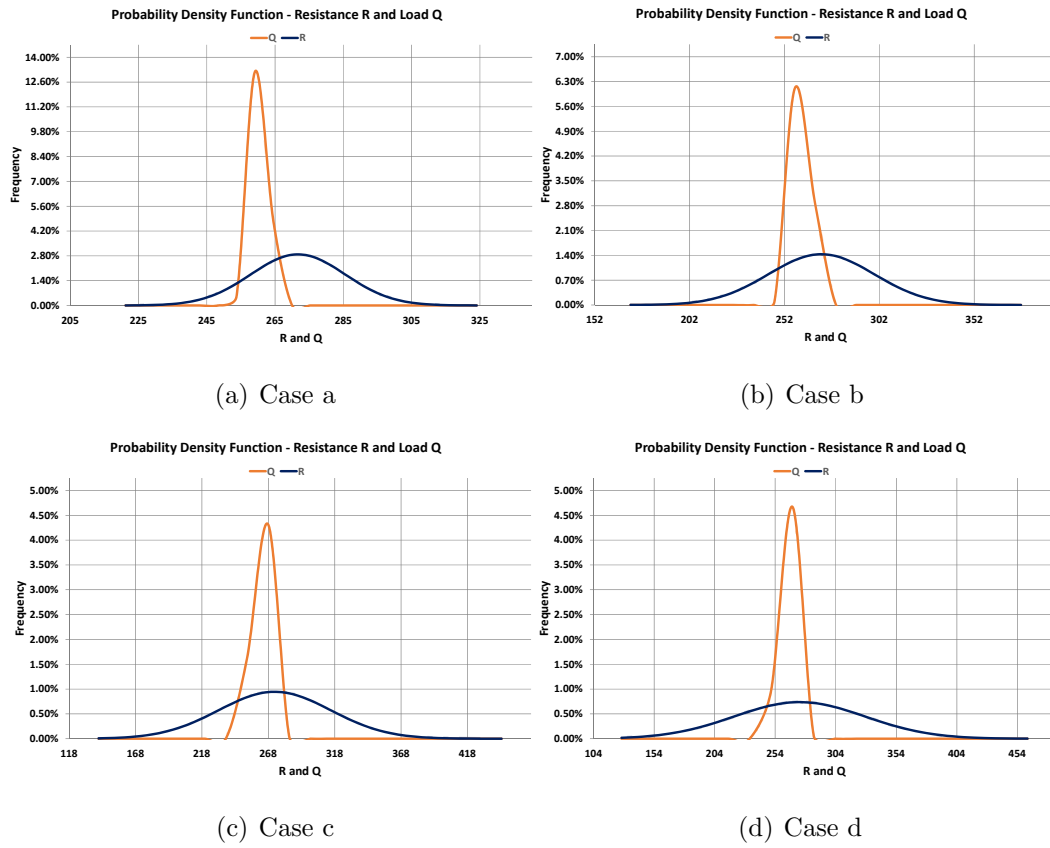


Fig. 3.19. Example 3.1 - PDF of R and Q

3.4.3 Example 3.2 - Homogeneous soil varying parameters

This example is composed of a short homogeneous slope. The objective is to study the relationship between the factor of safety and the variability of the soil parameters. Each parameter is varying independently about its respective mean value, while the other properties are fixed at their deterministic values. For example, when cohesion is considered in terms of mean and SD, internal friction angle and unit weight are represented only by a single deterministic value. This problem was adapted from Malkawi [40] and is shown in Fig.3.20. The statistic parameters are shown in Tab.3.6.

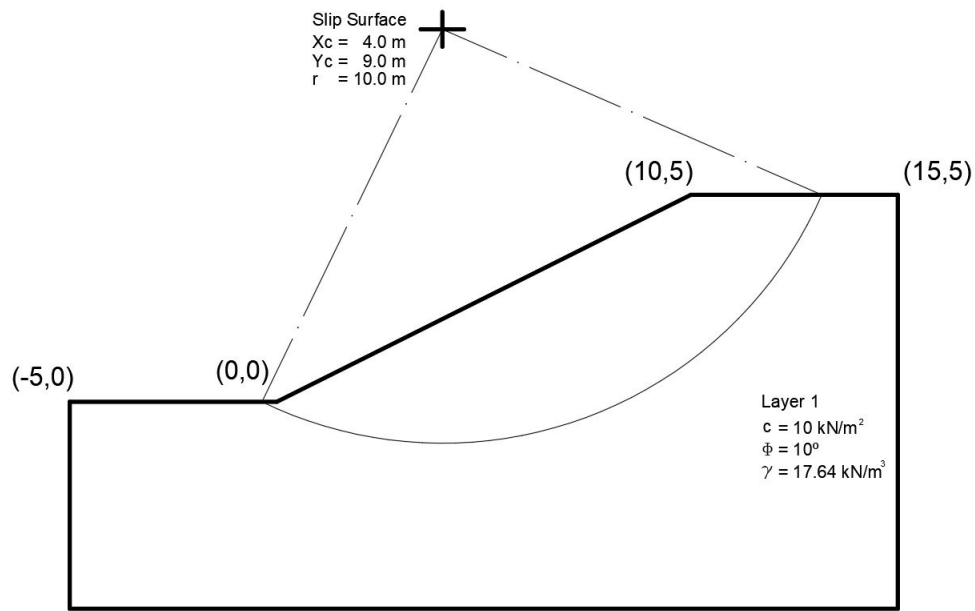


Fig. 3.20. Example 3.2 - Geometry

Table 3.6.
 Example 3.2 - Parameters

Soil parameter	Mean	Max	Min	CV
$c \text{ (kN/m}^2\text{)}$	10.0	20.0	0.0	0.40
$\Phi' \text{ (}^\circ\text{)}$	10.0	20.0	0.0	0.20
$\gamma \text{ (kN/m}^3\text{)}$	17.64	18.5	16.5	0.04

3.4.4 Example 3.2 - Results

The results for the four situations considered are presented in Table 3.7. In this example a homogeneous slope was analyzed with different assumptions for the parameters. The probability of failure was larger for the situations which cohesion was considered. The results obtained by PNW-SLOPE were approximately the same of SLIDE.

Table 3.7.
Example 3.2 - Results

Case	Program	Probability of failure (%)	Reliability index β (Normal)
a	PNW-SLOPE	15.52	1.033
	SLIDE	15.70	1.041
b	PNW-SLOPE	0.18	2.873
	SLIDE	0.12	2.960
c	PNW-SLOPE	0.00	22.454
	SLIDE	0.00	13.796
d	PNW-SLOPE	18.04	0.935
	SLIDE	17.84	0.792

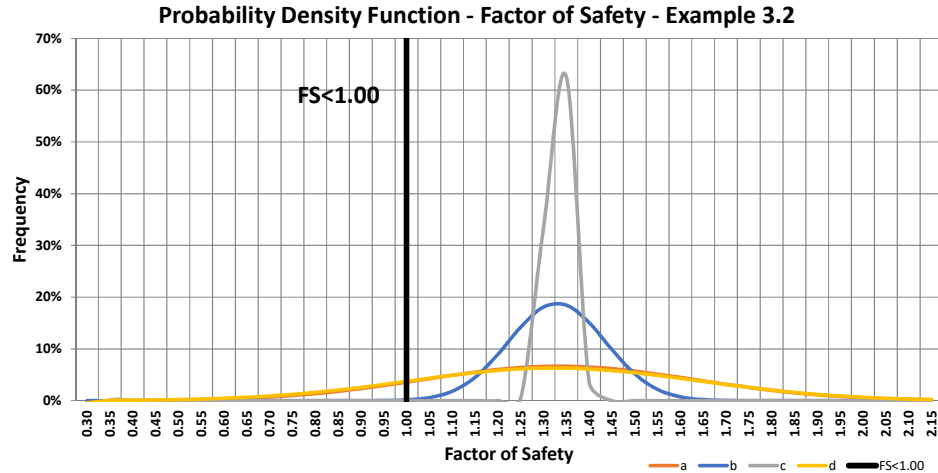


Fig. 3.21. Example 3.2 - PDF - Cases a, b, c, d

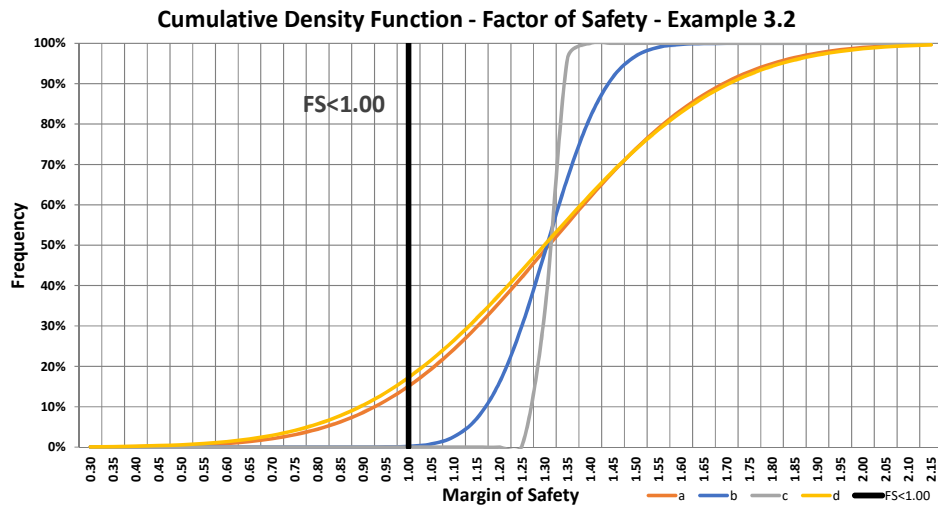


Fig. 3.22. Example 3.2 - CDF - Cases a, b, c, d

The probability density function charts clearly shows the effect of each parameter in the analysis (Figs.3.21-3.22). The situation with only unit weight considered had a smaller impact in the PDF of the factor of safety, with values varying from 1.27 to 1.36. On the other hand, the PDF for case a (only cohesion) was almost the same of the PDF for case d (All parameters), showing that cohesion is the parameter impacting more in the analysis for this situation.

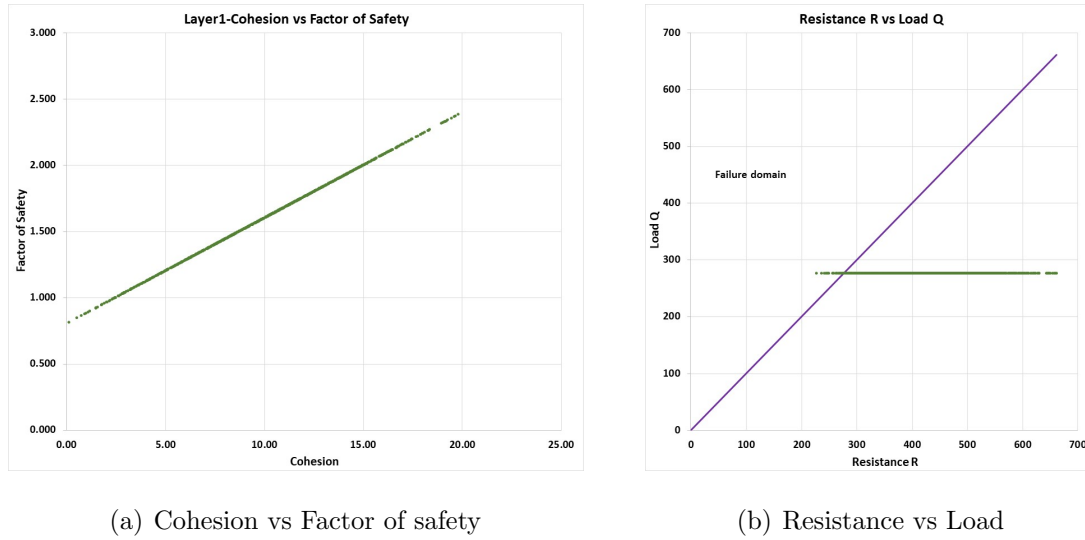
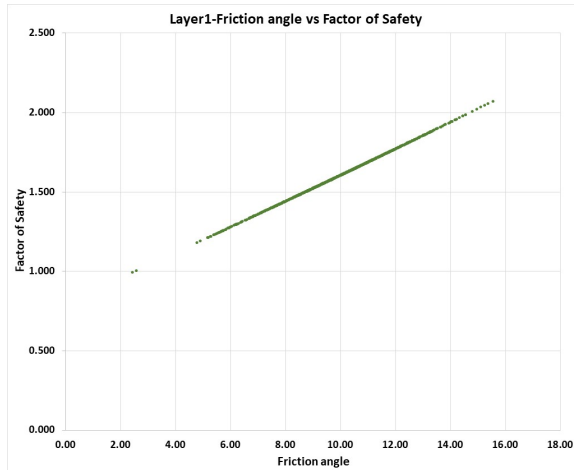
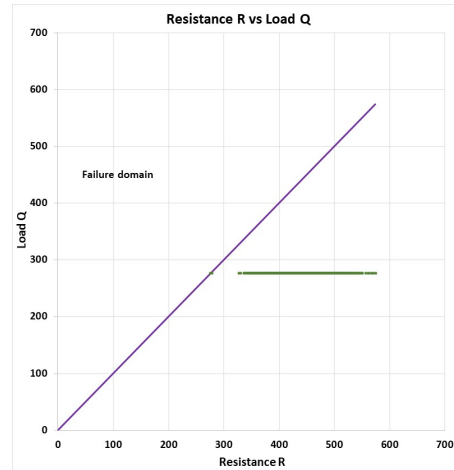


Fig. 3.23. Example 3.2 - Case a (Only cohesion)

The relationship between each parameter and the factor of safety for each case is presented in Figs.3.23-3.26. It is possible to see the linear relation between each parameter the factor of safety. This is happening because Ordinary Method of Slices was considered in the limit equilibrium method for each iteration, therefore equation Eq.2.7 was used. In this equation, the factor of safety is direct calculated, and each parameter is linearly proportional to it.

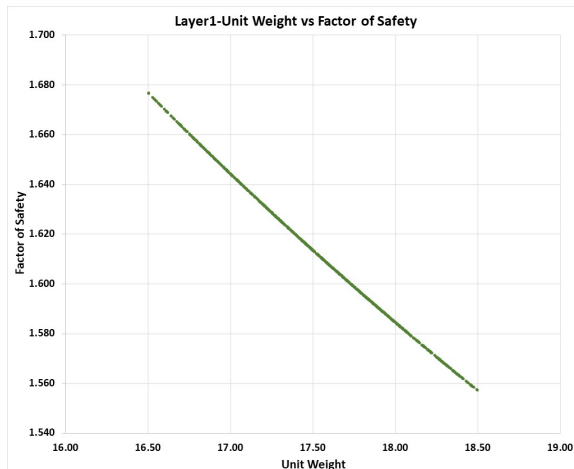


(a) Internal friction angle vs Factor of safety

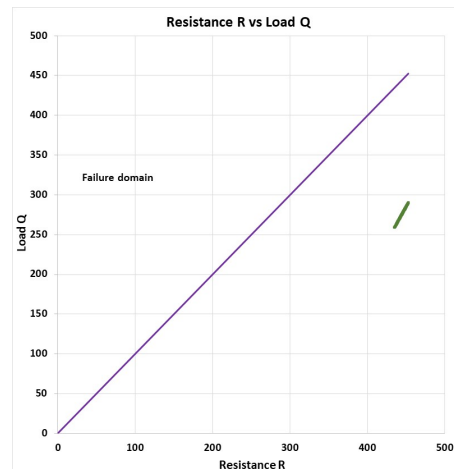


(b) Resistance vs Load

Fig. 3.24. Example 3.2 - Case b (Only internal friction angle)

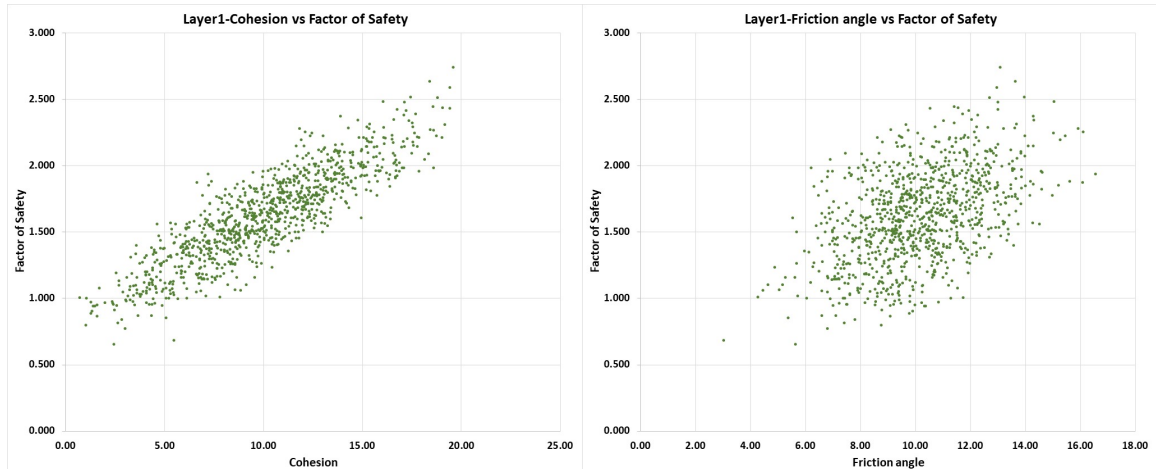


(a) Unit weight vs Factor of safety



(b) Resistance vs Load

Fig. 3.25. Example 3.2 - Case c (Only unit weight)



(a) Cohesion vs Factor of safety

(b) Internal friction angle vs Factor of safety



(c) Unit weight vs Factor of safety

(d) Resistance vs Load

Fig. 3.26. Example 3.2 - Case d (All parameters varying)

3.4.5 Example 3.3 - Cohesive soil – Spatial variation (Horizontal)

This example is composed of a short term analysis of homogeneous cohesive slope. The undrained shear strength, S_u , of the soil is modeled by a one-dimensional random field spatially varying along the horizontal direction. The geometry is presented in Fig.3.27. The objective of this example is to compare the results when spatial variability is considered in the horizontal direction. Unit weight is represented by its deterministic value of 20 kN/m^3 .

Table 3.8.
Example 3.3 - λ_h values considered for each case

Case	a	b	c	d	e	f
λ_h	1	2	3	4	8	∞

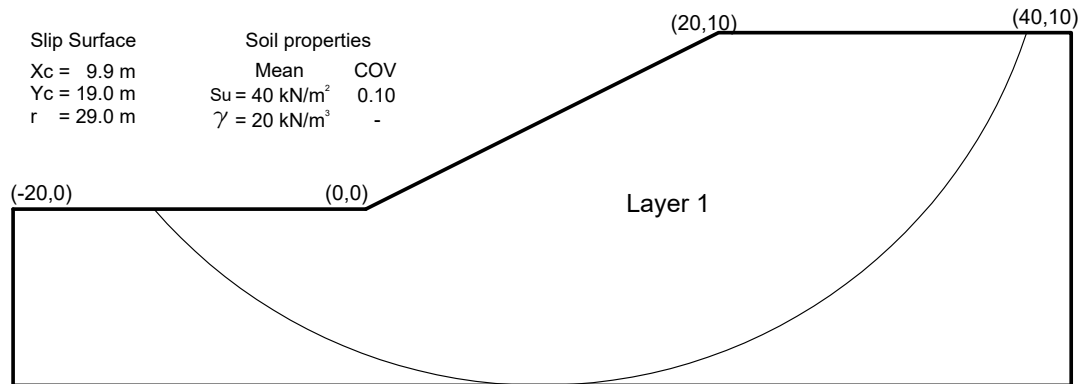
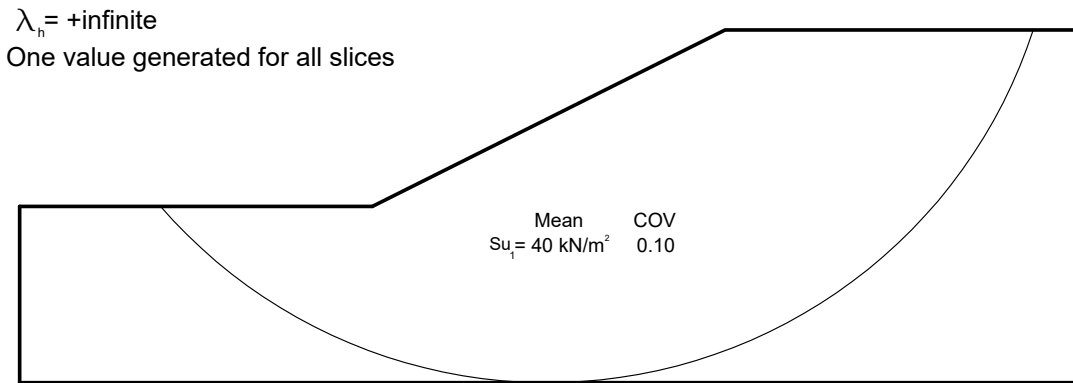


Fig. 3.27. Example 3.3 - Geometry

Fig. 3.28. Example 3.3 - $\lambda_h = 1$ Fig. 3.29. Example 3.3 - $\lambda_h = \infty$

In this case, λ_h is represented in terms of number of slices to be grouped and varies from 1 slice to $+\infty$. When $\lambda_h = 1$, every slice is represented by an independent value with identically distributed random variables (Fig.3.28). When $\lambda_h = \infty$, all slices are considered as a single random variable, since each slice is fully correlated with each other (Fig.3.29). Six cases are compared with different λ_h (Table 3.8).

3.4.6 Example 3.3 - Results

The results for the six situations analyzed are presented in Table 3.9.

Table 3.9.
Example 3.3 - Results

Case	λ_h	Program	Probability of failure (%)	Reliability index (Normal)
a	1	PNW-SLOPE	0.00	3.320
		SLIDE	0.02	3.362
b	2	PNW-SLOPE	1.20	2.186
		SLIDE	1.14	2.383
c	3	PNW-SLOPE	2.60	1.881
		SLIDE	2.28	2.002
d	4	PNW-SLOPE	5.40	1.678
		SLIDE	3.88	1.765
e	8	PNW-SLOPE	11.00	1.241
		SLIDE	9.48	1.306
f	∞	PNW-SLOPE	25.70	0.629
		SLIDE	25.60	0.649

The results clearly show a tendency of increase in the probability of failure when less random values are considered. In this example, factor of safety variance increases when the soil properties are characterized by a single random variable only (Case f: λ_h), leading to an overestimation of the probability of failure. When more field investigation and soil testing are available, and when more statistical data become available to justify the use of smaller lambda values, the probability of failure can be more accurately estimated.

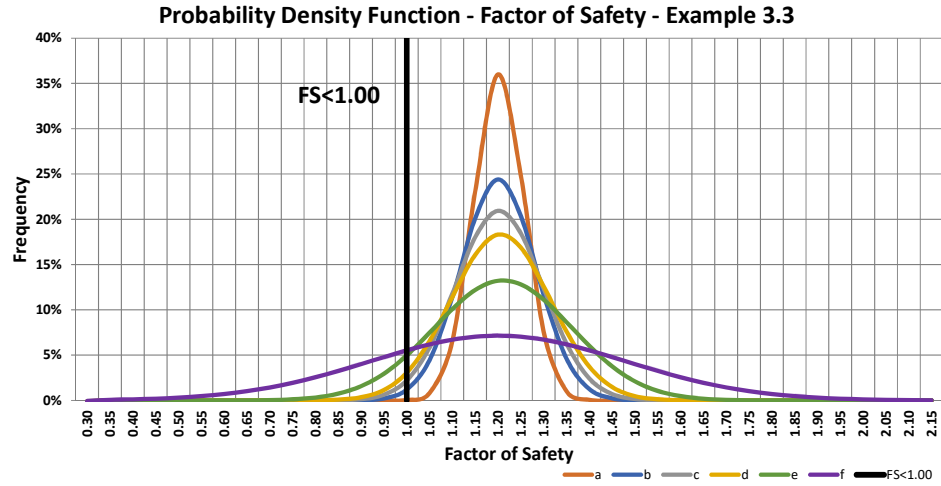


Fig. 3.30. Example 3.3 - PDF - Cases a, b, c, d, e, f

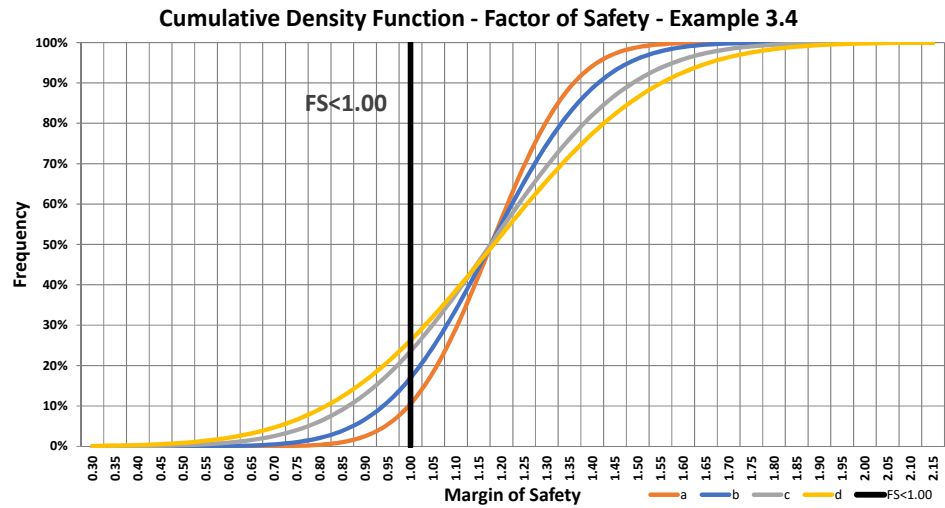


Fig. 3.31. Example 3.3 - CDF - Cases a, b, c, d, e, f

3.4.7 Example 3.4 - Cohesive soil – Spatial variation (Vertical)

This example is composed by the same geometry of Example 3.3, but now the undrained shear strength, S_u , of the soil is modeled by a one-dimensional random field spatially varying along the vertical direction. This problem was adapted from Cao [23]. The objective of this example is to compare the results when spatial variability is considered in the vertical direction. In this case, four situations are compared. In the first case, the slope geometry is divided in 8 layers. Each layer is considered uncorrelated, therefore they have independent values calculated from their probabilistic parameters. The four cases are presented in Fig.3.32. The probabilistic parameters for S_u are the same of Example 3.3. Unit weight is represented by its deterministic value of 20 kN/m³.

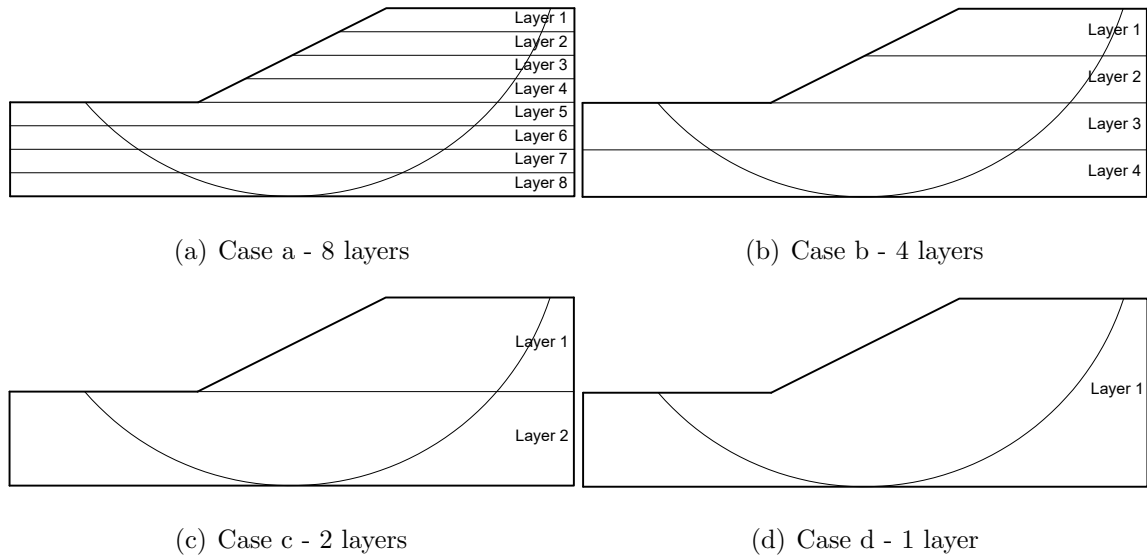


Fig. 3.32. Example 3.4 - Slope geometry at each case

3.4.8 Example 3.4 - Results

The results for the four situations analysed are presented in Table 3.10 and Fig.3.35.

Table 3.10.
Example 3.4 - Results

Program	Case	Probability of failure (%)	Reliability index (Normal)
a	PNW-SLOPE	10.46	1.253
	SLIDE	9.68	1.297
b	PNW-SLOPE	16.78	0.956
	SLIDE	17.00	0.969
c	PNW-SLOPE	24.00	0.722
	SLIDE	22.82	0.765
d	PNW-SLOPE	26.80	0.636
	SLIDE	25.66	0.659
	From Cao [23]	30.40	0.510

The results clearly show a tendency of increase in the probability of failure when less layers are considered. In this example, factor of safety variance increases when the soil properties are characterized by a single random variable only (Case d - no spacial variability), leading to an overestimation of the probability of failure. However it is important to note that overestimation of the FS variance may result in either over (conservative) or under (unconservative) estimation of P_f (i.e., probability of $FS < 1$). Griffiths and Fenton [41] reported that when FS is relatively low and the spatial variability is ignored by assuming perfect correlation, the value of P_f is underestimated and unconservative. The PDF and CDF of the four situations are presented in Figs.3.33-3.34. The scatter chart with resistance vs load forces is

presented in Fig.3.35. It is possible to see that from cases a to d there is an increase in the number points in the failure domain.

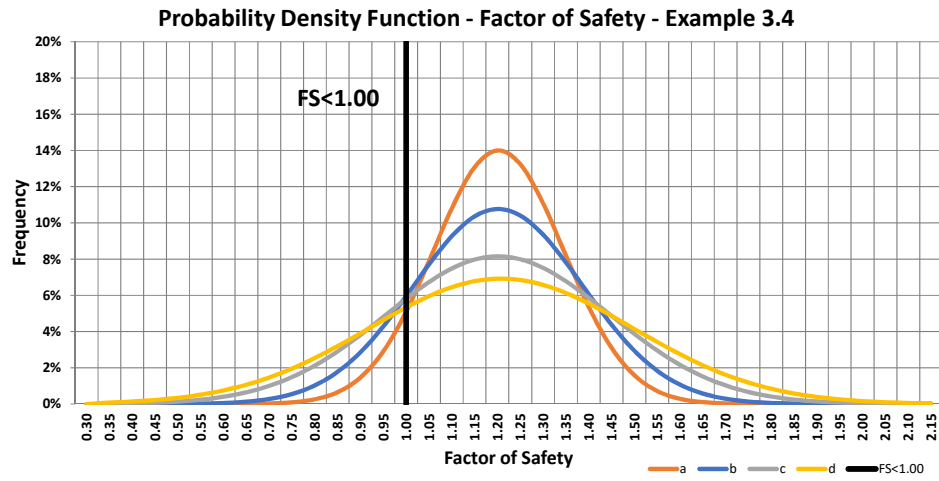


Fig. 3.33. Example 3.4 - PDF - Cases a, b, c, d

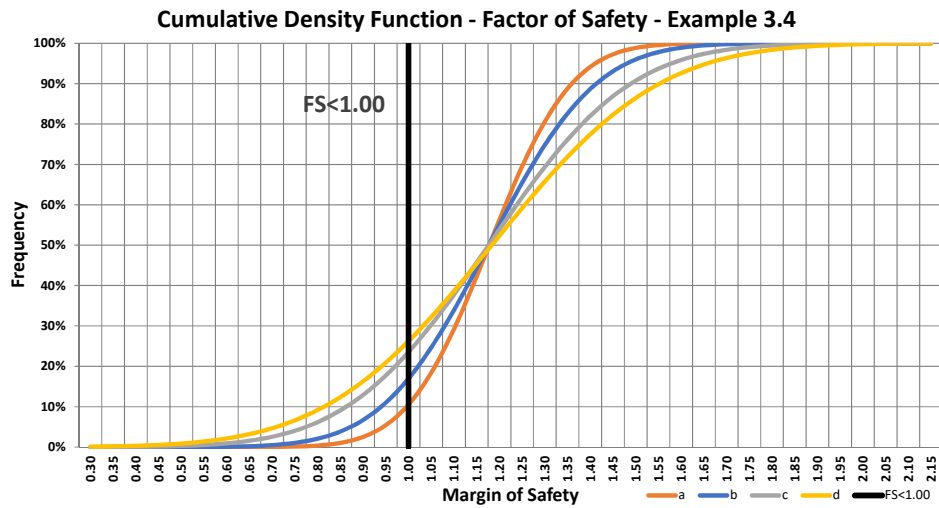
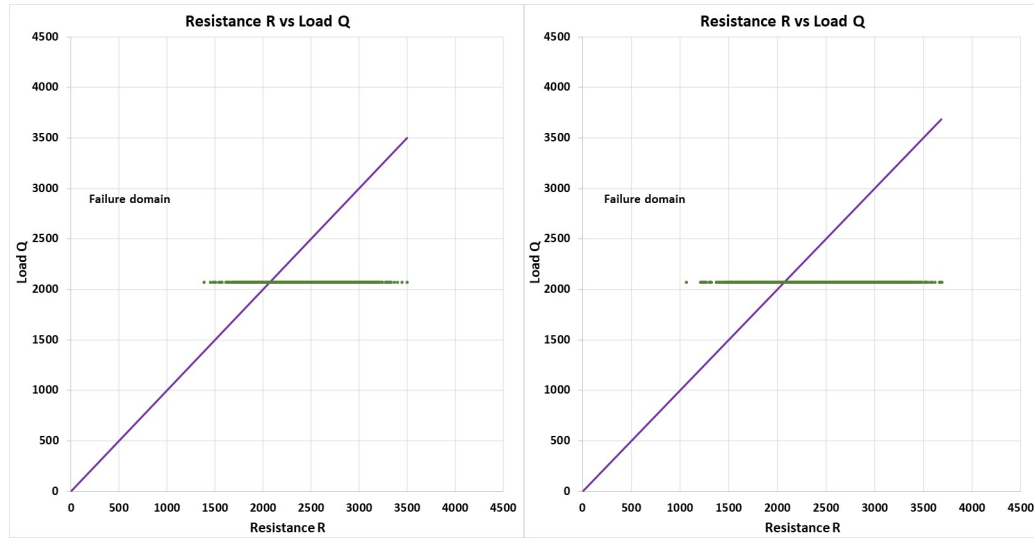
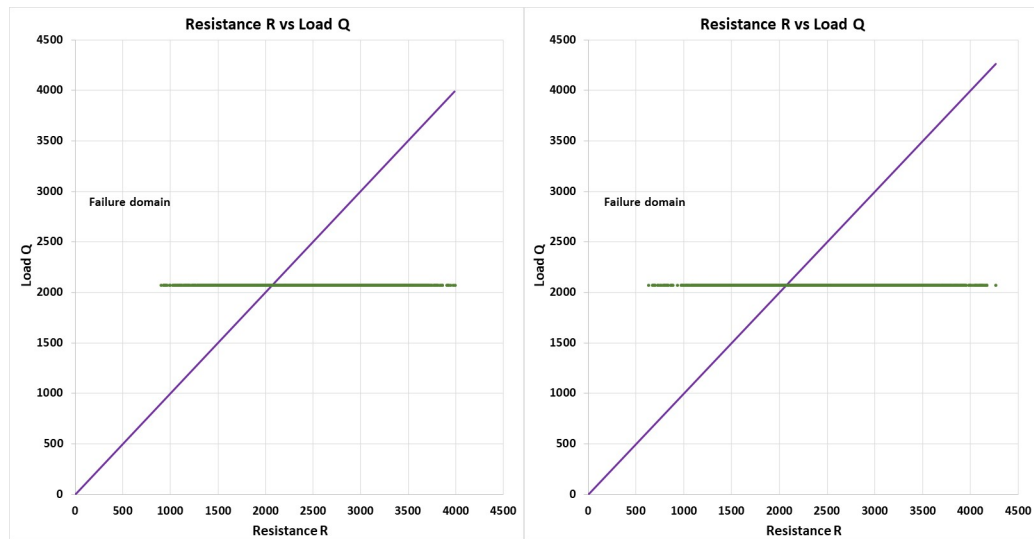


Fig. 3.34. Example 3.4 - CDF - Cases a, b, c, d



(a) Case a - 8 layers

(b) Case b - 4 layers



(c) Case c - 2 layers

(d) Case d - 1 layer

Fig. 3.35. Example 3.4 - Resistance R vs Load Q

3.5 Conclusion

In this part, a new tool to solve 2D slope problems considering probabilistic approach was presented. Some relevant aspects of reliability analysis in geotechnical engineering were demonstrated with the use of PNW-SLOPE. This is a relevant topic today, since one of the main challenges for engineers is to use the maximum of data provided by investigations in a more realistic approach. While in deterministic analysis the slope is evaluated based on a single factor of safety, in probabilistic approach the factor of safety is represented by a range of values, usually represented as a probability density function. More information is required to properly account the variability of parameters. Several examples were presented to demonstrate some of the capabilities of PNW-SLOPE. In the first example different levels of variability were analyzed to understand their impact in the probability of failure. The results showed that more variability increased the probability of failure. The second example compared the influence of each soil parameter in the analysis. Some charts generated by PNW-SLOPE were presented. The third example showed different situations of sampling in horizontal direction. The situation with a random value for each slice presented a lower probability of failure when compared with the situation with a single random value for the entire slope. The last example had the same idea of Example 3.3, but now sampling was compared in vertical direction. One conclusion when comparing deterministic and probabilistic analysis is that reliability theory does not invalidate the factor of safety calculations. Instead, it extends the meaning of deterministic analysis by giving additional information to help the engineer interpret the results. The current challenge to the geotechnical engineering profession is how to use probabilistic methods in practice, therefore this work contributed to clarify some of the theory behind this.

4. PLANE SLIDING ANALYSIS OF ROCK SLOPES

The main goal of this chapter is to present an additional module of PNW-SLOPE applied in rock mechanics. The goal of this part is to solve rock slope stability problems in which the failure results from sliding on a single planar surface dipping into the excavation. It consists of an Excel spreadsheet with a graphic interface created with the aid of VBA (Visual Basic for Applications) to help in the data entry and formulation of the problem. With this spreadsheet, students can easily solve 2D problems of plane sliding, with and without reinforcement. This research also presents a probabilistic approach to deal with this kind of problem using Monte Carlo simulations. Some example problems are presented to demonstrate the capability of the program. Each parameter can be selected to be represented by its probability distribution, therefore it is possible to examine the effect of variability on slope stability. The spreadsheet consists of a good learning tool for basic theory related to rock mechanics, also how probabilistic analysis can be applied in geotechnical engineering.

4.1 Introduction

Excavation of rocks has been considered one of the most important geotechnical engineering activities throughout the world. It is applicable in both civil and mining engineering. For civil engineering, usually transportation systems require some activity for construction of highways and railways, and for mining engineering open pits are necessary for mineral production. Rock and soil instability has been widely studied by researches in the last decades, with the development of different methodologies to deal with all the aspects in this kind of analysis. However much more improvement necessary yet in this area, most partly related to decision making, risk assessment and risk management applied in geotechnical engineering.

Rock slopes can assume different mechanics of failure, therefore the first step in the analysis is to understand all of possible ways which instability may occur in rocks. With this information it is possible to identify what are the most appropriate cases to approach with the slope being studied. Although most soil slopes are of continuous nature, the majority of rock slope instability is caused by individual discontinuities. When the slip follows a major discontinuity in a plane, it is possible to perform a 2D analysis of a typical section, in which the failure have a planar form. Although plane failure is not the most common situation for real slopes, yet this study has important applications. It is possible to understand the effect of some factors in the stability, such as presence of pore water pressure in the analysis.

The main goal of rock slopes projects is to determine the maximum safe cut face angle compatible with the planned maximum height. The required stability conditions of rock slopes will vary depending on the type of project and the consequence of failure. Usually two approaches can be used to deal with this situation: deterministic and probabilistic. In deterministic analysis, the factor of safety is calculated from the mean or most likely values of the input variables, while in probabilistic analysis the distribution of the factor of safety is obtained when input variables are expressed as

probability density functions. The final results is therefore the probability of failure of the slope.

The probabilistic approach must be used for a reliability analysis. The uncertainty is considered in one or more parameters in the stability model, and then some analysis method should be used, such as limit equilibrium analysis integrated with Monte Carlo simulations. Simulation methods are popular for reliability analysis of engineering systems [42]. One important aspect of reliability analysis is the proper consideration of the probability functions for each parameter. [43] presents a methodology involves the construction of posterior probability distributions that combine prior information on the parameter values with typical data from laboratory tests and site investigations used in design. Even though much work has been devoted to establishing the appropriate values of the input parameters and to identifying the sources and types of uncertainty, most people have considerable difficulty understanding the meaning of results expressed as probabilities of failure [36].

The objective of this chapter is to create a spreadsheet for calculations of a specific kind of instability: plane sliding. The procedures and equations which are being considered as reference are obtained from Willie and Mah [44].

4.2 Literature Review

4.2.1 Slope Instability in Rocks

The two types of slope failure mechanisms are when the rock is behaving as an equivalent continuum or discontinuum (Fig.4.1). When the rock mass has a continuum behavior, the failure surface is created through the rock mass, and in the other case the failure follow the discontinuities. The failure mechanisms also can be represented more specifically in four basic types (Fig.4.2).

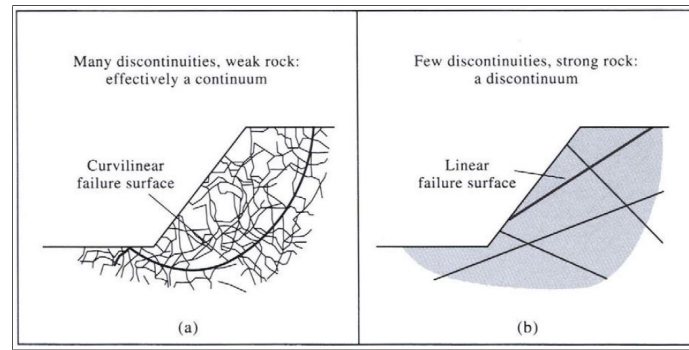


Fig. 4.1. Slope failure mechanisms in (a) a continuum and (b) a discontinuum. [44]

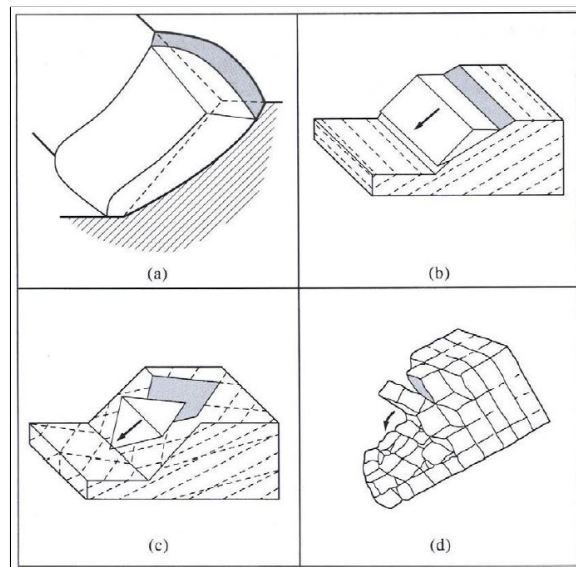


Fig. 4.2. The four basic mechanisms of rock slope instability: (a) circular slip; (b) plane sliding; (c) wedge sliding; and (d) toppling ((b), (c) and (d) [45].

4.2.2 Plane Sliding

Plane sliding is one of the simplest types of failure analyze. The factor of safety can be directly calculated by making suitable assumptions [44]. The solution consists of a straightforward 2D analysis of forces, with simple equations to describe the problem.

When the instability is dictated by the presence of pre-existing discontinuities, this is one possible form of instability, and the slip will usually have a planar form if it occurs on a major discontinuity (Fig.4.3). Plane failure is particularly useful for demonstrating the sensitivity of the slope to changes in shear strength and ground water conditions [44].

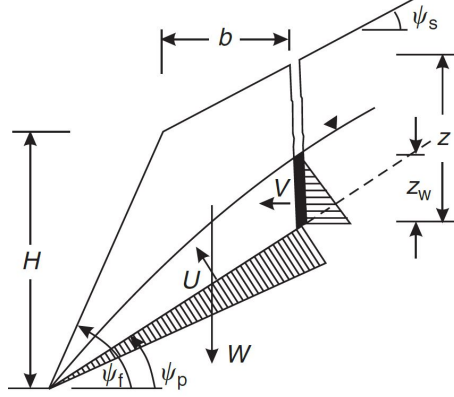


Fig. 4.3. Geometry of slope exhibiting plane failure [44].

It is possible to consider a reinforcement when it has been established that a slope is potentially unstable. Some of these methods to increase the factor of safety are the installation of tensioned anchors or fully grouted, untensioned dowels, or the construction of a toe buttress. In this work only the installation of tensioned anchors is being considered.

The equations used for calculations are presented below [46]:

- Forces due to pore water pressure:

$$U = \frac{1}{2} \gamma_w z_w (H + b \tan \psi_s - z) \operatorname{cosec} \psi_p \quad (4.1)$$

$$V = \frac{1}{2} \gamma_w z_w^2 \quad (4.2)$$

- Weight for the tension crack in the inclined upper slope surface:

$$W = \gamma_r \left[(1 - \cot \psi_f \tan \psi_p) (bH + \frac{1}{2} H^2 \cot \psi_f) + \frac{1}{2} b^2 (\tan \psi_s - \tan \psi_p) \right] \quad (4.3)$$

- Weight for the tension crack in the slope face:

$$W = \frac{1}{2} \gamma_r H^2 \left[\left(1 - \frac{z}{H} \right)^2 \cot \psi_p (\cot \psi_p \tan \psi_f - 1) \right] \quad (4.4)$$

- Factor of safety without reinforcement:

$$FS = \frac{cA + (W \cos \psi_p - U - V \sin \psi_p) \tan \phi}{W \sin \psi_p + V \cos \psi_p} \quad (4.5)$$

- Factor of safety with reinforcement using tensioned rock bolt

$$FS = \frac{cA + (W \cos \psi_p - U - V \sin \psi_p + T \sin(\psi_T + \psi_p)) \tan \phi}{W \sin \psi_p + V \cos \psi_p - T \cos(\psi_T + \psi_p)} \quad (4.6)$$

- Optimum installation angle for a tensioned bolt:

$$\phi = (\psi_{T(opt)} + \psi_p) \text{ or } \psi_{T(opt)} = (\phi - \psi_p) \quad (4.7)$$

4.3 Spreadsheet

4.3.1 Deterministic Analysis

The main features presented in the spreadsheet for deterministic analysis are:

- Define the geometric parameter of the slope. The failure plane angle can be defined or automatically calculated when the height of the vertical tension crack is defined.
- Select both situations of tension crack in upper surface or in the face of the slope.
- Consideration of reinforcement with tensioned rock bolt. When this option is selected, it is necessary to define the tension force and bolt angle considered. The optimum angle for the rock bolt is also calculated and suggested.
- Generation of the images for geometry and free body diagram

Static analysis of plane stability

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Static analysis of plane stability - Deterministic approach

Units	Geometry	Reinforcement	Results										
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W <input type="text" value=""/> kN	With anchor												
	FS <input type="text" value=""/>												

Case

☒ Tension crack in upper surface

(a) Face, Tension crack in upper surface of slope, Slide plane

☐ Tension crack in face

(b) Face, Tension crack in face, Slide plane

Equations

$$A = (H + b \tan \psi_f - z) \operatorname{cosec} \psi_p$$

$$U = \frac{1}{2} \gamma_w z_w (H + b \tan \psi_f - z) \operatorname{cosec} \psi_p$$

$$V = \frac{1}{2} \gamma_w z_w^2$$

(a) $W = \gamma \left[(1 - \cot \psi_f \tan \psi_p) \left(bH + \frac{1}{2} H^2 \cot \psi_f \right) + \frac{1}{2} b^2 (\tan \psi_f - \tan \psi_p) \right]$

(b) $W = \frac{1}{2} \gamma H^2 \left[\left(1 - \frac{z}{H} \right) \cot \psi_p \times (\cot \psi_p \tan \psi_f - 1) \right]$

$$FS = \frac{cA + (W \cos \psi_p - U - V \sin \psi_p) \tan \phi}{W \sin \psi_p + V \cos \psi_p}$$

(Anc) $FS = \frac{cA + (W \cos \psi_p - U - V \sin \psi_p + T \sin(\psi_f + \psi_p)) \tan \phi}{W \sin \psi_p + V \cos \psi_p - T \cos(\psi_f + \psi_p)}$

Free body diagram

Fig. 4.4. Plane Sliding - Deterministic Analysis

4.3.2 Probabilistic Analysis

After completing the deterministic plane sliding analysis it is possible to open the probabilistic window to perform a reliability analysis for the case being studied. The factor of safety will be expressed as a probability distribution, rather than a single value. The probabilistic analysis part of the software offers the following features:

- Choose the number of Monte Carlo iterations to be simulated.
- Choose each parameter to be considered as a random variable. When the parameter is selected, all the textboxes are showed for definition of probabilistic parameters.
- Calculation of probability of failure and reliability index.
- Generation of histograms for all the parameters considered as variable.

- Generation of scatter data, in which it is possible to choose the parameter for each axis.

PURDUE UNIVERSITY NORTHWEST **Static analysis of plane stability - Probabilistic approach**

Results from Deterministic Factor of Safety: <input type="text"/> <input type="button" value="Return to Det. analysis"/>		Monte Carlo Simulation N of iterations: <input type="text"/> <input type="button" value="Run analysis"/>		Results Probability of failure: <input type="text"/> % Reliability index (Normal): <input type="text"/> Reliability index (Lognormal): <input type="text"/> Factor of safety (Mean): <input type="text"/> Factor of safety (Max): <input type="text"/> Factor of safety (Min): <input type="text"/> Margin of safety (Mean): <input type="text"/> Margin of safety (Max): <input type="text"/> Margin of safety (Min): <input type="text"/>																																																									
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Fig. 4.5. Plane Sliding - Probabilistic Analysis

4.4 Examples

Two examples are being presented in this study to demonstrate the use of the spreadsheet. In the first example (Example 4.1), a rock slope will be analyzed considering deterministic analysis. Four situations (a,b,c,d) are being considered with different values of cohesion and water level in the crack (z_w), and one case with reinforcement (e). The geometry and parameters are presented in Fig.4.6.

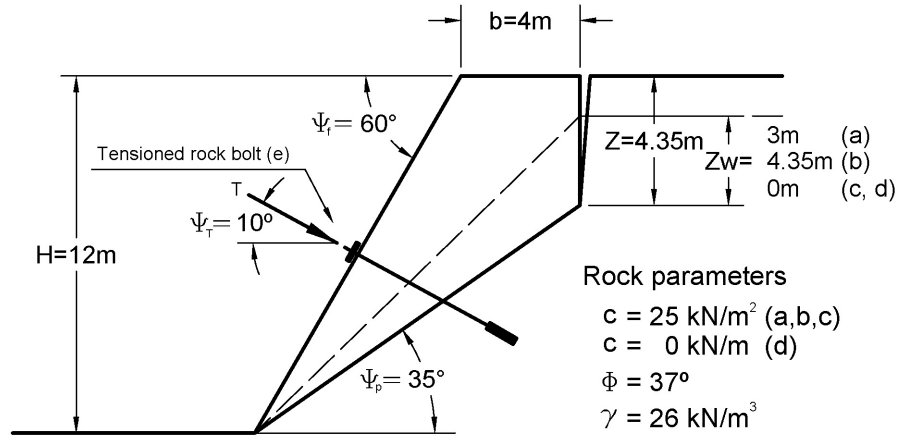


Fig. 4.6. Examples 4.1 and 4.2 [Adapted from [44]]

Table 4.1.
Example 4.2 - Probabilistic parameters

Parameter	Mean	Standard Deviation	Min	Max	Distribution
$\gamma_m \text{ (kN/m}^3\text{)}$	26	1	24	28	Normal
$c \text{ (kN/m}^2\text{)}$	25	5	15	35	Normal
$\phi \text{ (}^\circ\text{)}$	37	3	32	42	Normal
$z_w \text{ (m)}$	3	0.5	2	4	Normal

The second example (Example 4.2) is composed by Example 4.1a with a probabilistic approach. In this analysis it is necessary to specify the probabilistic parameters for the four parameters which are considered as random variables (Table 4.1).

4.5 Results

This section presents the results for the example problems described in the previous section. The first part contains the deterministic solution for factor of safety for each situation, with some pictures generated by the spreadsheet. The second part presents the Monte Carlo Simulations results. The four parameters considered as random variables are represented by their probability distributions.

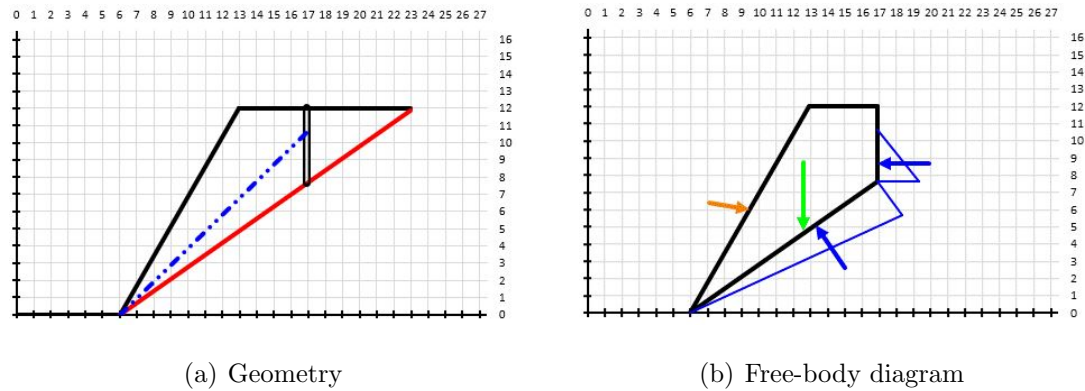


Fig. 4.7. Example 4.1e - Pictures generated by the spreadsheet

Results			
Forces		Factor of safety	
U	196.3 kN	Without anchor	
V	44.1 kN	FS	1.247
W	1,242.0 kN	With anchor	
		FS	1.266

Fig. 4.8. Example 4.1e - Results

Figures 4.7 and 4.8 present the results for deterministic analysis generated by the spreadsheet for Example 4.1e. In this example, the factor of safety without reinforcement was 1.247 and with reinforcement was 1.266. It is important to emphasize that the anchor angle influences this increase of factor of safety, with an optimum angle presented by Eq.4.7 to maximize its use. The results for the other situations studies in Example 4.1 are summarized in Table 4.2. From (a) to (b): reduction in factor of safety due to an increase of pore water pressure. From (a) to (c): increase in factor of

Table 4.2.
Example 4.1 - FS of each case

Case	FS
a	1.247
b	1.073
c	1.545
d	1.076
e	1.266

safety due to a reduction of pore water pressure. From (c) to (d): reduction in factor of safety due to reduction of cohesion.

For the second example, Figures 4.9 - 4.13 show the results and histograms generated by Monte Carlo simulations. The probability of failure was 1.00 % as the result of 1,000 iterations with values randomly selected from the input parameter distributions. The reliability index was 1.76 considering a normal distribution for Factor of Safety.

Results		
Probability of failure	1.00	%
Reliability index (Normal)	1.76	
Reliability index (Lognormal)	1.81	
Factor of safety (Mean)	1.244	
Factor of safety (Max)	1.625	
Factor of safety (Min)	0.906	
Margin of safety (Mean)	182.0	
Margin of safety (Max)	478.8	
Margin of safety (Min)	-70.4	

Fig. 4.9. Results - Example 4.2

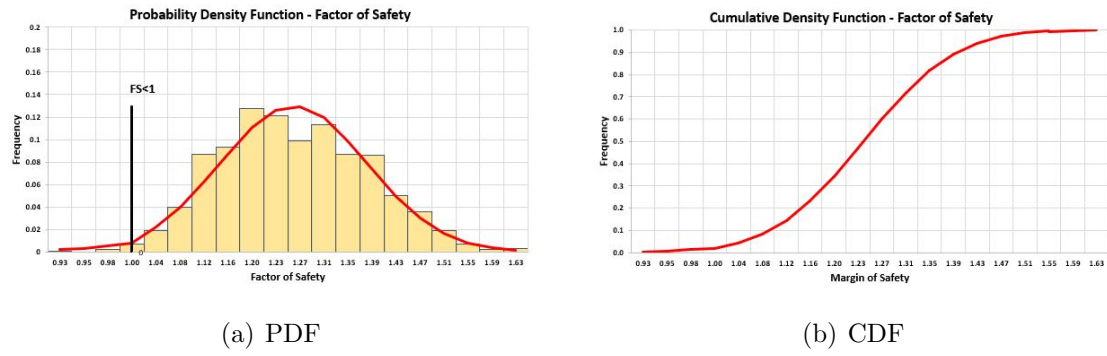


Fig. 4.10. Factor of safety

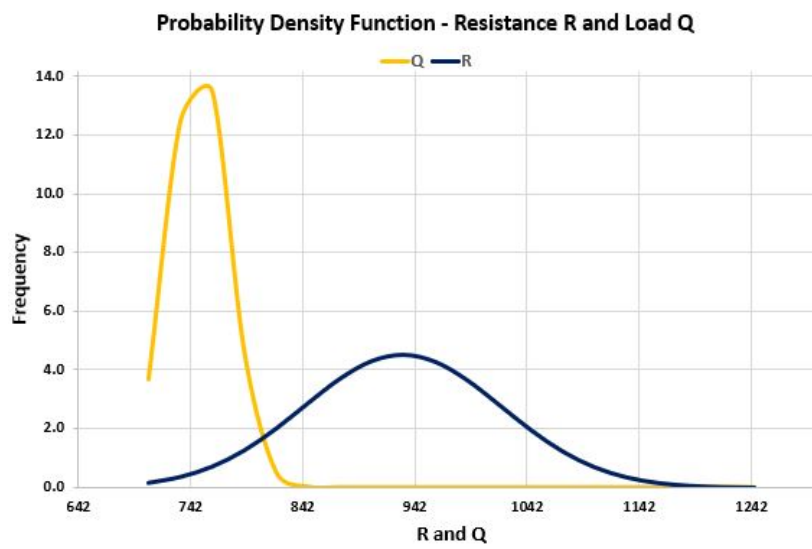


Fig. 4.11. Results of Probabilistic analysis

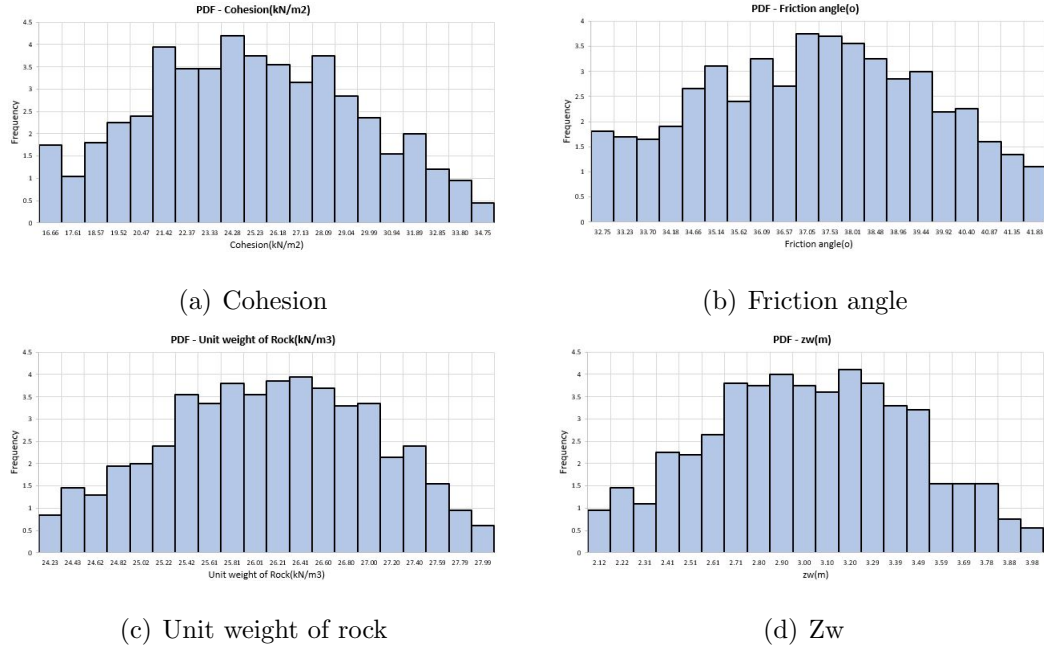


Fig. 4.12. PDF of parameters

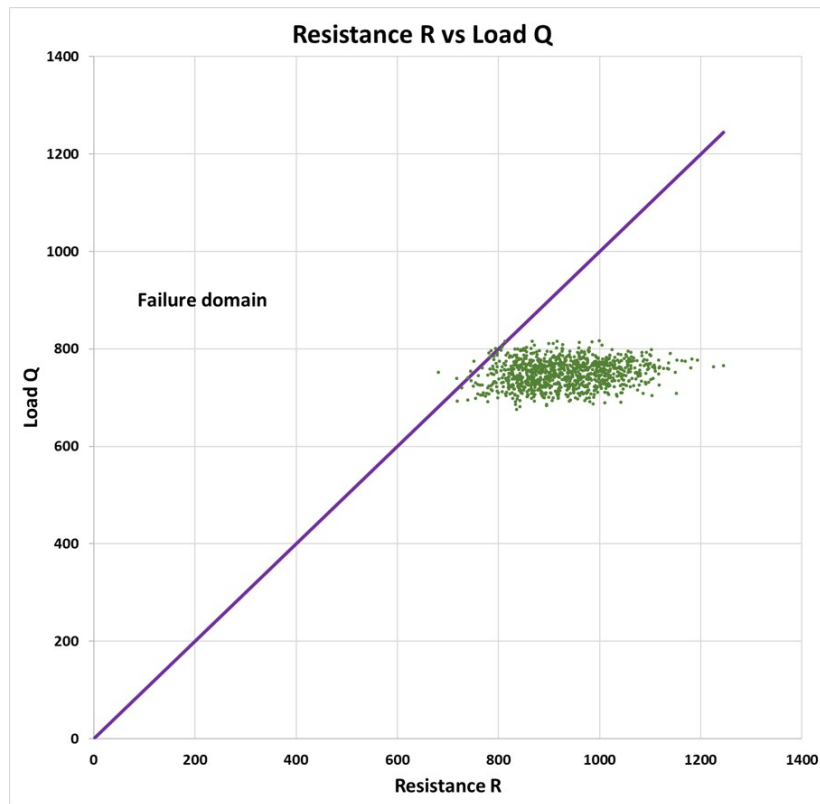


Fig. 4.13. Resistance vs Load

4.6 Conclusion

In this chapter, a new tool to solve simple slope stability problems of rock mechanics was presented. With this spreadsheet, students can solve 2D problems of plane sliding, with or without reinforcement. Different situations could be simulated and compared for understanding the impact of each parameter in the factor of safety. Another feature presented was the probabilistic analysis. Useful charts were automatically generated by the spreadsheet to help in the interpretation of results and to verify how reliable is the slope being studied. In the example considered in this study, which was simulated using the two approaches, the factor of safety was 1.266 and the probability of failure was 1.00 %. Even though commercial programs with the same purpose of this study are able to offer more complete features, this spreadsheet still is important to prepare students to deal with reliability theory.

5. CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

In this research, a new program for slope stability analysis was presented. PNW-SLOPE was created to help students learn important concepts in geotechnical engineering. The idea was to create a simple tool to solve this kind of problem, with clear instructions and results easy to be checked. Another goal of this work was to show an application of reliability methods more focused on geotechnical engineering problems. Several examples were presented for different situations and method of analysis. Chapter 2 focused on the first two modules of the program: geometry definition and deterministic slope stability analysis, while chapter 3 presented the probabilistic analysis. Chapter 4 presented an application for one specific type of failure in rock slopes. It is important to note that reliability theory does not invalidate the factor of safety calculations. Instead, it extends the meaning of deterministic analysis by giving additional information to help the engineer interpret the results. The advantages when considering a programming language in the development of spreadsheets were demonstrated with PNW-SLOPE. VBA has an immense potential to increase the capacity of spreadsheets developed in Excel by students and researches, and programming skills plays a fundamental role in this task.

5.2 Recommendations for Future Study

This research presented the first version of PNW-SLOPE. Even though this version of PNW-SLOPE was able to accomplish the main goals of this study, there is space for improvement. Some ideas for future studies with the goal to increase the resources of PNW-SLOPE are presented in the following:

- Add more method of slices, such as Jambu method;
- Include option for non-circular slip surfaces;
- Add more computational methods for reliability analysis, such as FORM;
- Include correlation between parameters;
- Include the processing time of the spreadsheet when performing Monte Carlo simulations;
- Include more complex geometries for the slope.

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APPENDICES

Appendix A. Complete Results

<div><div>PURDUE</div><div>UNIVERSITY</div><div>NORTHWEST</div></div>				PNW-Slope - Results																							
				Project: Example 1 Author: Thiago Date: 8/28/2019										Units													
														Coordinates		ft		Surcharge		lb/ft							
														Cohesion		psf		Forces		kip							
Coordinates at the midpoint of the slice				Width		Length of the Base		Angle at the Base		Angle at the Top		Cohesion and Friction		Weight		Pore water pressure		External Forces and Moments				Analysis		Sliding Forces		Resisting Forces for each method	
Xm Ytm Ybm				b		ΔL α		β		c' Φ'		W		u		P Q Mp		T or E		Q - OMS Q - BMS		R - OMS R - BMS					
1	306.00	91.00	72.83	33.00	49.08	47.76	0.00	1780.00	0.09	80930	0.00	0	0	0	0	T	59911	59911	92126	91288							
2	273.00	91.00	41.61	33.00	42.08	38.35	0.00	1780.00	0.09	220010	0.00	0	0	0	0	T	136511	136511	89991	94523							
3	248.25	87.70	23.40	16.50	19.46	32.01	21.80	1780.00	0.09	143226	0.00	0	0	0	0	T	75910	75910	45260	47461							
4	231.75	81.10	13.84	16.50	18.70	28.07	21.80	1780.00	0.09	149811	0.00	0	0	0	0	T	70499	70499	44851	46509							
5	215.25	74.50	5.72	16.50	18.10	24.28	21.80	1780.00	0.09	153200	0.00	0	0	0	0	T	62994	62994	44438	45572							
6	198.75	67.90	-1.10	16.50	17.63	20.60	21.80	1600.00	0.03	153549	0.00	0	0	0	0	T	54018	54018	33222	33600							
7	182.25	61.30	-6.72	16.50	17.25	17.00	21.80	1600.00	0.03	150631	0.00	0	0	0	0	T	44046	44046	32637	32844							
8	161.57	53.86	-12.02	24.86	25.47	12.60	18.43	1600.00	0.03	218681	0.00	0	0	0	0	T	47697	47697	48205	48294							
9	136.71	45.57	-16.41	24.86	25.07	7.39	18.43	1600.00	0.03	204733	0.00	0	0	0	0	T	26329	26329	47195	47153							
10	111.86	37.29	-18.51	24.86	24.88	2.24	18.43	1600.00	0.03	183551	0.00	0	0	0	0	T	7176	7176	46207	46169							
11	87.00	29.00	-18.37	24.86	24.89	-2.89	18.43	1600.00	0.03	155302	0.00	0	0	0	0	T	-7829	-7829	45238	45312							
12	62.14	20.71	-15.98	24.86	25.10	-8.04	18.43	1600.00	0.03	119973	0.00	0	0	0	0	T	-16786	-16786	44315	44563							
13	37.29	12.43	-11.30	24.86	25.54	-13.26	18.43	1600.00	0.03	77375	0.00	0	0	0	0	T	-17752	-17752	43491	43909							
14	12.43	4.14	-4.19	24.86	26.23	-18.60	18.43	1600.00	0.03	27118	0.00	0	0	0	0	T	-8650	-8650	42861	43345							
15	0.00	0.00	0.00	0.00	0.01	-21.30	0.00	1600.00	0.03	0	0.00	0	0	0	0	T	0	0	9	9							
16	-0.01	0.00	0.00	0.00	0.01	-21.31	0.00	1600.00	0.03	0	0.00	0	0	0	0	T	0	0	9	9							

Fig. A.1. Results PNW-SLOPE - Example 2.1 - Short term stability

<div><div>PURDUE</div><div>UNIVERSITY</div><div>NORTHWEST</div></div>			PNW-Slope - Results																	
			Project: Example 2										Units							
			Author: Thiago										Coordinates		ft		Surcharge			
			Date: 8/26/2019										Cohesion		psf		Forces			
Coordinates of the slope			Soil layers							Pore water pressure					Unit Weight					
Point	x	y	Layer	ytop	ybottom	y	c'	Φ'	ru	a	Method:	Piezometric line		Critical Slip Surface						
											Point	x	Y	Xc	Yc	r				
1	-50.00	0.00	Soil 1	112.00	0.00	130.00	0.00	34.00		E	P1	-50.00	22.50	131.00	248.00	280.00				
3	220.50	73.50									P2	260.00	22.50	Results						
4	316.75	112.00									P3	300.00	70.00	Method		Q	R	FS		
5	400.00	112.00									P4	350.00	99.50	Ordinary Method of Slices		767762	1410444	1.837		
											P5	400.00	99.50	Bishop Simplified Method		767762	1573063	2.049		
											P6			Surcharge						
											q	xi	xf	Spencer Method		-	-	2.025		
Table of Results for each slice																				
Slice	Coordinates at the midpoint of the slice			Width	Length of the Base	Angle at the Base	Angle at the Top	Cohesion and Friction Angle at Base		Weight	Pore water pressure	External Forces and Moments				Analysis	Sliding Forces		Resisting Forces for each method	
	Xm	Ytm	Ybm					b	ΔL			α	β	c'	Φ'		W	u	P	Q
1	365.92	112.00	96.73	19.67	36.33	57.22	0.00	0.00	0.59	39046	172.95	0	0	0	E	32829	32829	13017	29386	
2	346.25	112.00	69.59	19.67	30.82	50.35	0.00	0.00	0.59	108428	1728.24	0	0	0	E	83481	83481	32040	56313	
3	326.58	112.00	48.11	19.67	27.52	44.38	0.00	0.00	0.59	163364	2344.91	0	0	0	E	114250	114250	56526	83685	
4	307.13	108.15	30.68	19.25	24.78	39.02	21.80	0.00	0.59	193860	2715.67	0	0	0	E	122061	122061	74193	97035	
5	287.88	100.45	16.36	19.25	23.25	34.11	21.80	0.00	0.59	210424	2448.41	0	0	0	E	117996	117996	91196	108773	
6	268.63	92.75	4.41	19.25	22.11	29.47	21.80	0.00	0.59	221076	1768.06	0	0	0	E	108748	108748	109842	122179	
7	249.38	85.05	-5.52	19.25	21.24	25.03	21.80	0.00	0.61	226130	1748.71	0	0	0	E	95669	95669	122112	128267	
8	230.13	77.35	-13.66	19.25	20.58	20.75	21.80	0.00	0.61	226449	2256.66	0	0	0	E	80221	80221	119833	121323	
9	209.48	69.83	-20.53	22.05	22.97	16.29	18.43	0.00	0.61	256746	2685.23	0	0	0	E	72018	72018	132764	131017	
10	187.43	62.48	-26.02	22.05	22.51	11.64	18.43	0.00	0.61	250815	3027.94	0	0	0	E	50585	50585	126224	122926	
11	165.38	55.13	-29.66	22.05	22.22	7.06	18.43	0.00	0.61	239766	3254.77	0	0	0	E	29459	29459	116742	113720	
12	143.33	47.78	-31.51	22.05	22.07	2.52	18.43	0.00	0.61	223799	3370.28	0	0	0	E	9859	9859	104568	103216	
13	121.28	40.43	-31.61	22.05	22.06	-1.99	18.43	0.00	0.61	203013	3376.68	0	0	0	E	-7057	-7057	89962	91155	
14	99.23	33.08	-29.97	22.05	22.19	-6.52	18.43	0.00	0.61	177414	3274.12	0	0	0	E	-20149	-20149	73199	77170	
15	77.18	25.73	-26.55	22.05	22.47	-11.09	18.43	0.00	0.61	146914	3060.60	0	0	0	E	-28264	-28264	54577	60744	
16	55.13	18.38	-21.28	22.05	22.91	-15.74	18.43	0.00	0.61	111325	2474.48	5676	0	820674	E	-33123	-33123	41544	50028	
17	33.08	11.03	-14.05	22.05	23.54	-20.49	18.43	0.00	0.61	70339	1564.91	15789	0	2649939	E	-34086	-34086	32104	43542	
18	11.03	3.68	-4.70	22.05	24.41	-25.40	18.43	0.00	0.61	23487	522.57	25902	0	4949312	E	-27750	-27750	20651	33806	
19	0.17	0.06	0.45	-0.34	-0.38	-27.86	18.43	0.00	0.59	17	0.00	-474	0	-95998	E	335	335	-211	-400	
20	0.51	0.17	0.27	-0.34	-0.38	-27.78	18.43	0.00	0.59	4	0.00	-472	0	-95347	E	339	339	-218	-409	
21	0.85	0.28	0.09	-0.34	-0.38	-27.70	18.43	0.00	0.59	-9	12.06	-469	0	-94698	E	342	342	-222	-414	

Fig. A.2. Results PNW-SLOPE - Example 2.2 - Long term stability

PURDUE UNIVERSITY NORTHWEST			PNW-Slope - Results								Units								
Project: Example 3a											Coordinates		ft	Surcharge		lb/ft			
Author: Thiago											Cohesion		psf	Forces		kip			
Date: 8/28/2019											Unit Weight		pcf	Moments		kip-ft			
Coordinates of the slope			Soil layers						Pore water pressure										
Point	x	y	Layer	ytop	ybottom	γ	c'	Φ'	ru	a	Method:	Piezometric line	Critical Slip Surface						
1	-40.00	0.00	Soil 1	40.00	-20.00	120.00	600.00	20.00		E	Point	x	Y	Xc	Yc	r			
2	0.00	0.00									P1			20.00	70.00	80.00			
3	80.00	40.00									P2			Results					
4	140.00	40.00									P3			Method Q R FS					
											P4			Ordinary Method of Slices 84704 163587 1.931					
											P5			Bishop Simplified Method 84704 176193 2.080					
											P6			Spencer Method - - 2.076					
											Surcharge								
											q	xi	xf						
Table of Results for each slice																			
Slice	Coordinates at the midpoint of the slice			Width	Length of the Base	Angle at the Base	Angle at the Top	Cohesion and Friction		Weight	Pore water pressure	External Forces and Moments			Analysis	Sliding Forces		Resisting Forces for each method	
	Xm	Ytm	Ybm					b	ΔL			α	β	c'		Φ'	W	u	P
1	91.80	40.00	35.14	4.72	10.81	64.10	0.00	600.00	0.35	2754	0.00	0	0	0	E	2477	2477	6923	6454
2	87.08	40.00	26.63	4.72	8.69	57.11	0.00	600.00	0.35	7576	0.00	0	0	0	E	6362	6362	6714	8102
3	82.36	40.00	20.03	4.72	7.55	51.29	0.00	600.00	0.35	11312	0.00	0	0	0	E	8828	8828	7104	9122
4	77.33	38.67	14.34	5.33	7.66	45.85	26.57	600.00	0.35	15570	0.00	0	0	0	E	11171	11171	8541	10786
5	72.00	36.00	9.31	5.33	7.02	40.59	26.57	600.00	0.35	17084	0.00	0	0	0	E	11115	11115	8936	10785
6	66.67	33.33	5.10	5.33	6.57	35.72	26.57	600.00	0.35	18066	0.00	0	0	0	E	10548	10548	9280	10695
7	61.33	30.67	1.58	5.33	6.23	31.14	26.57	600.00	0.35	18618	0.00	0	0	0	E	9627	9627	9539	10541
8	56.00	28.00	-1.38	5.33	5.97	26.76	26.57	600.00	0.35	18803	0.00	0	0	0	E	8467	8467	9695	10337
9	50.67	25.33	-3.83	5.33	5.78	22.56	26.57	600.00	0.35	18666	0.00	0	0	0	E	7160	7160	9739	10088
10	45.33	22.67	-5.83	5.33	5.62	18.47	26.57	600.00	0.35	18238	0.00	0	0	0	E	5779	5779	9670	9800
11	40.00	20.00	-7.41	5.33	5.51	14.49	26.57	600.00	0.35	17543	0.00	0	0	0	E	4388	4388	9487	9472
12	34.67	17.33	-8.60	5.33	5.43	10.57	26.57	600.00	0.35	16596	0.00	0	0	0	E	3044	3044	9193	9103
13	29.33	14.67	-9.41	5.33	5.37	6.70	26.57	600.00	0.35	15408	0.00	0	0	0	E	1799	1799	8792	8690
14	24.00	12.00	-9.86	5.33	5.34	2.87	26.57	600.00	0.35	13987	0.00	0	0	0	E	700	700	8289	8229
15	18.67	9.33	-9.94	5.33	5.33	-0.96	26.57	600.00	0.35	12338	0.00	0	0	0	E	-206	-206	7690	7714
16	13.33	6.67	-9.68	5.33	5.35	-4.78	26.57	600.00	0.35	10460	0.00	0	0	0	E	-872	-872	7005	7136
17	8.00	4.00	-9.05	5.33	5.39	-8.63	26.57	600.00	0.35	8351	0.00	0	0	0	E	-1253	-1253	6242	6483
18	2.67	1.33	-8.05	5.33	5.46	-12.52	26.57	600.00	0.35	6007	0.00	0	0	0	E	-1302	-1302	5412	5740
19	-1.87	0.00	-6.93	3.75	3.89	-15.87	0.00	600.00	0.35	3114	0.00	0	0	0	E	-852	-852	3427	3699
20	-5.62	0.00	-5.76	3.75	3.95	-18.68	0.00	600.00	0.35	2590	0.00	0	0	0	E	-830	-830	3266	3579
21	-9.36	0.00	-4.39	3.75	4.03	-21.54	0.00	600.00	0.35	1973	0.00	0	0	0	E	-724	-724	3084	3425
22	-13.11	0.00	-2.80	3.75	4.12	-24.46	0.00	600.00	0.35	1257	0.00	0	0	0	E	-521	-521	2886	3229
23	-16.86	0.00	-0.97	3.75	4.22	-27.44	0.00	600.00	0.35	437	0.00	0	0	0	E	-202	-202	2674	2983

Fig. A.3. Results PNW-SLOPE - Example 2.3a - Homogeneous soil - Dry

<div><div>PURDUE</div><div>UNIVERSITY</div><div>NORTHWEST</div></div>			PNW-Slope - Results																				
			Project: Example 3b Author: Thiago Date: 8/28/2019										Units										
													Coordinates		ft	Surcharge		lb/ft	Cohesion		psf	Forces	
Coordinates of the slope			Soil layers							Pore water pressure							Critical Slip Surface						
Point	x	y	Layer	ytot	ybottom	γ	c'	Φ'	ru	a	Method	ore pressure ratio	x	y	Xc	Yc	r						
1	-40.00	0.00	Soil 1	40.00	-20.00	120.00	600.00	20.00	0.25	E	Point				20.00	70.00	80.00						
2	0.00	0.00									P1												
3	80.00	40.00									P2												
4	140.00	40.00									P3												
											P4												
											P5												
											P6												
											Surcharge												
											q	xi	xf										
Table of Results for each slice																							
Slice	Coordinates at the midpoint of the slice			Width	Length of the Base	Angle at the Base	Angle at the Top	Cohesion and Friction Angle at Base		Weight	Pore water pressure	External Forces and Moments				Analysis	Sliding Forces		Resisting Forces for each method				
	Xm	Ytm	Ybm					b	ΔL			α	β	c'	Φ'		W	u	P	Q	Mp	T or E	Q - OMS
1	91.80	40.00	35.14	4.72	10.81	64.10	0.00	600.00	0.35	2754	145.84	0	0	0	E	2477	2477	6813	5758				
2	87.08	40.00	26.63	4.72	8.69	57.11	0.00	600.00	0.35	7576	401.20	0	0	0	E	6362	6362	6339	6841				
3	82.36	40.00	20.03	4.72	7.55	51.29	0.00	600.00	0.35	11312	599.08	0	0	0	E	8828	8828	6461	7529				
4	77.33	38.67	14.34	5.33	7.66	45.85	26.57	600.00	0.35	15570	729.85	0	0	0	E	11171	11171	7555	8820				
5	72.00	36.00	9.31	5.33	7.02	40.59	26.57	600.00	0.35	17084	800.80	0	0	0	E	11115	11115	7755	8799				
6	66.67	33.33	5.10	5.33	6.57	35.72	26.57	600.00	0.35	18066	846.87	0	0	0	E	10548	10548	7945	8721				
7	61.33	30.67	1.58	5.33	6.23	31.14	26.57	600.00	0.35	18618	872.72	0	0	0	E	9627	9627	8089	8603				
8	56.00	28.00	-1.38	5.33	5.97	26.76	26.57	600.00	0.35	18803	881.39	0	0	0	E	8467	8467	8167	8453				
9	50.67	25.33	-3.83	5.33	5.78	22.56	26.57	600.00	0.35	18666	874.97	0	0	0	E	7160	7160	8171	8273				
10	45.33	22.67	-5.83	5.33	5.62	18.47	26.57	600.00	0.35	18238	854.93	0	0	0	E	5779	5779	8096	8067				
11	40.00	20.00	-7.41	5.33	5.51	14.49	26.57	600.00	0.35	17543	822.32	0	0	0	E	4388	4388	7942	7833				
12	34.67	17.33	-8.60	5.33	5.43	10.57	26.57	600.00	0.35	16596	777.92	0	0	0	E	3044	3044	7709	7572				
13	29.33	14.67	-9.41	5.33	5.37	6.70	26.57	600.00	0.35	15408	722.25	0	0	0	E	1799	1799	7399	7280				
14	24.00	12.00	-9.86	5.33	5.34	2.87	26.57	600.00	0.35	13987	655.66	0	0	0	E	700	700	7017	6955				
15	18.67	9.33	-9.94	5.33	5.33	-0.96	26.57	600.00	0.35	12338	578.33	0	0	0	E	-206	-206	6568	6592				
16	13.33	6.67	-9.68	5.33	5.35	-4.78	26.57	600.00	0.35	10460	490.30	0	0	0	E	-872	-872	6057	6183				
17	8.00	4.00	-9.05	5.33	5.39	-8.63	26.57	600.00	0.35	8351	391.47	0	0	0	E	-1253	-1253	5491	5722				
18	2.67	1.33	-8.05	5.33	5.46	-12.52	26.57	600.00	0.35	6007	281.56	0	0	0	E	-1302	-1302	4879	5196				
19	-1.87	0.00	-6.93	3.75	3.89	-15.87	0.00	600.00	0.35	3114	207.81	0	0	0	E	-852	-852	3154	3421				
20	-5.62	0.00	-5.76	3.75	3.95	-18.68	0.00	600.00	0.35	2590	172.84	0	0	0	E	-830	-830	3042	3353				
21	-9.36	0.00	-4.39	3.75	4.03	-21.54	0.00	600.00	0.35	1973	131.65	0	0	0	E	-724	-724	2917	3261				
22	-13.11	0.00	-2.80	3.75	4.12	-24.46	0.00	600.00	0.35	1257	83.92	0	0	0	E	-521	-521	2782	3141				
23	-16.86	0.00	-0.97	3.75	4.22	-27.44	0.00	600.00	0.35	437	29.18	0	0	0	E	-202	-202	2639	2987				

Fig. A.4. Results PNW-SLOPE - Example 2.3b - Homogeneous soil - Pore water pressure defined by R_u

<div><div>PURDUE</div><div>UNIVERSITY</div><div>NORTHWEST</div></div>			PNW-Slope - Results																
			Project: Example 3c Author: Thiago Date: 8/28/2019										Units						
													Coordinates	ft	Surcharge	lb/ft			
			Coordinates of the slope			Soil layers							Pore water pressure		Cohesion	psf	Forces	kip	
Point	x	y	Layer	ytop	ybottom	y	c'	Φ'	ru	a	Method:	Piezometric line	Unit Weight	pcf	Moments	kip-ft			
Critical Slip Surface																			
1	-40.00	0.00	Soil 1	40.00	-20.00	120.00	600.00	20.00		E	Point	x	Y	Xc	Yc	r			
2	0.00	0.00									P1	-40.00	0.00	20.00	70.00	80.00			
3	80.00	40.00									P2	0.00	0.00						
4	140.00	40.00									P3	140.00	20.00						
											P4								
											P5								
											P6								
											Surcharge								
											q	xi	xf						
											Bishop Simplified Method		84704	155322	1.834				
											Spencer Method		-	-	1.834				
Table of Results for each slice																			
Slice	Coordinates at the midpoint of the slice			Width	Length of the Base	Angle at the Base	Angle at the Top	Cohesion and Friction Angle at Base		Weight	Pore water pressure	External Forces and Moments			Analysis	Sliding Forces		Resisting Forces for each method	
	Xm	Ytm	Ybm					b	ΔL			α	β	c'		Φ'	W	u	P
1	91.80	40.00	35.14	4.72	10.81	64.10	0.00	600.00	0.35	2754	0.00	0	0	0	E	2477	2477	6923	6232
2	87.08	40.00	26.63	4.72	8.69	57.11	0.00	600.00	0.35	7576	0.00	0	0	0	E	6362	6362	6714	7877
3	82.36	40.00	20.03	4.72	7.55	51.29	0.00	600.00	0.35	11312	0.00	0	0	0	E	8828	8828	7104	8908
4	77.33	38.67	14.34	5.33	7.66	45.85	26.57	600.00	0.35	15570	0.00	0	0	0	E	11171	11171	8541	10569
5	72.00	36.00	9.31	5.33	7.02	40.59	26.57	600.00	0.35	17084	61.09	0	0	0	E	11115	11115	8846	10466
6	66.67	33.33	5.10	5.33	6.57	35.72	26.57	600.00	0.35	18066	275.77	0	0	0	E	10548	10548	8846	9960
7	61.33	30.67	1.58	5.33	6.23	31.14	26.57	600.00	0.35	18618	448.40	0	0	0	E	9627	9627	8794	9499
8	56.00	28.00	-1.38	5.33	5.97	26.76	26.57	600.00	0.35	18803	585.30	0	0	0	E	8467	8467	8680	9069
9	50.67	25.33	-3.83	5.33	5.78	22.56	26.57	600.00	0.35	18666	690.80	0	0	0	E	7160	7160	8501	8656
10	45.33	22.67	-5.83	5.33	5.62	18.47	26.57	600.00	0.35	18238	767.96	0	0	0	E	5779	5779	8256	8254
11	40.00	20.00	-7.41	5.33	5.51	14.49	26.57	600.00	0.35	17543	819.00	0	0	0	E	4388	4388	7948	7855
12	34.67	17.33	-8.60	5.33	5.43	10.57	26.57	600.00	0.35	16596	845.50	0	0	0	E	3044	3044	7580	7454
13	29.33	14.67	-9.41	5.33	5.37	6.70	26.57	600.00	0.35	15408	848.56	0	0	0	E	1799	1799	7156	7046
14	24.00	12.00	-9.86	5.33	5.34	2.87	26.57	600.00	0.35	13987	828.91	0	0	0	E	700	700	6682	6624
15	18.67	9.33	-9.94	5.33	5.33	-0.96	26.57	600.00	0.35	12338	786.93	0	0	0	E	-206	-206	6163	6184
16	13.33	6.67	-9.68	5.33	5.35	-4.78	26.57	600.00	0.35	10460	722.69	0	0	0	E	-872	-872	5607	5719
17	8.00	4.00	-9.05	5.33	5.39	-8.63	26.57	600.00	0.35	8351	635.96	0	0	0	E	-1253	-1253	5021	5220
18	2.67	1.33	-8.05	5.33	5.46	-12.52	26.57	600.00	0.35	6007	526.21	0	0	0	E	-1302	-1302	4415	4677
19	-1.87	0.00	-6.93	3.75	3.89	-15.87	0.00	600.00	0.35	3114	432.25	0	0	0	E	-852	-852	2860	3076
20	-5.62	0.00	-5.76	3.75	3.95	-18.68	0.00	600.00	0.35	2590	359.50	0	0	0	E	-830	-830	2801	3055
21	-9.36	0.00	-4.39	3.75	4.03	-21.54	0.00	600.00	0.35	1973	273.84	0	0	0	E	-724	-724	2737	3024
22	-13.11	0.00	-2.80	3.75	4.12	-24.46	0.00	600.00	0.35	1257	174.55	0	0	0	E	-521	-521	2669	2979
23	-16.86	0.00	-0.97	3.75	4.22	-27.44	0.00	600.00	0.35	437	60.69	0	0	0	E	-202	-202	2600	2920

Fig. A.5. Results PNW-SLOPE - Example 2.3c - Homogeneous soil - Pore water pressure defined by piezometric line

PURDUE UNIVERSITY NORTHWEST				PNW-Slope - Results																					
<div>Project: Example 4</div> <div>Author: Thiago</div> <div>Date: 8/28/2019</div>																	Units								
				Coordinates			Cohesion			kN/m2			Surcharge			kN/m									
Coordinates of the slope				Soil layers							Pore water pressure			Unit Weight			kN/m3			Moments			N-m		
Point	x	y	Layer	ytop	ybottom	γ	c'	Φ'	ru	a	Method:	Point	x	Y	Critical Slip Surface										
1	-22.00	0.00	Soil 1	6.00	1.50	18.00	30.00	0.00		T	P1				Xc	Yc	r								
2	0.00	0.00	Soil 2	1.50	-3.00	18.00	20.00	0.00		T	P2				6.67	19.90	21.33								
3	18.00	6.00	Soil 3	-3.00	-8.00	18.00	150.00	0.00		T	P3				Results										
4	38.00	6.00									P4				Method		Q	R	FS						
											P5				Ordinary Method of Slices		338	588	1.743						
											P6				Bishop Simplified Method		338	588	1.743						
											Surcharge			Spencer Method		-	-	1.743							
											q	xi	xf												
Table of Results for each slice																									
Slice	Coordinates at the midpoint of the slice			Width	Length of the Base	Angle at the Base	Angle at the Top	Cohesion and Friction Angle at Base		Weight	Pore water pressure	External Forces and Moments			Analysis	Sliding Forces		Resisting Forces for each method							
	Xm	Ytm	Ybm					b	ΔL			α	β	c'		Φ'	W	u	P	Q	Mp	T or E	Q-OMS	Q-BMS	R-OMS
1	22.04	6.00	5.16	1.62	2.34	46.20	0.00	30.00	0.00	25	0.00	0	0	0	T	18	18	70	70						
2	20.42	6.00	3.63	1.62	2.12	40.21	0.00	30.00	0.00	69	0.00	0	0	0	T	44	44	63	63						
3	18.81	6.00	2.39	1.62	1.97	34.73	0.00	30.00	0.00	105	0.00	0	0	0	T	60	60	59	59						
4	17.18	5.73	1.36	1.64	1.88	29.56	18.43	20.00	0.00	129	0.00	0	0	0	T	63	63	38	38						
5	15.55	5.18	0.53	1.64	1.80	24.61	18.43	20.00	0.00	137	0.00	0	0	0	T	57	57	36	36						
6	13.91	4.64	-0.15	1.64	1.74	19.86	18.43	20.00	0.00	141	0.00	0	0	0	T	48	48	35	35						
7	12.27	4.09	-0.66	1.64	1.70	15.24	18.43	20.00	0.00	140	0.00	0	0	0	T	37	37	34	34						
8	10.64	3.55	-1.04	1.64	1.67	10.72	18.43	20.00	0.00	135	0.00	0	0	0	T	25	25	33	33						
9	9.00	3.00	-1.29	1.64	1.65	6.28	18.43	20.00	0.00	126	0.00	0	0	0	T	14	14	33	33						
10	7.36	2.45	-1.40	1.64	1.64	1.86	18.43	20.00	0.00	114	0.00	0	0	0	T	4	4	33	33						
11	5.73	1.91	-1.39	1.64	1.64	-2.54	18.43	20.00	0.00	97	0.00	0	0	0	T	-4	-4	33	33						
12	4.09	1.36	-1.26	1.64	1.65	-6.95	18.43	20.00	0.00	77	0.00	0	0	0	T	-9	-9	33	33						
13	2.45	0.82	-0.99	1.64	1.67	-11.41	18.43	20.00	0.00	53	0.00	0	0	0	T	-11	-11	33	33						
14	0.82	0.27	-0.59	1.64	1.70	-15.94	18.43	20.00	0.00	26	0.00	0	0	0	T	-7	-7	34	34						
15	-0.25	0.00	-0.27	0.50	0.53	-18.94	0.00	20.00	0.00	2	0.00	0	0	0	T	-1	-1	11	11						
16	-0.76	0.00	-0.09	0.50	0.54	-20.38	0.00	20.00	0.00	1	0.00	0	0	0	T	0	0	11	11						

Fig. A.6. Results PNW-SLOPE - Example 2.4 - Heterogeneous soil

PURDUE UNIVERSITY NORTHWEST			PNW-Slope - Results																	
			Project: Example_5a										Units							
			Author: Thiago										Coordinates	m	Surcharge	kN/m				
			Date: 8/28/2019										Cohesion	kN/m2	Forces	kN				
													Unit Weight	kN/m3	Moments	N-m				
Coordinates of the slope			Soil layers										Pore water pressure							
Point	x	y	Layer	ytop	ybottom	γ	c'	Φ'	ru	a	Method:	x	y	Critical Slip Surface						
1	-10.00	0.00	Soil 1	10.00	-5.00	18.00	10.00	20.00		E	Point			Xc	Yc	r				
2	0.00	0.00									P1			1.09	10.99	11.08				
3	10.00	10.00									P2			Results						
4	25.00	10.00									P3			Method		Q	R	FS		
											P4			Ordinary Method of Slices	441	416	0.942			
											P5									
											P6									
											Surcharge			Bishop Simplified Method	412	415	1.007			
											q	xi	xf							
														Spencer Method		-	-	1.013		
Table of Results for each slice																				
Slice	Coordinates at the midpoint of the slice			Width	Length of the Base	Angle at the Base	Angle at the Top	Cohesion and Friction Angle at Base		Weight	Pore water pressure	External Forces and Moments				Analysis	Sliding Forces		Resisting Forces for each method	
	Xm	Ytm	Ybm	b	ΔL	α	β	c'	Φ'	W	u	P	Q	Mp	T or E	Q - OMS	Q - BMS	R - OMS	R - BMS	
1	11.77	10.00	8.49	0.71	3.11	76.83	0.00	10.00	0.35	19	0.00	0	0	0	E	19	13	33	19	
2	11.06	10.00	6.23	0.71	1.64	64.52	0.00	10.00	0.35	48	0.00	0	0	0	E	43	30	24	24	
3	10.35	10.00	4.95	0.71	1.30	56.91	0.00	10.00	0.35	64	0.00	0	0	0	E	54	38	26	26	
4	9.58	9.58	3.91	0.83	1.30	50.19	45.00	10.00	0.35	85	0.00	0	0	0	E	65	64	33	42	
5	8.75	8.75	3.01	0.83	1.16	43.84	45.00	10.00	0.35	86	0.00	0	0	0	E	60	60	34	40	
6	7.92	7.92	2.28	0.83	1.06	38.11	45.00	10.00	0.35	85	0.00	0	0	0	E	52	53	35	38	
7	7.08	7.08	1.68	0.83	0.99	32.81	45.00	10.00	0.35	81	0.00	0	0	0	E	44	45	35	36	
8	6.25	6.25	1.19	0.83	0.94	27.81	45.00	10.00	0.35	76	0.00	0	0	0	E	35	37	34	34	
9	5.42	5.42	0.80	0.83	0.91	23.03	45.00	10.00	0.35	69	0.00	0	0	0	E	27	28	32	31	
10	4.58	4.58	0.48	0.83	0.88	18.41	45.00	10.00	0.35	62	0.00	0	0	0	E	19	20	30	29	
11	3.75	3.75	0.24	0.83	0.86	13.92	45.00	10.00	0.35	53	0.00	0	0	0	E	13	13	27	26	
12	2.92	2.92	0.07	0.83	0.84	9.52	45.00	10.00	0.35	43	0.00	0	0	0	E	7	7	24	23	
13	2.08	2.08	-0.04	0.83	0.84	5.17	45.00	10.00	0.35	32	0.00	0	0	0	E	3	3	20	19	
14	1.25	1.25	-0.08	0.83	0.83	0.85	45.00	10.00	0.35	20	0.00	0	0	0	E	0	0	16	15	
15	0.42	0.42	-0.07	0.83	0.83	-3.47	45.00	10.00	0.35	7	0.00	0	0	0	E	0	0	11	11	
16	-0.06	0.00	-0.03	0.12	0.12	-5.93	0.00	10.00	0.35	0	0.00	0	0	0	E	0	0	1	0	
17	-0.18	0.00	-0.02	0.12	0.12	-6.54	0.00	10.00	0.35	0	0.00	0	0	0	E	0	0	1	0	
18	-0.29	0.00	-0.01	0.12	0.12	-7.15	0.00	10.00	0.35	0	0.00	0	0	0	E	0	0	1	0	

Fig. A.7. Results PNW-SLOPE - Example 2.5a - Homogeneous soil 1:1

PURDUE UNIVERSITY NORTHWEST				PNW-Slope - Results																	
Project: Example_5b															Units						
Author: Thiago															Coordinates	m	Surcharge	kN/m			
Date: 8/28/2019															Cohesion	kN/m2	Forces	kN			
Coordinates of the slope			Soil layers								Pore water pressure				Unit Weight	kN/m3	Moments	N-m			
Point	x	y	Layer	ytop	ybottom	γ	c'	Φ'	ru	a	Method:				Critical Slip Surface						
1	-10.00	0.00	Soil 1	10.00	-5.00	18.00	10.00	20.00		E	Point	x	Y	Xc	Yc	r					
2	0.00	0.00									P1			2.58	15.43	15.64					
3	15.00	10.00									P2			Results							
4	25.00	10.00									P3			Method		Q	R	FS			
											P4			Ordinary Method of Slices	485	548	1.130				
											P5				Bishop Simplified Method	491	591	1.206			
											P6			Surcharge							
											q	xi	xf		Spencer Method	-	-	1.202			
Table of Results for each slice																					
Slice	Coordinates at the midpoint of the slice			Width	Length of the Base	Angle at the Base	Angle at the Top	Cohesion and Friction Angle at Base			Weight	Pore water pressure	External Forces and Moments				Analysis	Sliding Forces		Resisting Forces for each method	
	Xm	Ytm	Ybm					b	ΔL	α			β	c'	Φ'	W		u	P	Q	Mp
1	16.88	10.00	9.15	0.75	1.87	66.28	0.00	10.00	0.35	12	0.00	0	0	0	E	11	11	20	18		
2	16.13	10.00	7.64	0.75	1.51	60.10	0.00	10.00	0.35	32	0.00	0	0	0	E	28	30	21	27		
3	15.38	10.00	6.45	0.75	1.31	54.94	0.00	10.00	0.35	48	0.00	0	0	0	E	39	42	23	32		
4	14.38	9.58	5.19	1.25	1.91	49.05	33.69	10.00	0.35	99	0.00	0	0	0	E	75	76	43	55		
5	13.13	8.75	3.90	1.25	1.69	42.45	33.69	10.00	0.35	109	0.00	0	0	0	E	74	74	46	56		
6	11.88	7.92	2.87	1.25	1.55	36.50	33.69	10.00	0.35	114	0.00	0	0	0	E	68	68	49	55		
7	10.63	7.08	2.03	1.25	1.46	30.98	33.69	10.00	0.35	114	0.00	0	0	0	E	59	58	50	53		
8	9.38	6.25	1.35	1.25	1.39	25.76	33.69	10.00	0.35	110	0.00	0	0	0	E	48	48	50	51		
9	8.13	5.42	0.82	1.25	1.34	20.77	33.69	10.00	0.35	104	0.00	0	0	0	E	37	36	49	48		
10	6.88	4.58	0.40	1.25	1.30	15.94	33.69	10.00	0.35	94	0.00	0	0	0	E	26	26	46	45		
11	5.63	3.75	0.10	1.25	1.27	11.22	33.69	10.00	0.35	82	0.00	0	0	0	E	16	16	42	41		
12	4.38	2.92	-0.10	1.25	1.26	6.58	33.69	10.00	0.35	68	0.00	0	0	0	E	8	8	37	36		
13	3.13	2.08	-0.19	1.25	1.25	1.99	33.69	10.00	0.35	51	0.00	0	0	0	E	2	2	31	31		
14	1.88	1.25	-0.19	1.25	1.25	-2.60	33.69	10.00	0.35	32	0.00	0	0	0	E	-1	-2	24	25		
15	0.63	0.42	-0.08	1.25	1.26	-7.20	33.69	10.00	0.35	11	0.00	0	0	0	E	-1	-1	17	17		
16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	10.00	0.35	0	0.00	0	0	0	E	0	0	0	0		
17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	10.00	0.35	0	0.00	0	0	0	E	0	0	0	0		
18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	10.00	0.35	0	0.00	0	0	0	E	0	0	0	0		

Fig. A.8. Results PNW-SLOPE - Example 2.5b - Homogeneous soil 1:1.5

<div><div>PURDUE</div><div>UNIVERSITY</div><div>NORTHWEST</div></div>			PNW-Slope - Results																				
			Project: Example_Sc										Units										
			Author: Thiago										Coordinates		m		Surcharge		kN/m				
			Date: 8/28/2019										Cohesion		kN/m2		Forces		kN				
Coordinates of the slope			Soil layers								Pore water pressure					Unit Weight		kN/m3		Moments		N-m	
Point	x	y	Layer	ytop	ybottom	γ	c'	Φ'	ru	a	Method:	Point	x	y	Critical Slip Surface								
1	-10.00	0.00	Soil 1	10.00	-5.00	18.00	10.00	20.00		E	P1				Xc	Yc	r						
2	0.00	0.00									P2				5.13	18.39	19.09						
3	20.00	10.00									P3				Results								
4	25.00	10.00									P4				Method		Q	R	FS				
											P5				Ordinary Method of Slices		562	754	1.341				
											P6				Bishop Simplified Method		520	739	1.422				
											Surcharge				Spencer Method		-	-	1.419				
											q	xi	xf										
Table of Results for each slice																							
Slice	Coordinates at the midpoint of the slice			Width	Length of the Base	Angle at the Base	Angle at the Top	Cohesion and Friction Angle at Base			Weight	Pore water pressure	External Forces and Moments				Analysis	Sliding Forces		Resisting Forces for each method			
	Xm	Ytm	Ybm					b	ΔL	α			β	c'	Φ'	W		U	P	Q	Mp	T or E	Q - OMS
1	21.90	10.00	9.30	0.76	1.59	61.54	0.00	10.00	0.35	10	0.00	0	0	0	0	E	8	9	18	16			
2	21.14	10.00	8.01	0.76	1.40	57.06	0.00	10.00	0.35	27	0.00	0	0	0	0	E	23	23	19	24			
3	20.38	10.00	6.92	0.76	1.26	53.07	0.00	10.00	0.35	42	0.00	0	0	0	0	E	34	35	22	30			
4	19.17	9.58	5.51	1.67	2.47	47.47	26.57	10.00	0.35	122	0.00	0	0	0	0	E	90	77	55	64			
5	17.50	8.75	3.89	1.67	2.19	40.48	26.57	10.00	0.35	146	0.00	0	0	0	0	E	95	80	62	68			
6	15.83	7.92	2.62	1.67	2.01	34.16	26.57	10.00	0.35	159	0.00	0	0	0	0	E	89	76	68	69			
7	14.17	7.08	1.60	1.67	1.89	28.30	26.57	10.00	0.35	164	0.00	0	0	0	0	E	78	67	72	69			
8	12.50	6.25	0.80	1.67	1.81	22.74	26.57	10.00	0.35	163	0.00	0	0	0	0	E	63	56	73	67			
9	10.83	5.42	0.19	1.67	1.75	17.41	26.57	10.00	0.35	157	0.00	0	0	0	0	E	47	43	72	65			
10	9.17	4.58	-0.25	1.67	1.71	12.23	26.57	10.00	0.35	145	0.00	0	0	0	0	E	31	30	69	61			
11	7.50	3.75	-0.53	1.67	1.68	7.15	26.57	10.00	0.35	129	0.00	0	0	0	0	E	16	19	63	56			
12	5.83	2.92	-0.67	1.67	1.67	2.12	26.57	10.00	0.35	108	0.00	0	0	0	0	E	4	9	56	50			
13	4.17	2.08	-0.66	1.67	1.67	-2.89	26.57	10.00	0.35	82	0.00	0	0	0	0	E	-4	1	47	42			
14	2.50	1.25	-0.50	1.67	1.68	-7.92	26.57	10.00	0.35	53	0.00	0	0	0	0	E	-7	-3	36	34			
15	0.83	0.42	-0.19	1.67	1.71	-13.01	26.57	10.00	0.35	18	0.00	0	0	0	0	E	-4	-2	24	23			
16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	10.00	0.35	0	0.00	0	0	0	0	E	0	0	0	0			
17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	10.00	0.35	0	0.00	0	0	0	0	E	0	0	0	0			
18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	10.00	0.35	0	0.00	0	0	0	0	E	0	0	0	0			

Fig. A.9. Results PNW-SLOPE - Example 2.5c - Homogeneous soil 1:2

<div><div><div>PURDUE</div><div>UNIVERSITY</div><div>NORTHWEST</div></div></div>			PNW-Slope - Results																		
			Project: Example 6										Units								
			Author: Thiago										Coordinates		m	Surcharge	kN/m				
			Date: 8/28/2019										Cohesion		kN/m ²	Forces	kN				
Coordinates of the slope			Soil layers								Pore water pressure					Unit Weight		kN/m ³		Moments	N-m
Point	x	y	Layer	ytop	ybottom	γ	c'	Φ'	ru	a	Method:	Piezometric line	Critical Slip Surface								
1	-20.00	0.00	Soil 1	30.00	20.00	19.00	8.00	18.00		E	Point	x	Y	Xc	Yc	r					
2	0.00	0.00	Soil 2	20.00	12.00	18.00	5.00	25.00		E	P1	-20.00	0.00	26.00	42.00	55.40					
3	20.00	15.00	Soil 3	12.00	3.00	18.50	10.00	20.00		E	P2	90.00	15.00	Results							
4	50.00	20.00	Soil 4	3.00	-15.00	19.00	8.00	18.00		E	P3	Method		Q	R	FS					
5	70.00	30.00									P4	Ordinary Method of Slices		8390	8088	0.964					
6	90.00	30.00									P5	Bishop Simplified Method		8390	9731	1.160					
											P6	Spencer Method		-	-	1.152					
											Surcharge			q	xi	xf					
											10.00	70.00	90.00								
Table of Results for each slice																					
Slice	Coordinates at the midpoint of the slice			Width	Length of the Base	Angle at the Base	Angle at the Top	Cohesion and Friction Angle at Base		Weight	Pore water pressure	External Forces and Moments			Analysis	Sliding Forces		Resisting Forces for each method			
	Xm	Ytm	Ybm	b	ΔL	α	β	c'	Φ'	W	u	P	Q	Mp	T or E	Q - OMS	Q - BMS	R - OMS	R - BMS		
1	78.40	30.00	24.86	3.36	10.81	71.89	0.00	8.00	0.31	328	0.00	0	34	-1762	E	344	344	123	231		
2	75.04	30.00	16.49	3.36	7.28	62.52	0.00	5.00	0.44	851	0.00	0	34	-1649	E	785	785	227	506		
3	71.68	30.00	10.80	3.36	5.96	55.67	0.00	10.00	0.35	1198	16.71	0	34	-1536	E	1017	1017	301	546		
4	68.00	29.00	6.00	4.00	6.15	49.40	26.57	10.00	0.35	1704	58.80	0	0	0	E	1294	1294	409	646		
5	64.00	27.00	1.78	4.00	5.50	43.37	26.57	8.00	0.31	1867	94.87	0	0	0	E	1282	1282	395	560		
6	60.00	25.00	-1.67	4.00	5.07	37.91	26.57	8.00	0.31	1977	123.33	0	0	0	E	1214	1214	421	535		
7	56.00	23.00	-4.51	4.00	4.76	32.82	26.57	8.00	0.31	2041	145.90	0	0	0	E	1106	1106	436	510		
8	52.00	21.00	-6.87	4.00	4.53	28.02	26.57	8.00	0.31	2068	163.64	0	0	0	E	971	971	442	484		
9	47.86	19.64	-8.85	4.29	4.66	23.26	9.46	8.00	0.31	2268	177.56	0	0	0	E	896	896	487	509		
10	43.57	18.93	-10.49	4.29	4.52	18.51	9.46	8.00	0.31	2347	187.90	0	0	0	E	745	745	511	516		
11	39.29	18.21	-11.74	4.29	4.41	13.89	9.46	8.00	0.31	2393	194.40	0	0	0	E	574	574	527	521		
12	35.00	17.50	-12.62	4.29	4.34	9.36	9.46	8.00	0.31	2410	197.33	0	0	0	E	392	392	536	526		
13	30.71	16.79	-13.16	4.29	4.30	4.89	9.46	8.00	0.31	2398	196.85	0	0	0	E	204	204	538	529		
14	26.43	16.07	-13.36	4.29	4.29	0.44	9.46	8.00	0.31	2360	193.08	0	0	0	E	18	18	532	531		
15	22.14	15.36	-13.22	4.29	4.30	-4.00	9.46	8.00	0.31	2294	186.04	0	0	0	E	-160	-160	519	532		
16	18.00	13.50	-12.78	4.00	4.04	-8.31	36.87	8.00	0.31	1973	176.17	0	0	0	E	-285	-285	440	468		
17	14.00	10.50	-12.05	4.00	4.10	-12.52	36.87	8.00	0.31	1698	163.60	0	0	0	E	-368	-368	364	406		
18	10.00	7.50	-11.00	4.00	4.18	-16.80	36.87	8.00	0.31	1397	147.98	0	0	0	E	-404	-404	284	335		
19	6.00	4.50	-9.62	4.00	4.29	-21.18	36.87	8.00	0.31	1070	129.11	0	0	0	E	-387	-387	202	255		
20	2.00	1.50	-7.88	4.00	4.44	-25.69	36.87	8.00	0.31	713	92.01	59	0	2560	E	-355	-355	145	204		
21	-1.69	0.00	-5.95	3.38	3.90	-30.01	0.00	8.00	0.31	381	58.30	83	0	2289	E	-232	-232	106	157		
22	-5.06	0.00	-3.83	3.38	4.08	-34.13	0.00	8.00	0.31	245	37.53	67	0	2095	E	-176	-176	83	130		
23	-8.44	0.00	-1.34	3.38	4.31	-38.47	0.00	8.00	0.31	86	13.15	52	0	1797	E	-86	-86	58	94		

Fig. A.10. Results PNW-SLOPE - Example 2.6 - Slope with more complex geometry

[illegible]

[illegible]

Fig. A.13. Results PNW-SLOPE - Example 3.1c - Heterogeneous soil with different COV

[illegible]

Fig. A.14. Results PNW-SLOPE - Example 3.1d - Heterogeneous soil with different COV

[illegible]

Fig. A.15. Results PNW-SLOPE - Example 3.2a - Homogeneous soil
- Only Cohesion as random variable

[illegible]

Fig. A.16. Results PNW-SLOPE - Example 3.2b - Homogeneous soil
- Only internal friction angle as random variable

[illegible]

Fig. A.17. Results PNW-SLOPE - Example 3.2c - Homogeneous soil
- Only unit weight as random variable

[illegible]

Fig. A.18. Results PNW-SLOPE - Example 3.2d - Homogeneous soil
- All soil parameters as random variables

[illegible]

Fig. A.19. Results PNW-SLOPE - Example 3.3a - Homogeneous cohesive soil - Spatial variation - $\lambda_h = 1$

[illegible]

Fig. A.20. Results PNW-SLOPE - Example 3.3b - Homogeneous cohesive soil – Spatial variation - $\lambda_h = 2$

[illegible]

Fig. A.21. Results PNW-SLOPE - Example 3.3c - Homogeneous cohesive soil – Spatial variation - $\lambda_h = 3$

[illegible]

Fig. A.22. Results PNW-SLOPE - Example 3.3d - Homogeneous cohesive soil – Spatial variation - $\lambda_h = 4$

[illegible]

Fig. A.23. Results PNW-SLOPE - Example 3.3e - Homogeneous cohesive soil – Spatial variation - $\lambda_b = 8$

[illegible]

Fig. A.24. Results PNW-SLOPE - Example 3.3f - Homogeneous cohesive soil – Spatial variation - $\lambda_h = \infty$

<div><div>PURDUE</div><div>UNIVERSITY</div><div>NORTHWEST</div></div>			PNW-Slope - Results - Probabilistic												Units				
			Project: Ch3_Example4a Author: Thiago Date: 9/3/2019										Coordinates		m	Surcharge	kN/m		
Coordinates of the slope			Soil layers - Deterministic								Pore water pressure				Cohesion		kN/m ²	Forces	kN
															Unit Weight		kN/m ³	Moments	N-m
			Critical Slip Surface																
Point	x	y	Layer	ytop	ybottom	y	c'	Φ'	ru	a	Method:			Xc	Yc	r			
1	-20.00	0.00	Soil 1	10.00	7.50	20.00	40.00	0.00		E	Point	x	Y	9.90	19.00	29.00			
2	0.00	0.00	Soil 2	7.50	5.00	20.00	40.00	0.00		E	P1								
3	20.00	10.00	Soil 3	5.00	2.50	20.00	40.00	0.00		E	P2								
4	40.00	10.00	Soil 4	2.50	0.00	20.00	40.00	0.00		E	P3								
			Soil 5	0.00	-2.50	20.00	40.00	0.00		E	P4								
			Soil 6	-2.50	-5.00	20.00	40.00	0.00		E	P5								
			Soil 7	-5.00	-7.50	20.00	40.00	0.00		E	P6								
			Soil 8	-7.50	-10.00	20.00	40.00	0.00		E									

Fig. A.25. Results PNW-SLOPE - Example 3.4a - Homogeneous cohesive soil – Spatial variation - 8 layers

PURDUE UNIVERSITY NORTHWEST			PNW-Slope - Results - Probabilistic													Units					
Project: Ch3_Example4b																Coordinates		m	Surcharge		kN/m
Author: Thiago																Cohesion		kN/m ²	Forces		kN
Date: 9/3/2019																Unit Weight		kN/m ³	Moments		N-m
Coordinates of the slope			Soil layers - Deterministic								Pore water pressure					Critical Slip Surface					
Point	x	y	Layer	ytop	ybottom	y	c'	Φ'	ru	a	Method:	x	Y	Xc	Yc	r					
1	-20.00	0.00	Soil 1	10.00	5.00	20.00	40.00	0.00		E	P1			9.90	19.00	29.00					
2	0.00	0.00	Soil 2	5.00	0.00	20.00	40.00	0.00		E	P2										
3	20.00	10.00	Soil 3	0.00	-5.00	20.00	40.00	0.00		E	P3										
4	40.00	10.00	Soil 4	-5.00	-10.00	20.00	40.00	0.00		E	P4										
											P5										
											P6										
							</														

Fig. A.26. Results PNW-SLOPE - Example 3.4b - Homogeneous cohesive soil – Spatial variation - 4 layers

[illegible]

Fig. A.27. Results PNW-SLOPE - Example 3.4c - Homogeneous cohesive soil – Spatial variation - 2 layers

[illegible]

Fig. A.28. Results PNW-SLOPE - Example 3.4d - Homogeneous cohesive soil – Spatial variation - 1 layer

Appendix B. PNW-SLOPE - Coding

This appendix presents some of the coding present in PNW-SLOPE. This modules can be accessed by using VBA Editor inside Microsoft Excel.

List of Sheets:

- Run (Sheet3)
- Image (Sheet6)
- Report_Det (Sheet2)
- Report_Prob (Sheet5)
- OMS_Calc (Sheet14)
- BMS_Calc (Sheet11)
- SPM_Calc (Sheet17)
- Prob_MC (Sheet16)
- Prob_R (Sheet13)
- Prob_Par (Sheet1)
- Scatter (Sheet12)
- Prob_Ref_OMS (Sheet15)
- Prob_Ref_OMS_SV (Sheet20)
- Geo (Sheet19)
- Last_Run (Sheet4)

List of Forms:

- Start
- Det1

- Det2
- Prob1
- Prob1_Histo
- Prob1_Scatter
- Prob1_Soil
- Prob1_Ref
- Prob1_Conv
- Progress

List of Modules:

- A_General
- B_Read_Last_Geo
- C_Read_Last_Ana
- D_Read_Last_Prob
- E_Write_Geo
- F_Write_Last_Ana
- G_Write_Prob
- H_Save
- I_Load
- J_Geometry
- K_Slices
- L_Run
- M_OMS
- N_BMS
- O_SPM

- P_MonteCarlo
- Q_MonteCarlo2
- R_QR_Open
- S_Hist_Parameters
- T_Scatter
- U_Convergence


```

J_Geometry - 1

Const pi = 3.14159265
Sub Draw_Geom2()

'On Error GoTo ProcError

.....'Verifications'.....
If Det1.Option1 = True Or Det1.Option2 = True Then
If Det1.H1.Value = "" Or Det1.S1.Value = "" Or Det1.Hor1.Value = "" Then
MsgBox ("Error: Verify values of H, slope and horizontal distance")
GoTo End_Run
End If
If Val(Det1.H1) > Val(Det1.Ymax) Then
Det1.Ymax = Det1.H1
End If

If Val(Det1.Hor1) > Val(Det1.Xmax) Then
Det1.Xmax = Det1.Hor1 + 5
End If

End If

If Det1.Option3 = True Then
If Val(Det1.plx) < Val(Det1.Xmin) Then
Det1.Xmin = Det1.plx
End If
If Val(Det1.Controls("p" & Det1.Number_Points_Top & "x")) > Val(Det1.Xmax) Then
Det1.Xmax = Det1.Controls("p" & Det1.Number_Points_Top & "x")
End If
If Val(Det1.Yt1) <> Val(Det1.Controls("p" & Det1.Number_Points_Top & "y")) Then
MsgBox ("Ymax in the geometry is different than in layers")
GoTo End_Run
End If
If Val(Det1.SS_c_xf) > Val(Det1.Controls("p" & Det1.Number_Points_Top & "x")) Then
Det1.Controls("p" & Det1.Number_Points_Top & "x") = Det1.SS_c_xf
End If

End If

If Det1.Option_Crit.Value = True And (Det1.SS_Yc_t.Value = "" Or Det1.SS_Yc_b.Value = "") Then
MsgBox ("Error: Verify values of center point of critical slip surface to be calculated")
GoTo End_Run
End If

If Val(Det1.Controls("Yb" & Det1.Number_Layers)) < Val(Det1.Ymin) Then
Det1.Ymin = Det1.Controls("Yb" & Det1.Number_Layers)
End If
If Val(Det1.SS_c_xf) > Val(Det1.Xmax) Then
Det1.Xmax = Det1.SS_c_xf
End If
If Val(Det1.SS_t_xi) < Val(Det1.Xmin) Then
Det1.Xmin = Det1.SS_t_xi
End If
If Val(Det1.SS_t_xi) > Val(Det1.SS_t_xf) Then
MsgBox ("Error: X initial of Toe is greater than X final")
GoTo End_Run
End If
If Val(Det1.SS_c_xi) > Val(Det1.SS_c_xf) Then

```

J_Geometry - 2

```

MsgBox ("Error: X initial of Crest is greater than X final")
GoTo End_Run
End If
For i = 1 To Val(Det1.Number_Layers)
If Val(Det1.Controls("Yb" & i)) < Val(Det1.Ymin) Then
Det1.Ymin = Det1.Controls("Yb" & i).Value
End If
Next
For i = 1 To Val(Det1.Number_Layers)
If Val(Det1.Controls("Yt" & i)) > Val(Det1.Ymax) Then
Det1.Ymax = Det1.Controls("Yt" & i)
End If
Next

If Det1.Cons_ppw.Value = True And Det1.Option_piez = True Then
For i = 1 To Val(Det1.Number_Layers)
If Val(Det1.Controls("PP" & i & "x")) < Val(Det1.Xmin) Then
Det1.Controls("PP" & i & "x") = Det1.Xmin
End If
Next
End If
If Det1.Cons_ppw.Value = True And Det1.Option_piez = True Then
For i = 1 To Val(Det1.Number_Layers)
If Val(Det1.Controls("PP" & i & "x")) > Val(Det1.Xmax) Then
Det1.Controls("PP" & i & "x") = Det1.Xmax
End If
Next
End If
If Det1.Cons_ppw.Value = True And Det1.Option_piez = True Then
For i = 1 To Val(Det1.Number_Layers)
If Val(Det1.Controls("PP" & i & "y")) > Val(Det1.Ymax) Then
Det1.Controls("PP" & i & "x") = Det1.Ymax
End If
Next
End If
If Det1.Cons_ppw.Value = True And Det1.Option_piez = True Then
For i = 1 To Val(Det1.Number_Layers)
If Val(Det1.Controls("PP" & i & "y")) < Val(Det1.Ymin) Then
Det1.Controls("PP" & i & "x") = Det1.Ymin
End If
Next
End If

If Det1.Number_Points_Top.Value < 10 Then
Det1.p10x.Value = ""
Det1.p10y.Value = ""
If Det1.Number_Points_Top.Value < 9 Then
Det1.p9x.Value = ""
Det1.p9y.Value = ""
If Det1.Number_Points_Top.Value < 8 Then
Det1.p8x.Value = ""
Det1.p8y.Value = ""
If Det1.Number_Points_Top.Value < 7 Then
Det1.p7x.Value = ""
Det1.p7y.Value = ""
If Det1.Number_Points_Top.Value < 6 Then
Det1.p6x.Value = ""
Det1.p6y.Value = ""

```

J_Geometry - 3

```

If Det1.Number_Points_Top.Value < 5 Then
Det1.p5x.Value = ""
Det1.p5y.Value = ""
If Det1.Number_Points_Top.Value < 4 Then
Det1.p4x.Value = ""
Det1.p4y.Value = ""
If Det1.Number_Points_Top.Value < 3 Then
Det1.p3x.Value = ""
Det1.p3y.Value = ""
End If
End If
End If
End If
End If
End If
End If

```

```

.....
.....

```

```

Dim H As Double
Dim S As Double
Dim Hor As Double

```

```

Dim p1xx As Double
Dim p2xx As Double
Dim p3xx As Double
Dim p4xx As Double
Dim p5xx As Double
Dim p6xx As Double
Dim p7xx As Double
Dim p8xx As Double
Dim p9xx As Double
Dim p10xx As Double
Dim p1yy As Double
Dim p2yy As Double
Dim p3yy As Double
Dim p4yy As Double
Dim p5yy As Double
Dim p6yy As Double
Dim p7yy As Double
Dim p8yy As Double
Dim p9yy As Double
Dim p10yy As Double

```

```

Dim Top_p As Double
Dim Bottom_p As Double
Dim NPY As Double
Dim Lxi As Double
Dim Lyi As Double
Dim Lxf As Double
Dim Lyf As Double

```

```

Application.ScreenUpdating = False
'Det1.Hide
Start.Hide

```

J_Geometry - 4

```
'ActiveWorkbook.Windows(1).Visible = False
'Application.Visible = False
```

```
Progress.Label1.Caption = "Running..."
Progress.Show vbModeless
Progress.Repaint
```

```
.....
'Clear previous data
```

```
For i = 1 To 10
Sheet6.Cells(12 + i, 2) = ""
Sheet6.Cells(12 + i, 3) = ""
Sheet6.Cells(12 + i, 4) = ""
Sheet6.Cells(12 + i, 5) = ""
Next
```

```
For i = 1 To 10
Sheet6.Cells(2, 27 + i) = ""
Sheet6.Cells(3, 27 + i) = ""
Next
.....
```

```
If Det1.Option1.Value = True Or Det1.Option2.Value = True Then
```

```
H = Det1.H1.Value
S = Det1.S1.Value
Hor = Det1.Hor1.Value
```

```
Sheet6.Cells(1, 27) = 4
Sheet6.Cells(2, 28) = Det1.Xmin.Value
Sheet6.Cells(3, 28) = 0
Sheet6.Cells(2, 29) = 0
Sheet6.Cells(3, 29) = 0
Sheet6.Cells(2, 30) = Hor
Sheet6.Cells(3, 30) = H
Sheet6.Cells(2, 31) = Det1.Xmax.Value
Sheet6.Cells(3, 31) = H
p1xx = Det1.Xmin
p1yy = 0
p2xx = 0
p2yy = 0
p3xx = Hor
p3yy = Det1.Yt1.Value
p4xx = Det1.Xmax
p4yy = Det1.Yt1.Value
NPY = 4
```

```
Det1.Number_Points_Top = 4
Det1.p1x.Value = p1xx
Det1.p1y.Value = p1yy
Det1.p2x.Value = p2xx
Det1.p2y.Value = p2yy
Det1.p3x.Value = p3xx
Det1.p3y.Value = p3yy
Det1.p4x.Value = p4xx
Det1.p4y.Value = p4yy
```

```
p1x = Det1.Xmin
p1y = 0
p2x = 0
p2y = 0
```

J_Geometry - 5

```
p3x = Hor
p3y = H
p4x = Det1.Xmax
p4y = H
```

```
Sheet6.Cells(8, 2) = p1x
Sheet6.Cells(8, 3) = p2x
Sheet6.Cells(8, 4) = p3x
Sheet6.Cells(8, 5) = p4x
Sheet6.Cells(9, 2) = p1y
Sheet6.Cells(9, 3) = p2y
Sheet6.Cells(9, 4) = p3y
Sheet6.Cells(9, 5) = p4y
```

Else

```
p1xx = 0
p2xx = 0
p3xx = 0
p4xx = 0
p5xx = 0
p6xx = 0
p7xx = 0
p8xx = 0
p9xx = 0
p10xx = 0
p1yy = 0
p2yy = 0
p3yy = 0
p4yy = 0
p5yy = 0
p6yy = 0
p7yy = 0
p8yy = 0
p9yy = 0
p10yy = 0
```

```
If Det1.Number_Points_Top >= 1 Then
p1xx = Det1.p1x.Value
p1yy = Det1.p1y.Value
Sheet6.Cells(2, 28) = p1xx
Sheet6.Cells(3, 28) = p1yy
End If
If Det1.Number_Points_Top >= 2 Then
p2xx = Det1.p2x.Value
p2yy = Det1.p2y.Value
Sheet6.Cells(2, 29) = p2xx
Sheet6.Cells(3, 29) = p2yy
End If
If Det1.Number_Points_Top >= 3 Then
p3xx = Det1.p3x.Value
p3yy = Det1.p3y.Value
Sheet6.Cells(2, 30) = p3xx
Sheet6.Cells(3, 30) = p3yy
End If
If Det1.Number_Points_Top >= 4 Then
p4xx = Det1.p4x.Value
p4yy = Det1.p4y.Value
```

J_Geometry - 6

```

Sheet6.Cells(2, 31) = p4xx
Sheet6.Cells(3, 31) = p4yy
End If
If Det1.Number_Points_Top >= 5 Then
p5xx = Det1.p5x.Value
p5yy = Det1.p5y.Value
Sheet6.Cells(2, 32) = p5xx
Sheet6.Cells(3, 32) = p5yy
End If
If Det1.Number_Points_Top >= 6 Then
p6xx = Det1.p6x.Value
p6yy = Det1.p6y.Value
Sheet6.Cells(2, 33) = p6xx
Sheet6.Cells(3, 33) = p6yy
End If
If Det1.Number_Points_Top >= 7 Then
p7xx = Det1.p7x.Value
p7yy = Det1.p7y.Value
Sheet6.Cells(2, 34) = p7xx
Sheet6.Cells(3, 34) = p7yy
End If
If Det1.Number_Points_Top >= 8 Then
p8xx = Det1.p8x.Value
p8yy = Det1.p8y.Value
Sheet6.Cells(2, 35) = p8xx
Sheet6.Cells(3, 35) = p8yy
End If
If Det1.Number_Points_Top >= 9 Then
p9xx = Det1.p9x.Value
p9yy = Det1.p9y.Value
Sheet6.Cells(2, 36) = p9xx
Sheet6.Cells(3, 36) = p9yy
End If
If Det1.Number_Points_Top >= 10 Then
p10xx = Det1.p10x.Value
p10yy = Det1.p10y.Value
Sheet6.Cells(2, 37) = p10xx
Sheet6.Cells(3, 37) = p10yy
End If

NPY = Det1.Number_Points_Top.Value

p1x = Det1.p1x.Value
p1y = Det1.p1y.Value

p4x = Det1.Controls("p" & Det1.Number_Points_Top.Value & "x").Value
p4y = Det1.Controls("p" & Det1.Number_Points_Top.Value & "y").Value
Sheet6.Cells(8, 2) = p1x
Sheet6.Cells(8, 5) = p4x
Sheet6.Cells(9, 2) = p1y
Sheet6.Cells(9, 5) = p4y

End If

If Det1.Option_Sur.Value = True Then
Sheet6.Cells(15, 10) = Det1.Sur_q.Value

```

J_Geometry - 7

```
Sheet6.Cells(16, 10) = Det1.Sur_xi.Value
Sheet6.Cells(17, 10) = Det1.Sur_xf.Value
Else
Sheet6.Cells(15, 10) = ""
Sheet6.Cells(16, 10) = ""
Sheet6.Cells(17, 10) = ""
End If
```

```
If Det1.Cons_ppw.Value = True And Det1.Option_ppr.Value = True Then
Sheet6.Cells(12, 10) = "x"
Else
Sheet6.Cells(12, 10) = ""
End If
```

```
If Det1.Cons_ppw.Value = True And Det1.Option_piez.Value = True Then
Sheet6.Cells(26, 8) = Det1.PP1x.Value
Sheet6.Cells(26, 9) = Det1.PP2x.Value
Sheet6.Cells(26, 10) = Det1.PP3x.Value
Sheet6.Cells(26, 11) = Det1.PP4x.Value
Sheet6.Cells(26, 12) = Det1.PP5x.Value
Sheet6.Cells(26, 13) = Det1.PP6x.Value
Sheet6.Cells(27, 8) = Det1.PP1y.Value
Sheet6.Cells(27, 9) = Det1.PP2y.Value
Sheet6.Cells(27, 10) = Det1.PP3y.Value
Sheet6.Cells(27, 11) = Det1.PP4y.Value
Sheet6.Cells(27, 12) = Det1.PP5y.Value
Sheet6.Cells(27, 13) = Det1.PP6y.Value
Else
Sheet6.Cells(26, 8) = ""
Sheet6.Cells(26, 9) = ""
Sheet6.Cells(26, 10) = ""
Sheet6.Cells(26, 11) = ""
Sheet6.Cells(26, 12) = ""
Sheet6.Cells(26, 13) = ""
Sheet6.Cells(27, 8) = ""
Sheet6.Cells(27, 9) = ""
Sheet6.Cells(27, 10) = ""
Sheet6.Cells(27, 11) = ""
Sheet6.Cells(27, 12) = ""
Sheet6.Cells(27, 13) = ""
End If
```

```
If Det1.Option_Crit.Value = True Then
For cc = 0 To 10
Sheet6.Cells(8 + cc, 7) = Det1.SS_c_xi.Value + (Det1.SS_c_xf.Value - Det1.SS_c_xi.Value) / 10 * cc
Sheet6.Cells(8 + cc, 9) = Det1.SS_t_xi.Value + (Det1.SS_t_xf.Value - Det1.SS_t_xi.Value) / 10 * cc
Sheet6.Cells(8 + cc, 8) = Calc_y_slope(NPY, p1xx, p2xx, p3xx, p4xx, p5xx, p6xx, p7xx, p8xx, p9xx, p10xx, _
    p1yy, p2yy, p3yy, p4yy, p5yy, p6yy, p7yy, p8yy, p9yy, p10yy, Sheet6.Cells(8 + cc, 7))
Sheet6.Cells(8 + cc, 10) = Calc_y_slope(NPY, p1xx, p2xx, p3xx, p4xx, p5xx, p6xx, p7xx, p8xx, p9xx, p10xx, _
    p1yy, p2yy, p3yy, p4yy, p5yy, p6yy, p7yy, p8yy, p9yy, p10yy, Sheet6.Cells(8 + cc, 9))
Next
End If
```

```
For i = 1 To Det1.Number_Layers
Top = Det1.Controls("Yt" & i).Value
```

J_Geometry - 8

```

    bottom = Det1.Controls("Yb" & i).Value
    Top_p = Top
    Bottom_p = bottom
    If bottom >= 0 Then
        Lxi = Calc_x_slope(NPY, p1xx, p2xx, p3xx, p4xx, p5xx, p6xx, p7xx, p8xx, p9xx, p10xx, _
            p1yy, p2yy, p3yy, p4yy, p5yy, p6yy, p7yy, p8yy, p9yy, p10yy, Bottom_p)

        Lyi = bottom
        Lxf = p4x
        Lyf = bottom
    Else
        Lxi = p1x
        Lyi = bottom
        Lxf = p4x
        Lyf = bottom
    End If

    Sheet6.Cells(12 + i, 2) = Lxi
    Sheet6.Cells(12 + i, 3) = Lxf
    Sheet6.Cells(12 + i, 4) = Lyi
    Sheet6.Cells(12 + i, 5) = Lyf

Next

Dim cht As ChartObject
Dim ser As Series
Dim sc As SeriesCollection
Dim myBuilder As FreeformBuilder
Dim myShape As Shape
Dim Top_Val As Double
Sheet6.Activate

Set cht = ActiveSheet.ChartObjects(1)

cht.Chart.ChartArea.ClearContents

For Each myShape In cht.Chart.Shapes
    myShape.Delete
Next myShape

Top_Val = Det1.Number_Points_Top.Value

Set srsNew = cht.Chart.SeriesCollection.NewSeries
With srsNew
    .Name = "Top"
    .Values = ActiveSheet.Range(Cells(3, 28), Cells(3, 28 + Top_Val - 1))
    .XValues = ActiveSheet.Range(Cells(2, 28), Cells(2, 28 + Top_Val - 1))
    .Format.Line.Weight = 3
    .Format.Line.ForeColor.RGB = RGB(0, 0, 0)
    .MarkerStyle = xlMarkerStyleNone
End With

If Det1.Cons_ppw.Value = True And Det1.Option_piez.Value = True Then

    Set srsNew = cht.Chart.SeriesCollection.NewSeries

```


J_Geometry - 9

```

        With srsNew
            .Name = "Piez"
            .Values = ActiveSheet.Range(Cells(27, 8), Cells(27, 13))
            .XValues = ActiveSheet.Range(Cells(26, 8), Cells(26, 13))
            .Format.Line.Weight = 3
            .Format.Line.ForeColor.RGB = RGB(0, 0, 204)
            .MarkerStyle = xlMarkerStyleNone
            .Format.Line.DashStyle = msoLineDashDotDot
        End With

    End If

    Sheet19.Activate

    'Resetting spreadsheet'.....
    Cells(2, 28) = ""
    Cells(2, 29) = ""
    Cells(3, 28) = ""
    Cells(3, 29) = ""
    Cells(4, 28) = ""
    Cells(4, 29) = ""
    Cells(5, 28) = ""
    Cells(5, 29) = ""
    Cells(6, 28) = ""
    Cells(6, 29) = ""
    Cells(7, 28) = ""
    Cells(7, 29) = ""
    Cells(8, 28) = ""
    Cells(8, 29) = ""
    Cells(9, 28) = ""
    Cells(9, 29) = ""
    Cells(10, 28) = ""
    Cells(10, 29) = ""
    Cells(11, 28) = ""
    Cells(11, 29) = ""
    Cells(6, 12) = ""
    Cells(6, 13) = ""

    For i = 1 To 10
        Cells(i + 7, 2) = ""
        Cells(i + 7, 3) = ""
        Cells(i + 7, 4) = ""
        Cells(i + 7, 5) = ""
        Cells(i + 7, 6) = ""
        Cells(i + 7, 7) = ""
        Cells(i + 7, 8) = ""
    Next

    For i = 1 To 6
        Cells(9, 9 + i) = ""
        Cells(10, 9 + i) = ""
    Next

    Cells(12, 10) = ""
    Cells(13, 10) = ""
    Cells(15, 10) = ""
    Cells(16, 10) = ""
    Cells(17, 10) = ""

```

J_Geometry - 10

```
Cells(1, 27) = Det1.Number_Points_Top.Value
Cells(2, 28) = Det1.p1x.Value
Cells(2, 29) = Det1.p1y.Value
Cells(3, 28) = Det1.p2x.Value
Cells(3, 29) = Det1.p2y.Value
Cells(4, 28) = Det1.p3x.Value
Cells(4, 29) = Det1.p3y.Value
Cells(5, 28) = Det1.p4x.Value
Cells(5, 29) = Det1.p4y.Value
Cells(6, 28) = Det1.p5x.Value
Cells(6, 29) = Det1.p5y.Value
Cells(7, 28) = Det1.p6x.Value
Cells(7, 29) = Det1.p6y.Value
Cells(8, 28) = Det1.p7x.Value
Cells(8, 29) = Det1.p7y.Value
Cells(9, 28) = Det1.p8x.Value
Cells(9, 29) = Det1.p8y.Value
Cells(10, 28) = Det1.p9x.Value
Cells(10, 29) = Det1.p9y.Value
Cells(11, 28) = Det1.p10x.Value
Cells(11, 29) = Det1.p10y.Value
```

```
For i = 1 To Det1.Number_Layers
Cells(7 + i, 2) = Det1.Controls("Yt" & i).Value
Cells(7 + i, 3) = Det1.Controls("Yb" & i).Value
Cells(7 + i, 4) = Det1.Controls("w" & i).Value
Cells(7 + i, 5) = Det1.Controls("c" & i).Value
Cells(7 + i, 6) = Det1.Controls("f" & i).Value
If Det1.Option_ppr.Value = True Then
Cells(7 + i, 7) = Det1.Controls("r" & i).Value
Else
Cells(7 + i, 7) = ""
End If
```

Next

If Det1.Option_Crit.Value = True Then

```
Cells(2, 20) = Det1.SS_t_xi.Value
Cells(2, 21) = Det1.SS_c_xf.Value
Cells(2, 22) = Det1.SS_Yc_b.Value
Cells(4, 22) = Det1.SS_t_xi.Value + (Det1.SS_c_xf.Value - Det1.SS_t_xi.Value) / 2
```

```
Cells(2, 2).Formula = "=Y15"
Cells(2, 3).Formula = "=Y16"
Cells(2, 4).Formula = "=Y8"
```

```
Cells(2, 32) = Det1.SS_t_xf.Value
Cells(2, 33) = Det1.SS_c_xi.Value
Cells(2, 34) = Det1.SS_Yc_t.Value
Cells(4, 34) = Det1.SS_t_xf.Value + (Det1.SS_c_xi.Value - Det1.SS_t_xf.Value) / 2
```

```
Cells(3, 2).Formula = "=AK15"
Cells(3, 3).Formula = "=AK16"
Cells(3, 4).Formula = "=AK8"
```

Else

J_Geometry - 11

```
Cells(2, 2) = Det1.Xc.Value
Cells(2, 3) = Det1.Yc.Value
Cells(2, 4) = Det1.r.Value
```

```
If Cells(2, 2) >= 0 Then
Cells(2, 20) = Cells(2, 2) - Sqr(Cells(2, 4) ^ 2 - Cells(2, 3) ^ 2)
Else
Cells(2, 20) = Cells(2, 2) + Sqr(Cells(2, 4) ^ 2 - Cells(2, 3) ^ 2)
End If
Cells(2, 21) = Cells(2, 2) + Sqr(Cells(2, 4) ^ 2 - (Cells(2, 3) - Cells(18, 2)) ^ 2)
```

End If

```
If Det2.Option_Constant.Value = False Then
Det2.N_Slices.Value = Val(Det2.n1.Value) + Val(Det2.n2.Value) + Val(Det2.n3.Value) + Val(Det2.n4.Value) + Val(Det2.n5.Value) + Val(Det2.n6.Value) + Val(Det2.n7.Value) + Val(Det2.n8.Value) + Val(Det2.n9.Value) + Val(Det2.n10.Value)
End If
```

Cells(2, 8) = 20

```
For i = 1 To 121
Cells(20 + i, 1) = ""
Cells(20 + i, 2) = ""
Next
```

```
For i = 1 To 20
Cells(20 + i, 1) = i
Range("B" & 20 + i).Formula = "=B" & 19 + i & " - ($F$2-$E$2)/$H$2"
Next
```

For i = 1 To 21

```
Sheet6.Cells(25 + i, 2) = Sheet19.Cells(19 + i, 2)
Sheet6.Cells(25 + i, 3) = Sheet19.Cells(19 + i, 3)
```

```
Sheet6.Cells(25 + i, 4) = Sheet19.Cells(19 + i, 11)
Sheet6.Cells(25 + i, 5) = Sheet19.Cells(19 + i, 12)
```

Next

Sheet6.Activate

Set srsNew = cht.Chart.SeriesCollection.NewSeries

```
With srsNew
.Name = "Slip1"
.Values = ActiveSheet.Range(Cells(26, 3), Cells(26 + 20, 3))
.XValues = ActiveSheet.Range(Cells(26, 2), Cells(26 + 20, 2))
.Format.Line.Weight = 3
```

J_Geometry - 12

```

        .Format.Line.ForeColor.RGB = RGB(255, 0, 0)
        .MarkerStyle = xlMarkerStyleNone
    End With

If Det1.Option_Crit.Value = True Then
Set srsNew = cht.Chart.SeriesCollection.NewSeries

    With srsNew
        .Name = "Slip2"
        .Values = ActiveSheet.Range(Cells(26, 5), Cells(26 + 20, 5))
        .XValues = ActiveSheet.Range(Cells(26, 4), Cells(26 + 20, 4))
        .Format.Line.Weight = 3
        .Format.Line.ForeColor.RGB = RGB(255, 0, 0)
        .MarkerStyle = xlMarkerStyleNone
    End With

Else

For i = 1 To cht.Chart.SeriesCollection.Count
    If cht.Chart.SeriesCollection(i).Name = "Slip2" Then
        cht.Chart.SeriesCollection(i).Delete
    End If
Next

End If

'End If

'Adjust y-axis Scale
cht.Chart.Axes(xlValue).MinimumScale = Det1.Ymin.Value
cht.Chart.Axes(xlValue).MaximumScale = Det1.Ymax.Value

'Adjust x-axis Scale
cht.Chart.Axes(xlCategory).MinimumScale = Det1.Xmin.Value
cht.Chart.Axes(xlCategory).MaximumScale = Det1.Xmax.Value

MUx = Int((Det1.Xmax.Value - Det1.Xmin.Value) / 10)
MUy = Int((Det1.Ymax.Value - Det1.Ymin.Value) / 10)

If Det1.space_x.Value <> "" Then
MUx = Det1.space_x.Value
Else
Det1.space_x.Value = MUx
End If
If Det1.space_y.Value <> "" Then
MUy = Det1.space_y.Value
Else
Det1.space_y.Value = MUy
End If

'Adjust y-axis Units
cht.Chart.Axes(xlValue).MajorUnit = MUy
cht.Chart.Axes(xlValue).MinorUnit = MUy

'Adjust x-axis Units
cht.Chart.Axes(xlCategory).MajorUnit = MUx
cht.Chart.Axes(xlCategory).MinorUnit = MUx

```

J_Geometry - 13

```
FactorX = 680
FactorY = 400
```

```
If (Det1.Ymax.Value - Det1.Ymin.Value) / (Det1.Xmax.Value - Det1.Xmin.Value) < 0.6 Then
cht.Height = (Det1.Ymax.Value - Det1.Ymin.Value) / (Det1.Xmax.Value - Det1.Xmin.Value) * FactorX
cht.Width = FactorX
Else
cht.Height = FactorY
cht.Width = (Det1.Xmax.Value - Det1.Xmin.Value) / (Det1.Ymax.Value - Det1.Ymin.Value) * FactorY
End If
```

```
.....
.....
```

```
Xleft = cht.Chart.PlotArea.InsideLeft
Xwidth = cht.Chart.PlotArea.InsideWidth
Ytopp = cht.Chart.PlotArea.InsideTop
Yheight = cht.Chart.PlotArea.InsideHeight
'Xleft = cht.Chart.PlotArea.Left
'Xwidth = cht.Chart.PlotArea.Width
'Ytopp = cht.Chart.PlotArea.Top
'Yheight = cht.Chart.PlotArea.Height
```

```
Xmin = cht.Chart.Axes(1).MinimumScale
Xmax = cht.Chart.Axes(1).MaximumScale
Ymin = cht.Chart.Axes(2).MinimumScale
Ymax = cht.Chart.Axes(2).MaximumScale
```

```
.....
.....
```

```
Dim Pjj_x As Double
Dim Pjj_y As Double
```

```
For i = 1 To Det1.Number_Layers
```

```
Set srsNew = cht.Chart.SeriesCollection.NewSeries
With srsNew
.Name = "L" & i
.Values = ActiveSheet.Range(Cells(12 + i, 4), Cells(12 + i, 5))
.XValues = ActiveSheet.Range(Cells(12 + i, 2), Cells(12 + i, 3))
.Format.Line.Weight = 3
.Format.Line.ForeColor.RGB = RGB(0, 0, 0)
.MarkerStyle = xlMarkerStyleNone
.Format.Line.DashStyle = msolineSolid
End With
```

```
Cells(8, 4) = Det1.Controls("p" & Det1.Number_Points_Top.Value - 1 & "x").Value
Cells(9, 4) = Det1.Controls("p" & Det1.Number_Points_Top.Value - 1 & "y").Value
Cells(8, 5) = Det1.Controls("p" & Det1.Number_Points_Top.Value & "x").Value
Cells(9, 5) = Det1.Controls("p" & Det1.Number_Points_Top.Value & "y").Value
```

```
If i = 1 Then
Xnode1 = Xleft + (Cells(8, 4) - Xmin) * Xwidth / (Xmax - Xmin)
Ynode1 = Ytopp + (Ymax - Cells(9, 4)) * Yheight / (Ymax - Ymin)
```

J_Geometry - 14

```

Xnode2 = Xleft + (Cells(8, 5) - Xmin) * Xwidth / (Xmax - Xmin)
Ynode2 = Ytopp + (Ymax - Cells(9, 5)) * Yheight / (Ymax - Ymin)

Else
  If Cells(11 + i, 5) <= 0 Then
    Xnode1 = Xleft + (Cells(8, 2) - Xmin) * Xwidth / (Xmax - Xmin)
  Else
    Xnode1 = Xleft + (Cells(11 + i, 2) - Xmin) * Xwidth / (Xmax - Xmin)
  End If
  Ynode1 = Ytopp + (Ymax - Cells(11 + i, 4)) * Yheight / (Ymax - Ymin)
  Xnode2 = Xleft + (Cells(11 + i, 3) - Xmin) * Xwidth / (Xmax - Xmin)
  Ynode2 = Ytopp + (Ymax - Cells(11 + i, 5)) * Yheight / (Ymax - Ymin)

End If

Xnode3 = Xleft + (Cells(12 + i, 3) - Xmin) * Xwidth / (Xmax - Xmin)
Ynode3 = Ytopp + (Ymax - Cells(12 + i, 5)) * Yheight / (Ymax - Ymin)
Xnode4 = Xleft + (Cells(12 + i, 2) - Xmin) * Xwidth / (Xmax - Xmin)
Ynode4 = Ytopp + (Ymax - Cells(12 + i, 4)) * Yheight / (Ymax - Ymin)
Xnode5 = Xleft + (Cells(12 + i, 2) - Xmin) * Xwidth / (Xmax - Xmin)
Ynode5 = Ytopp + (Ymax - 0) * Yheight / (Ymax - Ymin)

Top = Det1.Controls("Yt" & i).Value
bottom = Det1.Controls("Yb" & i).Value
Top_p = Top
Bottom_p = bottom

Set myBuilder = cht.Chart.Shapes.BuildFreeform(msoEditingAuto, Xnode1, Ynode1)
With myBuilder
  .AddNodes msoSegmentLine, msoEditingAuto, Xnode2, Ynode2
  .AddNodes msoSegmentLine, msoEditingAuto, Xnode3, Ynode3
  .AddNodes msoSegmentLine, msoEditingAuto, Xnode4, Ynode4

For jj = 1 To NPY
  Pjj_x = Det1.Controls("p" & jj & "x").Value
  Pjj_y = Det1.Controls("p" & jj & "y").Value

  If Bottom_p < Pjj_y And Top_p > Pjj_y Then

    Xnode5 = Xleft + (Pjj_x - Xmin) * Xwidth / (Xmax - Xmin)

    Ynode5 = Ytopp + (Ymax - Pjj_y) * Yheight / (Ymax - Ymin)
    .AddNodes msoSegmentLine, msoEditingAuto, Xnode5, Ynode5

  End If
  If Bottom_p = Pjj_y Then
    Xnode5 = Xleft + (Pjj_x - Xmin) * Xwidth / (Xmax - Xmin)
    Ynode5 = Ytopp + (Ymax - Pjj_y) * Yheight / (Ymax - Ymin)
    .AddNodes msoSegmentLine, msoEditingAuto, Xnode5, Ynode5
  End If

Next

.AddNodes msoSegmentLine, msoEditingAuto, Xnode1, Ynode1

```

J_Geometry - 15

End With

Set myShape = myBuilder.ConvertToShape

Select Case i

```
Case 1
  Fill = 3
  Line = 3
Case 2
  Fill = 4
  Line = 4
Case 3
  Fill = 5
  Line = 5
Case 4
  Fill = 6
  Line = 6
Case 5
  Fill = 7
  Line = 7
Case 6
  Fill = 10
  Line = 10
Case 7
  Fill = 11
  Line = 11
Case 8
  Fill = 12
  Line = 12
Case 9
  Fill = 13
  Line = 13
Case 10
  Fill = 14
  Line = 14
```

End Select

With myShape

```
.Fill.ForeColor.SchemeColor = Fill
.Fill.Transparency = 0.6
.Line.ForeColor.SchemeColor = Line
.Line.Weight = 1.5
.Line.Visible = msoFalse
```

End With

Next

```
Det2.p1x.Value = Det1.p1x.Value
Det2.p1y.Value = Det1.p1y.Value
Det2.p2x.Value = Det1.p2x.Value
```

J_Geometry - 16

```

Det2.p2y.Value = Det1.p2y.Value
Det2.p3x.Value = Det1.p3x.Value
Det2.p3y.Value = Det1.p3y.Value
Det2.p4x.Value = Det1.p4x.Value
Det2.p4y.Value = Det1.p4y.Value
Det2.p5x.Value = Det1.p5x.Value
Det2.p5y.Value = Det1.p5y.Value
Det2.p6x.Value = Det1.p6x.Value
Det2.p6y.Value = Det1.p6y.Value
Det2.p7x.Value = Det1.p7x.Value
Det2.p7y.Value = Det1.p7y.Value
Det2.p8x.Value = Det1.p8x.Value
Det2.p8y.Value = Det1.p8y.Value
Det2.p9x.Value = Det1.p9x.Value
Det2.p9y.Value = Det1.p9y.Value
Det2.p10x.Value = Det1.p10x.Value
Det2.p10y.Value = Det1.p10y.Value

```

```

For i = 1 To cht.Chart.SeriesCollection.Count
    If Left(cht.Chart.SeriesCollection(i).Name, 5) = "Slice" Then
        cht.Chart.SeriesCollection(i).Format.Line.Visible = msoFalse
    End If
Next

```

```

If Dir(ThisWorkbook.Path & "\" & Start.Name_Project.Text, vbDirectory) <> vbNullString Then
Else
    MkDir ThisWorkbook.Path & "\" & Start.Name_Project.Text
End If

```

```

relativePath = ThisWorkbook.Path & "\" & Start.Name_Project.Text & "\" & Start.Name_Project.Text & "
_geo.jpg"

```

```

If Dir(relativePath) <> "" Then
    Kill relativePath
End If
With cht
    ActiveWindow.Zoom = 175
    .Chart.Export relativePath
    ActiveWindow.Zoom = 100
End With
Application.Wait (Now + TimeValue("0:00:3"))
Det1.Image1.Picture = LoadPicture(relativePath)

```

```

For Each myShape In cht.Chart.Shapes
    myShape.Fill.Transparency = 0.99
Next myShape

```

```

'relativePath = ThisWorkbook.Path & "\" & Start.Name_Project.Text & "\" & Start.Name_Project.Text &

```



```

J_Geometry - 17

"_geo_slices.jpg"
'cht.Chart.Export (relativePath)
'Det2.Image2.Picture = LoadPicture(relativePath)

Sheet3.Activate
'MsgBox ("Geometry Generated")

Progress.Label1.Caption = "Geometry generated!"
Progress.Repaint
Application.Wait (Now + TimeValue("0:00:3"))

ProcExit:
Application.ScreenUpdating = True
Progress.Hide
'Det1.Show
'ActiveWorkbook.Windows(1).Visible = True
'Application.Visible = True
Exit Sub

ProcError:
MsgBox Err.Description
Application.ScreenUpdating = True
Det1.Show
' ActiveWorkbook.Windows(1).Visible = True
Resume ProcExit

End_Run:
Application.ScreenUpdating = True

End Sub

```

```

L_Run - 1

Sub Run()

Application.ScreenUpdating = False

If Dir(ThisWorkbook.Path & "\" & Start.Name_Project.Text, vbDirectory) <> vbNullString Then
Else
    Mkdir ThisWorkbook.Path & "\" & Start.Name_Project.Text
End If

Progress.Label1.Caption = "Running"
Progress.Show vbModeless
Progress.Repaint
Progress.Label1.Caption = "Processing OMS analysis"
Progress.Repaint
Call Calc_OMS
Progress.Label1.Caption = "Processing BMS analysis"
Progress.Repaint
Call Calc_BMS
Progress.Label1.Caption = "Processing SPM analysis"
Progress.Repaint
Call Calc_SPM
Progress.Label1.Caption = "Analysis completed!"
Progress.Repaint
Application.Wait (Now + TimeValue("0:00:3"))

Progress.Hide
Application.ScreenUpdating = True
.....

Sheet3.Activate

Application.ScreenUpdating = True

End Sub

```

```

M_OMS - 1

Sub Calc_OMS()

Application.ScreenUpdating = False
Sheet14.Activate

'Reseting spreadsheet'.....
Cells(2, 28) = ""
Cells(2, 29) = ""
Cells(3, 28) = ""
Cells(3, 29) = ""
Cells(4, 28) = ""
Cells(4, 29) = ""
Cells(5, 28) = ""
Cells(5, 29) = ""
Cells(6, 28) = ""
Cells(6, 29) = ""
Cells(7, 28) = ""
Cells(7, 29) = ""
Cells(8, 28) = ""
Cells(8, 29) = ""
Cells(9, 28) = ""
Cells(9, 29) = ""
Cells(10, 28) = ""
Cells(10, 29) = ""
Cells(11, 28) = ""
Cells(11, 29) = ""
Cells(6, 12) = ""
Cells(6, 13) = ""

For i = 1 To 10
Cells(i + 7, 2) = ""
Cells(i + 7, 3) = ""
Cells(i + 7, 4) = ""
Cells(i + 7, 5) = ""
Cells(i + 7, 6) = ""
Cells(i + 7, 7) = ""
Cells(i + 7, 8) = ""
Next

For i = 1 To 6
Cells(9, 9 + i) = ""
Cells(10, 9 + i) = ""
Next

Cells(12, 10) = ""
Cells(13, 10) = ""
Cells(15, 10) = ""
Cells(16, 10) = ""
Cells(17, 10) = ""

Cells(2, 2) = Sheet19.Cells(2, 2)
Cells(2, 3) = Sheet19.Cells(2, 3)
Cells(2, 4) = Sheet19.Cells(2, 4)
Cells(2, 8) = Det2.N_Slices.Value

Range("A1").Select
.....
'Setting spreadsheet
Range("BI18").Formula = "=SUM(BI21:BI121)"

```

M_OMS - 2

```
Range("BP18").Formula = "=SUM(BP21:BP121)"
Range("BQ18").Formula = "=SUM(BQ21:BQ121)"
Range("D18").Formula = "=MIN(C20:C121)"
Range("BJ18").Formula = "=SUM(BJ21:BJ121)"
Range("BU18").Formula = "=SUM(BU21:BU121)"
```

```
Cells(3, 84) = Start.Name_Project
Cells(4, 84) = Start.Author
Cells(5, 84) = Start.Date_Proj
```

```
For i = 1 To 121
Cells(20 + i, 1) = ""
Cells(20 + i, 2) = ""
Cells(20 + i, 3) = ""
Cells(20 + i, 4) = ""
Cells(20 + i, 5) = ""
Cells(20 + i, 6) = ""
Cells(20 + i, 7) = ""
Cells(20 + i, 8) = ""
Next
For i = 1 To Det2.N_Slices
Cells(20 + i, 1) = i
Range("B" & 20 + i).Formula = Sheet19.Range("B" & 20 + i).Formula
Range("C" & 20 + i).Formula = Sheet19.Range("C" & 20 + i).Formula
Range("D" & 20 + i).Formula = Sheet19.Range("D" & 20 + i).Formula
Range("E" & 20 + i).Formula = Sheet19.Range("E" & 20 + i).Formula
Range("F" & 20 + i).Formula = Sheet19.Range("F" & 20 + i).Formula
Range("G" & 20 + i).Formula = Sheet19.Range("G" & 20 + i).Formula
Range("H" & 20 + i).Formula = Sheet19.Range("H" & 20 + i).Formula
Next
```

```
For i = 1 To Det1.Number_Layers.Value
Cells(7 + i, 2) = Det1.Controls("Yt" & i).Value
Cells(7 + i, 3) = Det1.Controls("Yb" & i).Value
Cells(7 + i, 4) = Det1.Controls("w" & i).Value
Cells(7 + i, 5) = Det1.Controls("c" & i).Value
Cells(7 + i, 6) = Det1.Controls("f" & i).Value
If Det1.Option_ppr.Value = True Then
Cells(7 + i, 7) = Det1.Controls("r" & i).Value
Else
Cells(7 + i, 7) = ""
End If
If Det2.OptionChoose.Value = True Then
Cells(7 + i, 8) = Det2.Controls("a" & i).Value
Else
If Det2.OptionTotal.Value = True Then
Cells(7 + i, 8) = "T"
Else
Cells(7 + i, 8) = "E"
End If
End If
Next
```

```
If Det1.Option_Metric.Value = True Then
```

M_OMS - 3

```
Cells(6, 12) = "x"
Cells(6, 13) = ""
Else
Cells(6, 12) = ""
Cells(6, 13) = "x"
End If
```

```
Cells(4, 5) = 2
Cells(9, 10) = ""
Cells(9, 11) = ""
Cells(9, 12) = ""
Cells(9, 13) = ""
Cells(9, 14) = ""
Cells(9, 15) = ""
Cells(10, 10) = ""
Cells(10, 11) = ""
Cells(10, 12) = ""
Cells(10, 13) = ""
Cells(10, 14) = ""
Cells(10, 15) = ""
```

```
If Det1.Option_piez.Value = True And Det1.Cons_ppw.Value = True Then
```

```
For i = 1 To Det1.Number_Points.Value
```

```
Cells(9, 9 + i) = Det1.Controls("PP" & i & "x").Value
Cells(10, 9 + i) = Det1.Controls("PP" & i & "y").Value
```

```
Next
```

```
End If
```

```
If Det1.Option_ppr.Value = True And Det1.Cons_ppw.Value = True Then
```

```
Cells(12, 10) = "x"
Else
Cells(12, 10) = ""
End If
```

```
If Det1.Option_Sur = True Then
```

```
Cells(15, 10) = Det1.Sur_q.Value
Cells(16, 10) = Det1.Sur_xi.Value
Cells(17, 10) = Det1.Sur_xf.Value
Else
Cells(15, 10) = ""
Cells(16, 10) = ""
Cells(17, 10) = ""
End If
```

```
Cells(1, 27) = Det1.Number_Points_Top.Value
```

```
Cells(2, 28) = Det1.p1x.Value
Cells(2, 29) = Det1.p1y.Value
Cells(3, 28) = Det1.p2x.Value
Cells(3, 29) = Det1.p2y.Value
Cells(4, 28) = Det1.p3x.Value
Cells(4, 29) = Det1.p3y.Value
Cells(5, 28) = Det1.p4x.Value
```

M_OMS - 4

```
Cells(5, 29) = Det1.p4y.Value
Cells(6, 28) = Det1.p5x.Value
Cells(6, 29) = Det1.p5y.Value
Cells(7, 28) = Det1.p6x.Value
Cells(7, 29) = Det1.p6y.Value
Cells(8, 28) = Det1.p7x.Value
Cells(8, 29) = Det1.p7y.Value
Cells(9, 28) = Det1.p8x.Value
Cells(9, 29) = Det1.p8y.Value
Cells(10, 28) = Det1.p9x.Value
Cells(10, 29) = Det1.p9y.Value
Cells(11, 28) = Det1.p10x.Value
Cells(11, 29) = Det1.p10y.Value
```

```
Cells(2, 20) = Det1.SS_t_xi.Value
Cells(2, 21) = Det1.SS_c_xf.Value
Cells(2, 22) = Det1.SS_Yc_b.Value
Cells(4, 22).Formula = "=T2+(U2-T2)/2"
```

```
.....
'.....' Calculate Critical slip surface '.....'
.....
If Det1.Option_Crit.Value = True Then
Cells(2, 2).Formula = "=Y15"
Cells(2, 3).Formula = "=Y16"
Cells(2, 4).Formula = "=Y8"

Application.Run "SolverReset"
Application.Run "SolverAdd", "$T$2", 3, Det1.SS_t_xi.Value
Application.Run "SolverAdd", "$T$2", 1, Det1.SS_t_xf.Value
Application.Run "SolverAdd", "$U$2", 3, Det1.SS_c_xi.Value
Application.Run "SolverAdd", "$U$2", 1, Det1.SS_c_xf.Value
Application.Run "SolverAdd", "$V$2", 1, Det1.SS_Yc_t.Value
Application.Run "SolverAdd", "$V$2", 3, Det1.SS_Yc_b.Value
Application.Run "SolverAdd", "$Y$16", 3, "$B$18"

Application.Run "SolverOk", "$J$2", 2, , "$T$2:$V$2"

Application.Run "SolverSolve", True

End If
.....
If Det1.Option_Crit.Value = False Then

Cells(2, 2) = Det1.Xc.Value
Cells(2, 3) = Det1.Yc.Value
Cells(2, 4) = Det1.r.Value

If Cells(2, 2) >= 0 Then
Cells(2, 20) = Cells(2, 2) - Sqr(Cells(2, 4) ^ 2 - Cells(2, 3) ^ 2)
Else
Cells(2, 20) = Cells(2, 2) + Sqr(Cells(2, 4) ^ 2 - Cells(2, 3) ^ 2)
End If
Cells(2, 21) = Cells(2, 2) + Sqr(Cells(2, 4) ^ 2 - (Cells(2, 3) - Cells(18, 2)) ^ 2)

Application.Run "SolverReset"
```

M_OMS - 5

```

Application.Run "SolverAdd", "$E$4", 3, "0.01"

If Det2.OptionTotal = True Then
Application.Run "SolverAdd", "$E$6", 2, "0"
Application.Run "SolverOk", "$K$2", 2, , "$E$4"
Else
Application.Run "SolverAdd", "$E$5", 2, "0"
Application.Run "SolverOk", "$J$2", 2, , "$E$4"
End If
Application.Run "SolverSolve", True

End If
.....
' Transferring points of slip surface to Image

For i = 1 To Det2.N_Slices + 1
Sheet6.Cells(25 + i, 2) = Sheet14.Cells(19 + i, 2)
Sheet6.Cells(25 + i, 3) = Sheet14.Cells(19 + i, 3)

Sheet6.Cells(25 + i, 27) = Sheet14.Cells(20 + i, 1)
Sheet6.Cells(25 + i, 28) = Sheet14.Cells(20 + i, 2)
Sheet6.Cells(25 + i, 29) = Sheet14.Cells(20 + i, 2)
Sheet6.Cells(25 + i, 30) = Sheet14.Cells(20 + i, 4)
Sheet6.Cells(25 + i, 31) = Sheet14.Cells(20 + i, 3)
Next

.....
'' WRITING RESULTS
Det2.Xc_OMS = Format(Sheet14.Cells(2, 2), "0.000")
Det2.Yc_OMS = Format(Sheet14.Cells(2, 3), "0.000")
Det2.r_OMS = Format(Sheet14.Cells(2, 4), "0.000")
Det2.Fs_OMS = Format(Sheet14.Cells(2, 10), "0.000")

.....
Sheet6.Activate
Set cht = ActiveSheet.ChartObjects(1)

For i = 1 To cht.Chart.SeriesCollection.Count
If Left(cht.Chart.SeriesCollection(i).Name, 5) = "Slip2" Then
cht.Chart.SeriesCollection(i).Format.Line.Visible = msoFalse
End If
Next

For i = 1 To cht.Chart.SeriesCollection.Count
If Left(cht.Chart.SeriesCollection(i).Name, 5) = "Slice" Then
cht.Chart.SeriesCollection(i).Format.Line.Visible = msoTrue
End If
Next
Next
For Each myShape In cht.Chart.Shapes
myShape.Fill.Transparency = 0.99
Next myShape

relativePath = ThisWorkbook.Path & "\" & Start.Name_Project.Text & "\" & Start.Name_Project.Text & "
_geo_slices_OMS.jpg"
If Dir(relativePath) <> "" Then
Kill relativePath

```

```

M_OMS - 6

End If
With cht
    ActiveWindow.Zoom = 175
    .Chart.Export relativePath
    ActiveWindow.Zoom = 100
End With

If Det2.OptionOMS.Value = True Then
Det2.Image2.Picture = LoadPicture(relativePath)
End If

For i = 1 To cht.Chart.SeriesCollection.Count
    If Left(cht.Chart.SeriesCollection(i).Name, 5) = "Slice" Then
        cht.Chart.SeriesCollection(i).Format.Line.Visible = msoFalse
    End If
Next

For Each myShape In cht.Chart.Shapes
    myShape.Fill.Transparency = 0.5
Next myShape

relativePath = ThisWorkbook.Path & "\" & Start.Name_Project.Text & "\" & Start.Name_Project.Text & "
_OMS.jpg"
If Dir(relativePath) <> "" Then
    Kill relativePath
End If
With cht
    ActiveWindow.Zoom = 175
    .Chart.Export relativePath
    ActiveWindow.Zoom = 100
End With

If Det2.OptionOMS.Value = True Then
Det2.Image1.Picture = LoadPicture(relativePath)
End If

Application.ScreenUpdating = True

End Sub

```



```

P_MonteCarlo - 1

Sub MonteCarlo_Run()

Application.ScreenUpdating = False

If Dir(ThisWorkbook.Path & "\" & Start.Name_Project.Text, vbDirectory) <> vbNullString Then
Else
    Mkdir ThisWorkbook.Path & "\" & Start.Name_Project.Text
End If

Sheet16.Activate

Range(Cells(22, 1), Cells(10022, 100)).Select
Selection.ClearContents

Sheet15.Activate

Range("A1:CC200").Select
Selection.ClearContents

Sheet14.Activate
Cells.Select
Selection.Copy
Sheets("Prob_Ref_OMS").Select
Cells.Select
ActiveSheet.Paste

Progress.Label1.Caption = "0"
Progress.Show vbModeless

For j = 1 To 10
    Sheet15.Cells(7 + j, 2) = ""
    Sheet15.Cells(7 + j, 3) = ""
    Sheet15.Cells(7 + j, 4) = ""
    Sheet15.Cells(7 + j, 5) = ""
    Sheet15.Cells(7 + j, 6) = ""
    Sheet16.Cells(7 + j, 9) = ""
    Sheet16.Cells(7 + j, 10) = ""
    Sheet16.Cells(7 + j, 11) = ""
    Sheet16.Cells(7 + j, 13) = ""
    Sheet16.Cells(7 + j, 14) = ""
    Sheet16.Cells(7 + j, 15) = ""
    Sheet16.Cells(7 + j, 16) = ""
    Sheet16.Cells(7 + j, 17) = ""
    Sheet16.Cells(7 + j, 18) = ""
    Sheet16.Cells(7 + j, 19) = ""
    Sheet16.Cells(7 + j, 20) = ""
    Sheet16.Cells(7 + j, 21) = ""
    Sheet16.Cells(7 + j, 37) = ""
    Sheet16.Cells(7 + j, 38) = ""
    Sheet16.Cells(7 + j, 39) = ""
    Sheet16.Cells(7 + j, 40) = ""
    Sheet16.Cells(7 + j, 41) = ""
    Sheet16.Cells(7 + j, 42) = ""
    Sheet16.Cells(7 + j, 43) = ""
    Sheet16.Cells(7 + j, 44) = ""
    Sheet16.Cells(7 + j, 45) = ""

```

P_MonteCarlo - 2

```

Sheet16.Cells(7 + j, 46) = ""
Sheet16.Cells(7 + j, 47) = ""
Sheet16.Cells(7 + j, 48) = ""
Next

```

For i = 1 To Prob1.N.Value

For j = 1 To Det1.Number_Layers.Value

```

If j = 1 Then
    Sheet15.Cells(7 + j, 2) = Prob1.Controls("Yt" & j).Value
Else
    Sheet15.Cells(7 + j, 2) = Sheet15.Cells(6 + j, 3)
End If

```

If Prob1.Check_Ybot.Value = True Then

```

RET_MC1:
    If Prob1.Yb1_DIST.Value = "Normal" Then
        Sheet15.Cells(7 + j, 3) = WorksheetFunction.Norm_Inv(Rnd(), Prob1.Controls("Yb" & j).Value, Prob1.Controls("Yb" & j & "_SD").Value)
        If Sheet15.Cells(7 + j, 3).Value > Val(Prob1.Controls("Yb" & j & "_MAX").Value) Or Sheet15.Cells(7 + j, 3).Value < Val(Prob1.Controls("Yb" & j & "_MIN").Value) Then
            GoTo RET_MC1
        End If
    Else
        M = Prob1.Controls("Yb" & j).Value
        S = Prob1.Controls("Yb" & j & "_SD").Value
        scaled_mean = Application.WorksheetFunction.Ln(M ^ 2 / Sqr(M ^ 2 + S ^ 2))
        scaled_sd = Sqr(Application.WorksheetFunction.Ln((M ^ 2 + S ^ 2) / M ^ 2))
        Sheet15.Cells(7 + j, 3) = WorksheetFunction.LogNorm_Inv(Rnd(), scaled_mean, scaled_sd)
    End If

```

```

    If Sheet15.Cells(7 + j, 3).Value > Val(Prob1.Controls("Yb" & j & "_MAX").Value) Or Sheet15.Cells(7 + j, 3).Value < Val(Prob1.Controls("Yb" & j & "_MIN").Value) Then
        GoTo RET_MC1
    End If
End If
Else
    Sheet15.Cells(7 + j, 3) = Prob1.Controls("Yb" & j).Value
End If
If Prob1.Check_w.Value = True Then
RET_MC2:
    If Prob1.w1_DIST.Value = "Normal" Then
        Sheet15.Cells(7 + j, 4) = WorksheetFunction.Norm_Inv(Rnd(), Prob1.Controls("w" & j).Value, Prob1.Controls("w" & j & "_SD").Value)
        If Sheet15.Cells(7 + j, 4).Value > Val(Prob1.Controls("w" & j & "_MAX").Value) Or Sheet15.Cells(7 + j, 4).Value < Val(Prob1.Controls("w" & j & "_MIN").Value) Then
            GoTo RET_MC2
        End If
    End If

```

P_MonteCarlo - 3

```

        End If
    Else
        M = Prob1.Controls("w" & j).Value
        S = Prob1.Controls("w" & j & "_SD").Value
        scaled_mean = Application.WorksheetFunction.Ln(M ^ 2 / Sqr(M ^ 2 + S ^ 2))
        scaled_sd = Sqr(Application.WorksheetFunction.Ln((M ^ 2 + S ^ 2) / M ^ 2))
        Sheet15.Cells(7 + j, 4) = WorksheetFunction.LogNorm_Inv(Rnd(), scaled_mean, scaled_s
d)

        If Sheet15.Cells(7 + j, 4).Value > Val(Prob1.Controls("w" & j & "_MAX").Value) Or Sh
eet15.Cells(7 + j, 4).Value < Val(Prob1.Controls("w" & j & "_MIN").Value) Then
            GoTo RET_MC2
        End If
    End If
Else
    Sheet15.Cells(7 + j, 4) = Prob1.Controls("w" & j).Value
End If
If Prob1.Check_c.Value = True Then
RET_MC3:
    If Prob1.c1_DIST.Value = "Normal" Then
        Sheet15.Cells(7 + j, 5) = WorksheetFunction.Norm_Inv(Rnd(), Prob1.Controls("c" & j).
Value, Prob1.Controls("c" & j & "_SD").Value)
        If Sheet15.Cells(7 + j, 5).Value > Val(Prob1.Controls("c" & j & "_MAX").Value) Or Sh
eet15.Cells(7 + j, 5).Value < Val(Prob1.Controls("c" & j & "_MIN").Value) Then
            GoTo RET_MC3
        End If
    Else
        M = Prob1.Controls("c" & j).Value
        S = Prob1.Controls("c" & j & "_SD").Value
        scaled_mean = Application.WorksheetFunction.Ln(M ^ 2 / Sqr(M ^ 2 + S ^ 2))
        scaled_sd = Sqr(Application.WorksheetFunction.Ln((M ^ 2 + S ^ 2) / M ^ 2))
        Sheet15.Cells(7 + j, 5) = WorksheetFunction.LogNorm_Inv(Rnd(), scaled_mean, scaled_s
d)

        If Sheet15.Cells(7 + j, 5).Value > Val(Prob1.Controls("c" & j & "_MAX").Value) Or Sh
eet15.Cells(7 + j, 5).Value < Val(Prob1.Controls("c" & j & "_MIN").Value) Then
            GoTo RET_MC3
        End If
    End If
Else
    Sheet15.Cells(7 + j, 5) = Prob1.Controls("c" & j).Value
End If
If Prob1.Check_f.Value = True Then
RET_MC4:
    If Prob1.f1_DIST.Value = "Normal" Then
        Sheet15.Cells(7 + j, 6) = WorksheetFunction.Norm_Inv(Rnd(), Prob1.Controls("f" & j).
Value, Prob1.Controls("f" & j & "_SD").Value)
        If Sheet15.Cells(7 + j, 6).Value > Val(Prob1.Controls("f" & j & "_MAX").Value) Or Sh
eet15.Cells(7 + j, 6).Value < Val(Prob1.Controls("f" & j & "_MIN").Value) Then
            GoTo RET_MC4
        End If
    Else
        M = Prob1.Controls("f" & j).Value
        S = Prob1.Controls("f" & j & "_SD").Value
        scaled_mean = Application.WorksheetFunction.Ln(M ^ 2 / Sqr(M ^ 2 + S ^ 2))
        scaled_sd = Sqr(Application.WorksheetFunction.Ln((M ^ 2 + S ^ 2) / M ^ 2))
        Sheet15.Cells(7 + j, 6) = WorksheetFunction.LogNorm_Inv(Rnd(), scaled_mean, scaled_s
d)

        If Sheet15.Cells(7 + j, 6).Value > Val(Prob1.Controls("f" & j & "_MAX").Value) Or Sh

```

P_MonteCarlo - 4

```

eet15.Cells(7 + j, 6).Value < Val(Prob1.Controls("f" & j & "_MIN").Value) Then
    GoTo RET_MC4
End If
End If
Else
    Sheet15.Cells(7 + j, 6) = Prob1.Controls("f" & j).Value
End If

```

```

Sheet16.Cells(7 + j, 9) = Prob1.Controls("Yt" & j).Value
Sheet16.Cells(7 + j, 10) = Prob1.Controls("Yb" & j).Value
Sheet16.Cells(7 + j, 11) = Prob1.Controls("Yb" & j & "_SD").Value
Sheet16.Cells(7 + j, 13) = Prob1.Controls("w" & j).Value
Sheet16.Cells(7 + j, 14) = Prob1.Controls("w" & j & "_SD").Value
Sheet16.Cells(7 + j, 15) = Prob1.Controls("w" & j & "_COV").Value
Sheet16.Cells(7 + j, 16) = Prob1.Controls("c" & j).Value
Sheet16.Cells(7 + j, 17) = Prob1.Controls("c" & j & "_SD").Value
Sheet16.Cells(7 + j, 18) = Prob1.Controls("c" & j & "_COV").Value
Sheet16.Cells(7 + j, 19) = Prob1.Controls("f" & j).Value
Sheet16.Cells(7 + j, 20) = Prob1.Controls("f" & j & "_SD").Value
Sheet16.Cells(7 + j, 21) = Prob1.Controls("f" & j & "_COV").Value

```

```

Sheet16.Cells(21 + i, 5 + 5 * j) = Sheet15.Cells(7 + j, 2)
Sheet16.Cells(21 + i, 6 + 5 * j) = Sheet15.Cells(7 + j, 3)
Sheet16.Cells(21 + i, 7 + 5 * j) = Sheet15.Cells(7 + j, 4)
Sheet16.Cells(21 + i, 8 + 5 * j) = Sheet15.Cells(7 + j, 5)
Sheet16.Cells(21 + i, 9 + 5 * j) = Sheet15.Cells(7 + j, 6)

```

Next

```

Sheet16.Cells(21 + i, 1) = i
Sheet16.Cells(21 + i, 2) = Sheet15.Cells(2, 14)
Sheet16.Cells(21 + i, 3) = Sheet15.Cells(1, 14)
Sheet16.Cells(21 + i, 4) = Sheet15.Cells(2, 10)

```

```

If Sheet16.Cells(21 + i, 4) > 1 Then
    Sheet16.Cells(21 + i, 5) = 0
Else
    Sheet16.Cells(21 + i, 5) = 1
End If

```

```

Sheet16.Cells(21 + i, 6) = Sheet16.Cells(21 + i, 2) - Sheet16.Cells(21 + i, 3)

```

```

If Sheet16.Cells(21 + i, 6) > 0 Then
    Sheet16.Cells(21 + i, 7) = 0
Else
    Sheet16.Cells(21 + i, 7) = 1
End If

```

```

Sheet16.Cells(21 + i, 8) = Sheet16.Cells(21 + i, 4) + Sheet16.Cells(20 + i, 8)

```

```

Sheet16.Cells(21 + i, 9) = Sheet16.Cells(21 + i, 8) / i

```

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```

If i = 1 Then
Progress.Label1.Caption = "0% completed"
Progress.Repaint
End If
If i = Prob1.N.Value * 0.05 Then
Progress.Label1.Caption = "5% completed"
Progress.Repaint
End If
If i = Prob1.N.Value * 0.1 Then
Progress.Label1.Caption = "10% completed"
Progress.Repaint
End If
If i = Prob1.N.Value * 0.15 Then
Progress.Label1.Caption = "15% completed"
Progress.Repaint
End If
If i = Prob1.N.Value * 0.2 Then
Progress.Label1.Caption = "20% completed"
Progress.Repaint
End If
If i = Prob1.N.Value * 0.25 Then
Progress.Label1.Caption = "25% completed"
Progress.Repaint
End If
If i = Prob1.N.Value * 0.3 Then
Progress.Label1.Caption = "30% completed"
Progress.Repaint
End If
If i = Prob1.N.Value * 0.35 Then
Progress.Label1.Caption = "35% completed"
Progress.Repaint
End If
If i = Prob1.N.Value * 0.4 Then
Progress.Label1.Caption = "40% completed"
Progress.Repaint
End If
If i = Prob1.N.Value * 0.45 Then
Progress.Label1.Caption = "45% completed"
Progress.Repaint
End If
If i = Prob1.N.Value * 0.5 Then
Progress.Label1.Caption = "50% completed"
Progress.Repaint
End If
If i = Prob1.N.Value * 0.55 Then
Progress.Label1.Caption = "55% completed"
Progress.Repaint
End If
If i = Prob1.N.Value * 0.6 Then
Progress.Label1.Caption = "60% completed"
Progress.Repaint
End If
If i = Prob1.N.Value * 0.65 Then
Progress.Label1.Caption = "65% completed"
Progress.Repaint
End If
If i = Prob1.N.Value * 0.7 Then
Progress.Label1.Caption = "70% completed"

```

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```

Progress.Repaint
End If
If i = Prob1.N.Value * 0.75 Then
Progress.Label1.Caption = "75% completed"
Progress.Repaint
End If
If i = Prob1.N.Value * 0.8 Then
Progress.Label1.Caption = "80% completed"
Progress.Repaint
End If
If i = Prob1.N.Value * 0.85 Then
Progress.Label1.Caption = "85% completed"
Progress.Repaint
End If
If i = Prob1.N.Value * 0.9 Then
Progress.Label1.Caption = "90% completed"
Progress.Repaint
End If
If i = Prob1.N.Value * 0.95 Then
Progress.Label1.Caption = "95% completed"
Progress.Repaint
End If
If i = Prob1.N.Value Then
Progress.Label1.Caption = "100% completed"
Progress.Repaint
End If

```

Next

Progress.Hide

Sheet16.Activate

```

Sheet16.Cells(7, 25) = Prob1.N.Value
Range("Y8").Formula = "=SUM(E22:E" & Prob1.N.Value + 21 & ")"
Sheet16.Cells(9, 25) = Sheet16.Cells(8, 25) / Sheet16.Cells(7, 25)
Range("Y11").Formula = "=SUM(D22:D" & Prob1.N.Value + 21 & ")/Y7"
Range("Y12").Formula = "=MAX(D22:D" & Prob1.N.Value + 21 & ")"
Range("Y13").Formula = "=MIN(D22:D" & Prob1.N.Value + 21 & ")"

Range("AD9").Formula = "=SUM(B22:B" & Prob1.N.Value + 21 & ")/AG9"
Range("AD10").Formula = "=MAX(B22:B" & Prob1.N.Value + 21 & ")"
Range("AD11").Formula = "=MIN(B22:B" & Prob1.N.Value + 21 & ")"
Range("AD12").Formula = "=STDEV.S(B22:B" & Prob1.N.Value + 21 & ")"

Range("AE9").Formula = "=SUM(C22:C" & Prob1.N.Value + 21 & ")/AG9"
Range("AE10").Formula = "=MAX(C22:C" & Prob1.N.Value + 21 & ")"
Range("AE11").Formula = "=MIN(C22:C" & Prob1.N.Value + 21 & ")"
Range("AE12").Formula = "=STDEV.S(C22:C" & Prob1.N.Value + 21 & ")"

Range("AG10").Formula = "=SUM(E22:E" & Prob1.N.Value + 21 & ")"
Range("AG13").Formula = "=SUM(D22:D" & Prob1.N.Value + 21 & ")/AG9"
Range("AG14").Formula = "=MAX(D22:D" & Prob1.N.Value + 21 & ")"
Range("AG15").Formula = "=MIN(D22:D" & Prob1.N.Value + 21 & ")"
Range("AG16").Formula = "=STDEV.S(D22:D" & Prob1.N.Value + 21 & ")"

Range("AI10").Formula = "=SUM(G22:G" & Prob1.N.Value + 21 & ")"
Range("AI13").Formula = "=SUM(F22:F" & Prob1.N.Value + 21 & ")/AI9"
Range("AI14").Formula = "=MAX(F22:F" & Prob1.N.Value + 21 & ")"
Range("AI15").Formula = "=MIN(F22:F" & Prob1.N.Value + 21 & ")"

```

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Range("AI16").Formula = "=STDEV.S(F22:F" & Prob1.N.Value + 21 & ")"

Prob1.FS.Value = Det2.Fs_OMS.Value

Prob1.FS_Mean.Value = Format(Sheet16.Cells(11, 25), "###0.000")

Prob1.FS_Max.Value = Format(Sheet16.Cells(12, 25), "###0.000")

Prob1.FS_Min.Value = Format(Sheet16.Cells(13, 25), "###0.000")

Prob1.p.Value = Format(Sheet16.Cells(9, 25) * 100, "##0.00")

Prob1.RI_n.Value = Format(Sheet16.Cells(4, 35), "##0.000")

Prob1.RI_l.Value = Format(Sheet16.Cells(5, 35), "##0.000")

Sheet3.Activate

Application.ScreenUpdating = True

End Sub